Dynamic Models for International Environmental Agreements*

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Abstract

We develop a model to analyze, in a dynamic framework, how countries join environmental agreements. We assume that countries participating in an international environmental agreement decide on their level of emissions by maximizing the aggregate welfare of all signatory countries, whereas non-signatories decide on their emissions by maximizing their own individual welfare. Signatory countries are assumed to be able to punish the non-signatories at a cost. We consider both the dynamics of the pollution stock and of the number of signatory countries, which evolves over time according to a replicator dynamics. Our model captures situations in which some countries are in, and others are out, of an international environmental agreement which is stable over time. It also captures situations where all countries participate in a stable agreement, or situations where no stable agreement is feasible.

Keywords: International environmental agreements, dynamic games, dynamic systems.

1 Introduction

Environmental problems often share the feature of being international, that is, the welfare of a country depends not only on its own policy, but also on those of other countries. Examples of these problems are acid rains, pollution emissions, exploitation of the fishing grounds or rain forests. In the case of pollution emissions, air pollutants mix in the atmosphere and additions to the concentrations of these gases depend on the global total emissions. Countries have realized that these kinds of problems have to be solved on a global basis and that international environmental agreements (IEAs) are the only solutions.

The participation of countries in an international agreement to improve the quality of the environment is a complex question, for two main reasons. First, countries are sovereign and their participation to international agreements is voluntary. There is no supra-national authority that forces countries to participate to an agreement, as well as there is no international environmental judicial system powerful enough to guarantee compliance to an IEA [15, 30, 16]. Second, each country may have an incentive to free-ride. In fact, while the costs for reducing emissions are carried out exclusively by the country that is taking action, the benefits of a reduction in emissions are shared by all countries, so that each country has the incentive to wait for the others to reduce their emissions. This problem is commonly known as "The Tragedy of the Commons" [18].

Because of these interactions, global environmental problems can be modeled in a game theory framework. The literature based on this approach has developed following two streams: the cooperative and the non-cooperative approaches. In this paper, we take the non-cooperative point of view; the main concept in that case is that players cannot make binding agreements, they act as rivals and in their own best interest, so that sucessful agreements must be *self-enforcing*.[1]

The conditions for an IEA to work are that all countries collectively have to be better off

[1] Cooperative game theory assumes that players realize that if they act as a group and coordinate their actions, they can obtain mutual benefits. A particular agreement is the result of questions about the circumstances under which the agreement can be established, what can be achieved as a group, and the ways in which the benefits of the cooperation are redistributed among the participants [12, 30].

(the pie has to be bigger) and each single country has to be sure that at least it won't lose by participating (it will receive a piece of the pie at least as big as the piece it would have eaten alone). In non-cooperative static games this has been translated by saying that an IEA has to be self-enforcing [1]. The conditions for a self-enforcing agreement are profitability, which ensures that the accession to the agreement is individually rational for a country, and stability, which ensures that the group of signatory countries is an equilibrium. Two conditions define this equilibrium: internal stability, which implies that no member has an incentive to quit the group, and external stability, which implies that no non-member has an incentive to join the group. Behind these concepts of internal and external stability, there is the idea that governments can re-optimize their choice, but in static models this is only hypothetical, because since the game will not be played again, there is no adjustment towards a stable solution - and no change of state. Internal and external stability may be linked to the concepts of renegotiation-proof or dynamically consistent agreements in the context of repeated and dynamic games respectively.

The main body of the literature on non-cooperative games and self-enforcing agreements uses a static framework to describe pollution emissions, that is, environmental damage is assumed to depend on the flow of emissions. In this static framework, several stages in which specific decisions have to be made can be considered; a typical example is a two stage game, where countries decide wether or not to become a member of an IEA in the first stage and decide on their emissions in the second stage.

Many static models using this stability concept have reached the general conclusion that successful cooperation among a large number of countries is rare to occur without additional actions, and the pessimistic result that the size of the membership of a stable IEA is inversely related to the relative importance of the environmental damage cost. In order to explain the greater participation observed in reality,[2] static models have incorporated the ideas of

[2]The Montreal Protocol on Substances that Deplete the Ozone Layer was negotiated and signed by 24 countries and by the European Economic Community in September 1987. At present, 191 nations have become party to the Montreal Protocol. It includes trade sanctions to achieve the stated goals and it also offers major incentives for non-signatory nations to sign the agreement.

the "stick" and the "carrot". Some suggestions to increase the propensity to cooperate are leadership behaviors [2, 11], transfers [8, 19], reputation effects [19, 20], issue linkages [5, 6, 3, 21, 9, 10, 24], including a minimum participation clause [7], and considering modest emission reduction targets [17].

Although any model abstracts from reality, static games applied to pollution emissions and IEAs may be criticized on two important aspects. One aspect is the stock externality: transboundary environmental damage is usually related to the accumulation (stock) of pollution, rather than to the emissions (flow). The second aspect is that countries can revise their decisions of being in or out of an agreement at different points in time, possibly more so if the environmental damage cost is changing over time.

Most dynamic models considering the evolution of stock pollutants study IEAs as differential games, and compare different solution concepts (see for instance [23, 25, 13, 14] and [27, 28]. Few papers considering the evolution of stock pollutants also allow for the possibility that the membership of an IEA may change over time, or as a function of the pollution stock level.

The paper which is the closest to our work is Rubio and Ulph [26].This paper studies the formation and stability of IEAs over time in a difference game with stock pollutants. In each period, countries can join or quit an agreement, the IEA membership varying as the stock of pollution changes. The authors assume that the emissions of a country is either 0 or 1, and that signatories are randomly selected in each period.[3] The result is the existence of a unique steady-state of pollution stock, between the cooperative and the non-cooperative ones, to which corresponds a steady-state for the size of a stable IEA membership. As in the static case, the size of the self-enforcing IEA is negatively related to the importance of the damage costs.

In this paper, we study the formation and stability of IEAs when stock externality is considered and membership of an IEA is allowed to change endogenously over time. We want to capture both the possibility of full cooperation and (more realistic) situations in which

[3]This means that the incentive for a signatory to abide by its agreement is not addressed in the model.

some countries are in, and others are out, of an IEA. We consider a discrete time setting; this is motivated by the fact that, even if pollution evolves continuously over time, decisions from countries about joining or quitting an IEA are rather taken at discrete moments. We assume that countries choose the level of their emissions, by optimizing their welfare function.

As in Hoel and Schneider [19],we also consider a non-environmental cost in the welfare function of a country that doesn't joint the agreement, which is assumed proportional to the number of countries signatories of the IEA, and to the level of the accumulated pollution. This cost can be interpreted as a social norm, or a trade sanction[4] (e.g. a carbon tax) which becomes stronger with increasing levels of pollution and environmental damage. Finally, since a general punishment (or, more specifically, a trade sanction) is not without adverse consequences, we also include a cost suffered by signatory countries for punishing the non-signatories (as in [29] and [4] in the fishery context), which we assume proportional to the punishment.

The stability concept we adopt relies on an evolutionary process that might lead to a stable IEA. This process is based on a replicator dynamics providing evolutionary pressure in favor of groups obtaining the highest payoffs. Countries, instead of computing their welfare if they were inside or outside the coalition, compare the welfare obtained by each group during a given time period. Following the spirit of evolutionary games, the group that has performed better is joined by a fraction of new agents. The adjustment speed at which countries switch to the superior strategy is related to the difference in welfare, and reflects the imperfect diffusion of information or the "psychological" cost of changing behavior. In essence, instead of having countries able to re-optimize their choice continuously, we consider bounded rational countries that adapt and imitate the best.[5] The evolutionary process ends when the two groups perform equally well, that is when the welfare of signatory countries is

[4]In the Montreal Protocol, the treaty negotiators justified the trade sanctions because depletion of the ozone layer is most effectively addressed on the global level. Furthermore, without sanctions, there would be economic incentives for non-signatories to increase production, damaging the competitiveness of the industries in the signatory nations as well as decreasing the search for less damaging CFC alternatives.

[5]This is not uncommon in economics. For example, when new strategies or technologies are introduced on the market, firms will tend to imitate the most successful ones, or the ones that yields a 'satisficing' level of profits.

equal to the welfare of non-signatory countries. It is worthwhile noticing that in that case, external and internal stability conditions are satisfied for the coalition of signatory countries, and that the evolution of players' welfare depends not only on the dynamics of emissions and pollution, but also on the evolution of the different groups' composition.

The paper is organized as follow. Section 2 presents the model and the general dynamics that govern the evolution of pollution and of the proportion of signatory countries. In Section 3, we propose a repeated game in which countries choose their emissions without considering the dynamic evolution of pollution. We then study how the dynamics of pollution and of the groups' composition are influenced by the players' myopic behavior and find the long-run level of pollution and prevailing behavior. In Section 4, we reformulate the same problem, this time considering countries that are able to optimize their welfare over an infinite horizon by taking into account the evolution of the pollution stock. Section 5 presents steady-state and sensitivity analysis and Section 6 concludes the paper.

2 Model and notation

2.1 Players and welfare

Let us consider N countries. Each country has a production activity that generates benefits but also pollution. A fraction s of them, identified as "signatory countries", decides to join an international environmental agreement, according to which their production activity is decided by maximizing the aggregate welfare of the coalition. Denote S the set of signatory countries. The remaining fraction $(1-s)$, identified as "non-signatory countries" or "defectors", acts individually, each of them deciding its production activity by maximizing its individual welfare. Denote D the set of defectors.

Apart from their behavior with respect to welfare maximization, we assume that all countries are symmetric.[6] Production of country j gives rise to emissions, denoted e_j. We suppose that the net revenues (i.e., gross revenues minus costs) derived from country j's production

[6] As usual in papers considering more than two players, see for instance [25, 19, 2, 3, 26, 17, 28, 11].

activity are increasing concave functions of its emissions, and assume they are given by the quadratic function

$$R_j(e_j) = e_j \left(b - \frac{1}{2} e_j \right). \tag{1}$$

Countries suffer an environmental damage arising from (global) pollution, which is assumed linear[7] and given by

$$D_j(P) = dP, \tag{2}$$

where $d > 0$ is the constant marginal damage and P is the stock of pollution.

We assume that each signatory country can in some way punish a defector for its irresponsible behavior (e.g. by reducing trade with that country, or by imposing a carbon-tax on its exports), and that this punishment is proportional to the level of pollution, so that the non-environmental cost incurred by a defector as punishment is given by

$$\alpha N s P. \tag{3}$$

In the following we assume that $\alpha < d$, that is, we suppose that the punishment incurred by a defector country from each signatory is smaller than the damage its suffers from pollution.

However, punishing has itself a cost, proportional to the punishment αP imposed to the $N(1-s)$ non-signatory countries, so that each signatory incurs a non-environmental cost given by

$$\gamma N (1-s) P, \tag{4}$$

where we assume $\gamma < \alpha$. As a consequence, the welfare of a signatory country $j \in S$ is given by

$$W_j^S(e_j, P, s) = e_j \left(b - \frac{e_j}{2} \right) - dP - \gamma N (1-s) P, \tag{5}$$

[7] This simplification is not uncommon (see for example [19, 17] and is supported by some empirical estimations (see [22].

whereas the welfare of a defector $j \in D$ is given by:

$$W_i^D(e_j, P, s) = e_j\left(b - \frac{e_j}{2}\right) - dP - \alpha N s P. \tag{6}$$

Parameter values are assumed to be such that individual welfares remain non-negative along the emission and pollution stock paths over time. In the sequel, we will use the following convenient abbreviated notation

$$\begin{aligned} c_S &\equiv d + \gamma N(1-s) \\ c_D &\equiv d + \alpha N s \end{aligned}$$

to represent the marginal impact of pollution (environmental and non-environmental) on the welfare of signatory and defector countries respectively (which are however not constants, but linear functions of s). Notice that the difference in marginal costs $c_S - c_D$ is a linear function of s which is positive (higher costs for signatories) for $s < \frac{\gamma}{\alpha+\gamma}$ and negative (higher cost for non-signatories) for $s > \frac{\gamma}{\alpha+\gamma}$.

2.2 Dynamics

We assume that the evolution over time of the pollution level is given by the discrete time equation

$$P_t = P_{t-1}(1-\delta) + \sum_{i \in S} e_{it} + \sum_{k \in D} e_{kt} \tag{7}$$

where $\delta \in (0,1)$ is the natural decay, $\sum_{i \in S} e_{it}$ is the total emissions of signatory countries and $\sum_{k \in D} e_{kt}$ is the total emissions of defectors during time period t.

We also assume that the proportion of signatory countries evolves over time following a discrete time replicator dynamics

$$s_{t+1} = s_t \frac{W_t^S}{s_t W_t^S + (1-s_t) W_t^D} \tag{8}$$

where W_t^S and W_t^D are respectively the individual welfares of signatory countries and defectors during period t. The denominator of (8) represents the weighted average welfare observed at time t, where the weights are given by the current proportions of the two types of countries. Whenever the current welfare for one type of behavior is higher than for the other, a fraction of countries will join the group that is performing best. This equation captures the notion that a strategy yielding above *(below)* average profits increases *(decreases)* in share in the population. This update mechanism ensures that the change in shares is a gradual process, so that there is some inertia for countries to change their behaviors, the "speed" of changes depending on the relative welfare inequalities (and can be interpreted as the result of adjustment costs and delays). Notice that this dynamics is only defined when $1 \leq Ns_t \leq N-1$ (which does not however exclude full cooperation or full defection, see Section 5.3).

3 A repeated game

In this section, we assume that players are myopic, in the sense that when they solve their welfare maximization problem, they do not take into account the dynamic evolution of pollution, and they repeatedly play static games. The static welfare maximization problem for a signatory country $j \in S$ is then written

$$\begin{aligned} \max_{e_j} W^S &= \sum_{i \in S} \left(e_i \left(b - \frac{e_i}{2} \right) - c_S P \right) \\ \text{s.t.} & \\ P &= P_{t-1}(1-\delta) + \sum_{i \in S} e_i + \sum_{k \in D} e_k \end{aligned}$$

where P_{t-1} is the stock of pollution during the last period and W^S is the aggregate welfare of the signatory countries. From the first order (sufficient) conditions, the optimal emissions of a signatory country are given by

$$e^S = b - Nsc_S, \tag{9}$$

assuming $b > Nsc_S$. Note that these emissions endogenize the damage of the entire coalition and are convex in s.

Similarly, the welfare maximization problem for a defector $j \in D$ is written

$$\begin{aligned} \max_{e_j} W_j^D &= e_j \left(b - \frac{e_j}{2}\right) - c_D P \\ \text{s.t.} & \\ P &= P_{t-1}(1-\delta) + \sum_{i \in S} e_i + \sum_{k \in D} e_k \end{aligned}$$

so that the optimal emissions of a defector country are given by

$$e^D = b - c_D, \tag{10}$$

assuming $b > c_D$. A non-signatory country considers only its own damage and its optimal emissions are linear decreasing in s.[8]

Using the optimal emissions (9) and (10), the pollution evolves according to the discrete-time linear system

$$P_t = P_{t-1}(1-\delta) + Ns_t(b - Ns_t c_S) + N(1-s_t)(b - c_D)$$

and the corresponding equilibrium welfares are

$$\begin{aligned} W_t^S &= \frac{\left(b^2 - N^2 c_S^2 s_t^2\right)}{2} - c_S P_t \\ W_t^D &= \frac{\left(b^2 - c_D^2\right)}{2} - c_D P_t. \end{aligned}$$

If players are allowed to change their behavior according to the replicator dynamics (8), then

[8] Emissions of a defector are larger than those of a signatory for $Ns \in [1, N-1]$ iif $\frac{\gamma}{\gamma+\alpha} > \frac{1}{N}$.

the dynamic model is described by the two-dimensional system

$$\begin{cases} P_t = P_{t-1}\left(1-\delta\right) + N\left(b - Nc_S s_t^2 - c_D\left(1-s_t\right)\right) \\ s_{t+1} = s_t \frac{(b+Ns_t c_S)(b-Ns_t c_S)-2c_S P_t}{b^2-N^2 c_S^2 s_t^3 - c_D^2(1-s_t)-2cP_t} \end{cases} \tag{11}$$

where $c \equiv \left(c_D\left(1-s_t\right) + c_S s_t\right)$. Notice that the consequence of assuming constant marginal environmental damage is that optimal emissions of countries are independent of each other (orthogonal free-riding), but they still influence each other through the evolution of s.

The locus of points where $P_{t+1} = P_t$ is given by

$$P^*(s) = \frac{N}{\delta}\left(b - \left(c_D\left(1-s\right) + c_S N s^2\right)\right) \tag{12}$$

and indicates, for every value of s, the long run level of pollution.

For the dynamics of signatory countries, the steady state s^* satisfies $s_{t+1} = s_t$ which occurs if and only if $W_t^S = W_t^D$, or equivalently when

$$\bar{P}(s) = \frac{{c_D}^2 - N^2 s^2 {c_S}^2}{2\left(c_S - c_D\right)} \tag{13}$$

which is the locus of point where the welfare of signatory and non-signatory counties are equal.

4 A dynamic game

In this section, we assume that countries optimize their welfare by taking into account the evolution of the stock of pollution. The total discounted welfare of players is maximized over an infinite horizon, where β is the one-period discount factor, assumed common to all players.

The welfare maximization problem for a signatory country $j \in S$ is thus given by

$$\max_{e_j} W^S = \sum_{i \in S} \sum_{t=0}^{\infty} \beta^t \left(e_{it} \left(b - \frac{e_{it}}{2} \right) - P_t c_S \right) \tag{14}$$

s.t.

$$P_t = P_{t-1} (1 - \delta) + \sum_{i \in S} e_{it} + \sum_{k \in D} e_{kt}, \quad P_0 \text{ given}, \tag{15}$$

where e_{it} is the emissions of country i during period t and e_j denotes the sequence of emissions $\{e_{jt}\}_{t=0,\ldots\infty}$. In the same way, the welfare maximization problem for a defector country $j \in D$ is written

$$\max_{e_j} W^D = \sum_{t=0}^{\infty} \beta^t \left(e_{jt} \left(b - \frac{e_{jt}}{2} \right) - P_t c_D \right),$$

subject to (15).

In order to characterize the optimal reaction strategies of players, and the dynamic equilibrium, we use a dynamic programming formulation, where the state variable is the level of pollution of the preceding time period, denoted by P. We are seeking a Nash equilibrium in stationary strategies. Define the constant κ representing the combined effect of the discount factor and the natural pollution decay by

$$\kappa \equiv \frac{1}{1 - \beta(1 - \delta)} > 1.$$

We first obtain the optimal reaction of the set of signatory countries to a given stationary strategy vector for the defectors, denoting $E^D(P)$ the resulting total emissions of the non-signatory countries as a function of P. For each signatory country the value function $V^S(P; E^D)$ represents the optimal total welfare of a signatory country, given $E^D(P)$, and satisfies

$$\begin{aligned} V^S(P; E^D) = \max_e \Big\{ & e \left(b - \frac{e}{2} \right) - \left(P(1 - \delta) + Nse + E^D(P) \right) c_S \\ & + \beta V^S \left(P(1 - \delta) + Nse + E^D(P) \right) \Big\}. \end{aligned} \tag{16}$$

Proposition 1 *The value function of a signatory country is linear in P. The optimal reaction of signatory countries is independent of the level of pollution and of the strategy of the defectors and it is given by*

$$e^{SP} = b - Ns\kappa c_S, \tag{17}$$

assuming $b > Ns\kappa c_S$.

Proof. Assume that $V^S(P; E^D) = k^S - m^S P$. First order sufficient conditions then yield

$$e^{SP} = b - Ns\left(m^S\beta + c_S\right),$$

which achieves the maximum in (16) and which does not depend on P or $E^D(P)$. Replacing e^{SP} in (16) yields an expression linear in P, verifying our assumption. It is now straightforward to obtain by identification

$$m^S = c_S\kappa(1-\delta) \tag{18}$$

$$k^S = Ns\kappa c_S\frac{Ns\kappa c_S - 2b}{2(1-\beta)} + \frac{b^2}{2(1-\beta)} - \frac{\kappa E^D(P)c_S}{(1-\beta)}. \tag{19}$$

The optimal emissions of a signatory country are therefore:

$$e^{SP} = b - Ns\kappa c_S.$$

■

As for the repeated game, we find that the optimal reaction of a signatory country is convex in s. It is then immediate to find that the total emissions of signatory countries are given by $E^S = Ns(b - Ns\kappa c_S)$.

In the same way, we now express the optimal reaction of a defector to a given stationary strategy vector of the other countries, denoting by $E^d(P)$ the total emissions of all other non-signatory countries as a function of P, and by E^S the total emissions of the signatory countries. The value function of a defector country, denoted by $V^D(P; E^S, E^d)$, represents

the optimal total welfare of a defector, given $E^d(P)$ and E^S, and satisfies

$$\begin{aligned} V^D(P;E^S,E^d) &= \left\{\max_e e\left(b-\frac{e}{2}\right)-\left(P\left(1-\delta\right)+E^S+e+E^d\left(P\right)\right)c_D \right. \\ &\quad \left. +\beta V^D\left(P\left(1-\delta\right)+E^S+e+E^d\left(P\right)\right)\right\}. \end{aligned} \tag{20}$$

Proposition 2 *The value function for a defector country is linear in P.The optimal reaction of defector countries is independent of the level of pollution and of the strategy of the other players and is given by*

$$e^{DP}=b-\kappa c_D, \tag{21}$$

assuming $b>\kappa c_D$.

Proof. Assume that $V^D(P;E^S,E^d)=k^D-m^DP$. First order sufficient conditions yield

$$e^{DP}=b-c_D-m^D\beta.$$

Substituting in (20) yields a linear function of P and

$$m^D = c_D\kappa\left(1-\delta\right) \tag{22}$$

$$k^D = \frac{b^2-\kappa c_D\left(2b-\kappa c_D+2\left(E^S+E^d\left(P\right)\right)\right)}{2\left(1-\beta\right)}, \tag{23}$$

so that the optimal emissions for a defector country are

$$e^{DP}=b-\kappa c_D.$$

■

Notice that optimal emissions of a defector are linear decreasing in s, [9] independent of P and of the strategies of the other players. It is then immediate to compute the total emissions of other defector countries as $E^d=\left(N-Ns-1\right)\left(b-\kappa c_D\right)$.

[9] The condition for the emissions of a defector to be larger than those of a signatory for $Ns\in[1,N-1]$ is again $\frac{\gamma}{\gamma+\alpha}>\frac{1}{N}$.

Combining these results, the equilibrium strategies of both kinds of players are given by $\left(e^{SP}, e^{DP}\right)$. Using (17) and (21), the dynamics of the pollution stock is then described by

$$\begin{aligned} P_t &= P_{t-1}(1-\delta) + E^S + E^D \\ &= Ns\left(b - Ns\kappa c_S\right) + N(1-s)\left(b - \kappa c_D\right) \end{aligned} \tag{24}$$

and the current equilibrium welfares are given by

$$\begin{aligned} W_t^{SP} &= \frac{b^2 - N^2\kappa^2 c_S^2 s_t^2}{2} - c_S P_t \\ W_t^{DP} &= \frac{b^2 - \kappa^2 c_D^2}{2} - c_D P_t. \end{aligned}$$

In this case, the two-dimensional system (7) and (8) becomes

$$\begin{cases} P_t = P_{t-1}(1-\delta) + N\left(b - Ns_t^2\kappa c_S - (1-s_t)\kappa c_D\right) \\ s_{t+1} = s_t \frac{b^2 - N^2\kappa^2 c_S^2 s_t^2 - 2c_S P_t}{b^2 - N^2\kappa^2 c_S^2 s_t^3 - \kappa^2 c_D^2 (1-s_t) - 2cP_t}. \end{cases} \tag{25}$$

The set of points in the plane (P, s) that give one-period stationary pollution $P_{t+1} = P_t$ is then

$$P^*(s) = \frac{N}{\delta}\left(b - \kappa\left(c_D(1-s) + c_S N s^2\right)\right), \tag{26}$$

and the set of point where $s_{t+1} = s_t$, or equivalently $W_t^S = W_t^D$, is given by

$$\bar{P}(s) = \kappa^2 \frac{c_D^2 - N^2 s^2 c_S^2}{2\left(c_S - c_D\right)}. \tag{27}$$

5 Steady-state and sensitivity analysis

We now study what happens, in the long run, both to the dynamics of the pollution and of the countries' shares under the two types of behavior. In particular, we are interested in finding if full cooperation, coexistence of cooperators and defectors, or no cooperation at all are possible outcomes of these games, and under what conditions.

First notice by comparing (12)–(13) with (26)–(27) that the solution of the repeated game can be retrieved from the solution of the dynamic game by setting $\kappa = 1$, which would correspond to a limiting case where either pollution decay is equal to 1 (no stock accumulation) or the players' discount factor is equal to 0 (no value for the future). The general shape of functions $P^*(s)$ and $\bar{P}(s)$ does not depend on the parameter κ, so that the steady-state and sensitivity analysis for both cases are qualitatively similar.

When they exist, equilibrium steady-state values of the stock of pollution and the proportion of signatory countries will be indexed by $\upsilon \in \{n, l, h, m, c\}$ and denoted $\xi_\upsilon = (P_\upsilon, s_\upsilon)$, where

$$P^*(s_\upsilon) = \bar{P}(s_\upsilon) = P_\upsilon.$$

The boundary equilibria ξ_n corresponding to $s_n = 0$ and ξ_c corresponding to $s_c = 1$ always exist, that is, they are always computable, and given by

$$\begin{aligned} \xi_n &= \left(\frac{N(b-\kappa d)}{\delta}, 0\right) \\ \xi_c &= \left(\frac{N(b-\kappa dN)}{\delta}, 1\right), \end{aligned}$$

where $P_n > P_c$. However, the dynamics of s is not defined on these boundaries; indeed, if at any time a boundary is reached, then only one group of players remains, punishment is no longer relevant, and a comparison of current welfare between the two groups is no longer possible. We first study the existence of inner steady-states, that is, such that $Ns_\upsilon \in [1, N-1]$, and analyze boundary equilibria separately in Section 5.3.

5.1 Partial cooperation

Using (27), it is easy to show that the pollution stock corresponding to a proportion of signatories in $\left[\frac{1}{N}, \frac{N-1}{N}\right]$ is positive when $c_S < c_D < Nsc_S$. This means that at an inner steady-state, defectors are emitting more than signatories, and their marginal cost is higher, so that if s_υ is an inner steady-state, then $s_\upsilon > \frac{\gamma}{\gamma+\alpha}$. On the other hand, the function (26)

giving the steady-state pollution is a cubic function of s. From the shape of functions P^* and $\bar{P}$, there might be 0, 1 or 2 coexisting inner steady-states with partial cooperation.

Numerically solving for these, we find that changes in the values of the parameters determine the appearance of the two inner steady-states through a saddle-node bifurcation. In that case, the steady-state with the lower percentage of signatory countries, denoted ξ_l, is the saddle point and the one with the higher percentage of signatory countries, denoted ξ_h, is the stable node. The stable set of the saddle point separates the basin of attraction of the inner steady state ξ_h (or of ξ_c when $Ns_h > N-1$) from the one of ξ_n. With respect to the level of pollution, the following inequality holds: $P_c < P_h < P_l < P_n$.

Starting from a situation where no inner equilibrium exist, partial cooperation can be obtained in two ways: by using the "stick" (increasing the punishment) or the "carrot" (reducing the cost for punishing).

For instance, Figure 1 illustrates what happens to the long run values for the pollution stock and the share of signatory countries when the punishment α is increased. In Figure 1a (with $\alpha = 0.00033$) the dynamics generated by the 2-dimensional system (25), starting from any level of pollution and signatory percentage, converge to a situation where all countries defect. In Figure 1b (with $\alpha = 0.00036$) the saddle-node bifurcation has occurred, and if the initial fraction of signatory countries is large enough, then the result will be a situation with some countries in the agreement and others outside the agreement. An important observation is that the minimum size of the initial fraction of signatories which leads to an inner steady-state is decreasing with the initial pollution stock (see Figure 2 for a zoom-in). Increasing α decreases both $P^*(s)$ and and $\bar{P}(s)$ and results in a higher level of cooperation and a lower level of pollution at the steady-state, and a wider basin of initial states generating trajectories converging to ξ_h. In Figure 1c (with $\alpha = 0.00042$), a transcritical bifurcation has occurred, that is, ξ_h has merged with ξ_c (full cooperation) which has become stable. Further increases in the punishment do not have any effect on the long run values of the dynamic variables, but they make the full cooperation more robust (i.e. supported by a greater number of initial states).

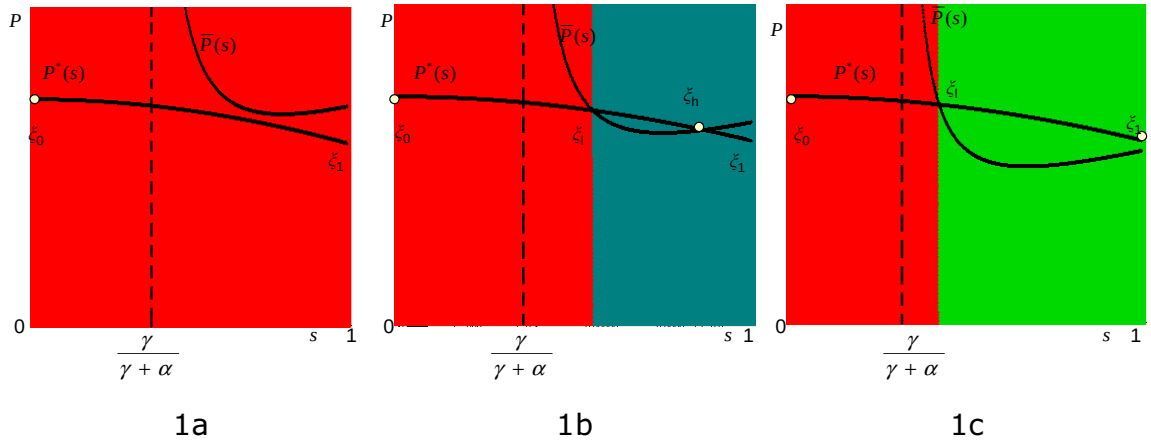

Figure 1: Impact of increasing punishment. The parameter values are: $N = 100$, $b = 200$, $d = 0.4$, $\gamma = 0.0002$, $\delta = 0.8$. In 1a, $\alpha = 0.00033$; in 1b, $\alpha = 0.00036$; In 1c, $\alpha = 0.00042$. The red area represents the set of initial conditions generating trajectories converging to a situation with all defectors. The blue area represents the set of initial states generating trajectories converging to the inner stable steady state. The green area represents the set of initial states generating trajectories converging to the full cooperation.

Figure 2: Zoom in on the boundary between the basin of attraction of ξ_n and ξ_h.

Figure 3 illustrates what happens when the cost for punishing γ is decreased. Starting from the same situation of complete defection as in Figure 1a, a decrease in γ determines the appearance of the two inner steady-states (Figure 3a). The blue area represents the set of initial states that, in the long run, will bring the trajectories of the system to converge to the stable inner equilibrium ξ_h. Further decreases in γ have small positive effects on the long run values of the dynamic variables and strong positive effects on the basin of E_h. An explanation

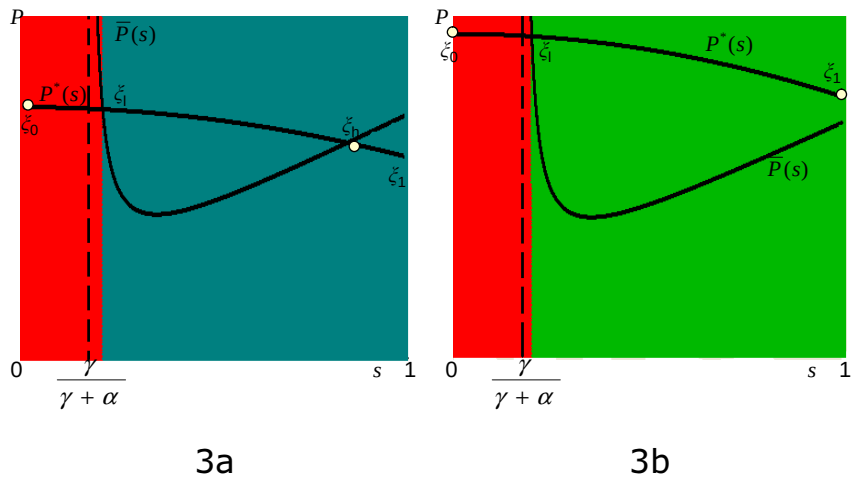

Figure 3: Impact of decreasing the cost for punishing γ. In Figure 3a the parameters values are: $N = 100$, $b = 200$, $d = 0.4$, $\alpha = 0.00033$, $\delta = 0.8$, $\gamma = 0.0007$; in 3b, $\delta = 0.6$. The red area represents the set of initial conditions generating trajectories converging to a situation with all defectors. The blue area represents the set of initial states generating trajectories converging to partial cooperation. The green area is the set of initial states generating trajectories converging to full cooperation.

for this is given in the next section, where we study the limiting case where $\gamma = 0$. Figure 3b illustrates a case where reducing γ generates full cooperation. Proposition 3 below shows that this happens when the value of δ is relatively small, all other parameters being equal.

Differentiating functions P^* and $\bar{P}$ with respect to the other parameters yield additionnal insight on the impact of their values on the appearance of inner steady-state equilibria. It is straightforward to verify that an increase in b shifts curve P^* upwards without affecting $\bar{P}$. Thus, increasing the profitability of emissions has a posive impact on the number of signatories at the steady-state as well as on the basin of initial states generating trajectories converging to partial or full cooperation, however at the expense of a higher steady-state pollution stock.

More interestingly, it can be verified that the impact of an increase of the environmental damage cost is a decrease in P^* and an increase in $\bar{P}$. As a result, an increase in the damage cost impacts negatively the size of the steady-state coalition and the size of the basin of initial states leading to it. This result is consistent with what has been observed in static games, as well as in the dynamic game of Rubio and Ulph [26]: when the potential gain from cooperation is large, then membership of an IEA is likely to be small. This can be

explained by the incitation to free-ride, which implies that a stable IEA is the smallest one where emitting less is welfare-enhancing.

Finally, an increase in the discount factor also result in a decrease in P^* and an increase in $\bar{P}$, thus also having a negative impact on the size of the steady-state coalition and of the basin leading to it. When players are myopic, an increase in the decay rate results in a decrease in P^* (decrease in the steady-state coalition, long-term pollution and basin). However an increase in the decay rate when players are not myopic can have opposite results, according to the value of the parameters, because of its impact on the value of κ.

5.2 No Cost for Punishing

We now consider the case where $\gamma = 0$ (as in many papers, see for instance [19]). Notice that in that case the marginal cost of pollution is always higher for a non-signatory country than for a signatory country, and that defectors emit more than signatories for $s > \frac{d}{N(d-\alpha)}$.

The locus of steady-state pollution and steady-state signatory proportion are then given by

$$\begin{aligned} P^*(s) &= \frac{N}{\delta}\left(b - \kappa\left(d - s\left(d - N\alpha\right) + Ns^2\left(d - \alpha\right)\right)\right) \\ \bar{P}(s) &= \kappa^2 \frac{\left(-d - Ns\alpha + Nds\right)\left(d + Ns\alpha + Nds\right)}{2Ns\alpha}. \end{aligned}$$

Notice that $\bar{P}(s)$ has an asymptote at $s = 0$ and is positive for $s > \frac{d}{N(d-\alpha)} > \frac{1}{N}$. We first show that in that case there is at most one inner equilibrium (coexistence of defectors and signatories), denoted ξ_m, with $s_m > \frac{d}{N(d-\alpha)}$.

Proposition 3 *If $\gamma = 0$, a necessary and sufficient condition for the existence of a unique inner steady-state with partial cooperation is*

$$\frac{\left(\alpha\left(N-1\right) + Nd\right)\left(d\left(N-2\right) - \alpha\left(N-1\right)\right)}{\alpha\left(N-1\right)\left(Nb - \kappa\left(\alpha\left(N-1\right) + d\left(N^2 - 2N + 2\right)\right)\right)} > \frac{2}{\delta\kappa^2}.$$

Proof. The proof is straightforward and based on the shape of the two functions. The

function $P^*(s)$ is a concave parabola, with $P^*(\frac{N-1}{N}) = \frac{\left(Nb-\kappa\left(\alpha(N-1)+d\left(N^2-2N+2\right)\right)\right)}{\delta}$. The function $\bar{P}(s)$ is increasing and concave for $s > 0$, with $\lim_{s\to 0^+} \bar{P}(s) = -\infty$ and $\bar{P}(\frac{N-1}{N}) = \kappa^2 \frac{(\alpha(N-1)+Nd)(d(N-2)-\alpha(N-1))}{2\alpha(N-1)}$. Under our assumptions on parameter values, equilibrium emissions are non-negative, so that P^* is also non-negative. As a consequence, whenever $P^*(\frac{N-1}{N}) < \bar{P}(\frac{N-1}{N})$, an inner steady-state with partial cooperation exists in $[\frac{d}{N(d-\alpha)}, \frac{N-1}{N}]$. ■

If punishment has no cost, the solution is qualitatively different: in that case, the two-dimensional dynamic system never admits as a solution the case of complete defection, and we observe either an inner steady-state ξ_m, or a situation with full cooperation ξ_c. These steady-states are locally and globally stable. In general, full-cooperation occurs for a sufficiently high level of punishment. Again, sensitivity analysis shows that increasing the marginal damage cost or the discount factor have negative effects on the size of the stable IEA.

Comparing with the previous case, if all other parameters are equal but punishment has no cost, the most important effect is that all initial conditions lead to stable coalitions, with more signatory countries, but also higher long-term pollution.

5.3 Stability and boundary equilibria

Consider an inner equilibrium steady-state ξ_υ, with $Ns_\upsilon \in [1, N-1]$; at ξ_υ, the stock of pollution and the number of signatories do not change over time. Total discounted welfares over an infinite horizon for signatories and defectors are given by

$$W_\upsilon^S = W_\upsilon^D = \frac{1}{2\left(1-\beta\right)}\left(b^2 + \kappa^2 \frac{c_S c_D \left(c_D - N^2 s^2 c_S\right)}{\left(c_D - c_S\right)}\right)$$

where it is obvious that myopic players enjoy a lower welfare at equilibrium (obtained with $\kappa = 1$), and that the long-term welfare in both groups are equal, even if only immediate welfare is considered in the dynamics of s_t. At such a point, internal and external stability of the coalition of signatories is satisfied: there is no incentive for a signatory to defect, and no incentive for a non-signatory to join the coalition, whether the criterion be immediate or long-term welfare, since welfare of all players are equal in all periods. As a consequence, if

an inner steady-state ξ_v exists and if the initial conditions are in the basin of attraction of ξ_v (i.e. if the set of initial signatories is large enough), then the players will converge in the long run to a stable IEA involving a fraction of the players.

Now consider boundary steady-states ξ_n, corresponding to $s = 0$, and ξ_c, corresponding to $s = 1$. It is straightforward to verify that the solution of the boundary games between N defectors or N signatories can be obtained by setting s either to 0 or to 1 in (17), (21), (26), (27), with $\kappa = 1$ when players are myopic. Total discounted welfares over an infinite horizon for signatories and defectors in both cases are given by:

$$W_c = \frac{\left(b^2 - N^2\kappa^2 d^2\right)}{2\left(1-\beta\right)} - \frac{d\left(N\left(b - N\kappa d\right)\right)}{\delta\left(1-\beta\right)}$$

$$W_n = \frac{\left(b^2 - \kappa^2 d^2\right)}{2\left(1-\beta\right)} - \frac{dN\left(b - \kappa d\right)}{\delta\left(1-\beta\right)}.$$

Since the replicator dynamics are only defined on $\left[\frac{1}{N}, \frac{N-1}{N}\right]$, we need different stability conditions for these boundary equilibria. The idea is that if at $s = \frac{N-1}{N}$, a single defector would have a welfare that is inferior to that of the signatories, there is no incentive for a player to defect from the full coalition, knowing that if he does so, the $N-1$ others will apply punishment α. The full coalition is then a stable IEA. In the same way, if at $s = \frac{1}{N}$, a single "signatory" country would have a welfare that is inferior to that of the non-signatories, then there is no incentive for a player to start punishing defectors. As a consequence, we find that when the initial conditions are in the set of initial states converging to ξ_c, then the players will converge in the long run to a stable IEA involving all players. If on the other hand the initial conditions are in the set of initial states converging to ξ_n, then the players will converge in the long run to complete defection.[10]

[10] Notice that if $\frac{\gamma}{\gamma+\alpha} > \frac{1}{N}$, it requires at least two players to initiate a stable IEA - which is more reasonable - because initial coalitions in the basin of ξ_c are always larger than $\frac{\gamma}{\gamma+\alpha}$. Recall that this condition is equivalent to stating that emissions of defectors are always higher than those of signatories.

6 Conclusions

In this paper we considered the problem of stability of international environmental agreements about pollution emissions when stock externalities are considered and when players can decide to abide by the agreement or to defect at any time. We assumed that signatory countries agree to punish non-signatory countries at a (proportional) cost, and that this punishment can be costly. The problem has been modeled by adopting a dynamic approach and the evolution to a self-enforcing agreement was modeled by an evolutionary mechanism based on the imitation of the best. This evolutionnary process leads to a steady-state equilibrium where the concept of internal and external stability of the IEA is satisfied.

We have developed a model in which countries are myopic, and a model where countries optimize their welfare over an infinite horizon, taking into account the evolution of the stock of pollution. In the dynamic case, emissions and long-term pollution stock are lower, and individual welfares are higher, than when players repeatedly play static strategies, however the dynamics of the corresponding system are qualitatively similar.

In particular, when punishment has a cost, it is always possible to observe a solution where no country joins an environmental agreement, along with full or partial cooperation. The possibility to end up in one solution or the other depends on the initial conditions: if the initial coalition is not large enough given an initial level of pollution, then the equilibrium solution is full defection. The initial number of signatory countries necessary to lead to a stable IEA is decreasing with the initial level of pollution, indicating that it is easier to endogenously bring other countries to join in an IEA when the stock of pollution is high. On the other hand, as in the static models, the number of signatories to a stable IEA is negatively related to the environmental cost, or equivalently to the benefit of cooperation.

Sensitivity analysis shows that partial or full cooperation in a stable IEA can be obtained either by using the "stick", that is increasing the punishment, or the "carrot", that is reducing the cost for punishing.

Finally, to complete the analysis, we have considered the case in which the punishment has

no cost. In that case, complete defection can never be observed. There is is only one possible outcome, independent of the initial conditions, which is either partial or full cooperation, according to the value of the parameters. Again, the number of signatories to a stable IEA is negatively related to the environmental cost.

Our assumption of linear damage cost makes the problem tractable, since the strategies of the players become independent of the state and of the actions of others. A linear damage cost may also be interpreted as a marginal approximation of the damage function by the players. Further development is to consider non-linear damage costs, which will require a numerical approach.

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