Dollevoet, Spliet

Crew scheduling problem

MILP formulations

Duty formulation Partial duty formulation Numerical comparison

Valid inequalities Main idea

Relaxation

Outlook

A partial duty formulation for a crew scheduling problem

Twan Dollevoet Remy Spliet

Erasmus University Rotterdam, the Netherlands

May 17, 2023

Dollevoet, Spliet

Crew scheduling problem

MILP formulations

Duty formulation Partial duty formulation Numerical comparison

Valid inequalitie Main idea

Outlook

But first...

Dollevoet, Spliet

Crew scheduling problem

MILP formulations

Duty formulation Partial duty formulation Numerical comparison

Valid inequalitie Main idea

Relaxation

Outlool

・ロト ・ 同ト ・ ヨト ・ ヨー・ つへぐ

Theorem

Computing the LP bound of the set partitioning formulation of the CVRP is strongly NP-hard.

A p-step Formulation for the Capacitated Vehicle Routing Problem, Pedro Munari, Twan Dollevoet, Remy Spliet, *working paper*. Theorem

The pricing problem of the set partitioning formulation of vehicle routing problems is strongly NP-hard.



Partial duty formulation

Dollevoet, Spliet

Crew scheduling problem

MILP formulations

Duty formulation Partial duty formulation Numerical comparison

Valid inequalitie Main idea

Outlook

The complexity of the Pricing Problem of the Set Partitioning Formulation of Vehicle Routing Problems, Remy Spliet, *Operations Research, 2023*.

Dollevoet, Spliet

Crew scheduling problem

MILP formulations

Duty formulation Partial duty formulation Numerical comparison

/alid inequalities

Main idea

Relaxation

Outlook

Crew scheduling problem

Crew scheduling problem - Tasks

Tasks V

Start time

- End time
- Work duration

Partial duty formulation

Dollevoet, Spliet

Crew scheduling problem

MILP formulations

Duty formulation Partial duty formulation Numerical comparison

/alid inequalities

Main idea

Outlook

▲□▶ ▲□▶ ▲目▶ ▲目▶ 目 のへぐ

Crew scheduling problem - Tasks

Tasks V

Start time

- End time
- Work duration

• Compatible tasks $A \subseteq V \times V$

Partial duty formulation

Dollevoet, Spliet

Crew scheduling problem

MILP formulations

Duty formulation Partial duty formulation Numerical comparison

/alid inequalities

Relaxation

Outlook

Crew scheduling problem - Tasks

Tasks V

- Start time
- End time
- Work duration
- Compatible tasks $A \subseteq V \times V$
- Acyclic graph G = (V, A)

Partial duty formulation

Dollevoet, Spliet

Crew scheduling problem

MILP formulations

Duty formulation Partial duty formulation Numerical comparison

/alid inequalities

Relayation

Outlook

◆□▶ ◆□▶ ◆□▶ ◆□▶ □ のQ@

Crew scheduling problem - Duty

Duty:

- Path in G
- Duty length ≤ maximum duty length
- Cumulative work duration ≤ maximum work duration
- Includes no or one break

Partial duty formulation

Dollevoet, Spliet

Crew scheduling problem

MLP formulations

Duty formulation Partial duty formulation Numerical comparison

/alid inequalitie

Main idea

Kelaxation

Outlook

Crew scheduling problem - Duty

Duty:

- Path in G
- Duty length ≤ maximum duty length
- Cumulative work duration ≤ maximum work duration
- Includes no or one break

Break:

- Mandatory if duty length is at least L⁰
- Starts at latest L⁻ since duty start
- Ends at latest L⁺ before duty end
- Occurs between two tasks in $A_B \subseteq A$

Partial duty formulation

Dollevoet, Spliet

Crew scheduling problem

MLP formulations

Duty formulation Partial duty formulation Numerical comparison

alid inequalities

Main idea

. . .

◆□▶ ◆□▶ ◆□▶ ◆□▶ ○□ のQ@

Crew scheduling problem - Duty

Duty:

- Path in G
- Duty length ≤ maximum duty length
- Cumulative work duration ≤ maximum work duration
- Includes no or one break

Break:

- Mandatory if duty length is at least L⁰
- Starts at latest L⁻ since duty start
- Ends at latest L⁺ before duty end
- Occurs between two tasks in $A_B \subseteq A$
- Duty cost:
 - Fixed cost
 - Variable cost for duty length

Partial duty formulation

Dollevoet, Spliet

Crew scheduling problem

MLP formulations

Duty formulation Partial duty formulation Numerical comparison

/alid inequalities

Main idea

Outlook

Crew scheduling problem

Partial duty formulation

Dollevoet, Spliet

Crew scheduling problem

MILP formulations

Duty formulation Partial duty formulation Numerical comparison

Valid inequalities

Relaxation

Outlook

・ロト ・ 同ト ・ ヨト ・ ヨー・ つへぐ

Find a collection of duties that covers all tasks at minimum costs.

Dollevoet, Spliet

Crew scheduling problem

MILP formulations

Duty formulation Partial duty formulation Numerical comparison

/alid inequalities

Main idea

Outlook

MILP formulations

Duty formulation

Partial duty formulation

Dollevoet, Spliet

Crew scheduling problem

MILP formulations

Duty formulation Partial duty formulation Numerical comparison

Valid inequalities

Main idea

Relaxation

Outlook

$$\min \sum_{d \in D} c_d x_d$$

 $\sum_{d \in D} a_{dv} x_d = 1 \quad \forall v \in V$
 $x_d \in \{0,1\} \; \forall d \in D$

▲□▶▲圖▶▲圖▶▲圖▶ ■ のへで

Duty formulation - Pricing problem

Shortest Path Problem with Resource Constraints

- Duty length
- Cumulative work duration

Partial duty formulation

Dollevoet, Spliet

Crew scheduling problem

MILP formulations

Duty formulation Partial duty formulation Numerical comparison

Valid inequalities

Iviain idea

Outlook

・ロト・西ト・田・・田・・日・ シック

Duty formulation - Pricing problem

Shortest Path Problem with Resource Constraints

- Duty length
- Cumulative work duration
- Pseudopolynomial labeling algorithm

Partial duty formulation

Dollevoet, Spliet

Crew scheduling problem

MILP formulations

Duty formulation Partial duty formulation Numerical comparison

Valid inequalitie

Delevention

Outlook

Partial duty formulation - Partial duty

Partial duty:

- Part of a duty $d \in D$ before the break
- Part of a duty $d \in D$ after the break
- Duty $d \in D$ with no break

Partial duty formulation

Dollevoet, Spliet

Crew scheduling problem

MILP formulations

Duty formulation

Partial duty formulation

Numerical comparison

Valid inequalities

Main idea

Relaxation

Outlook

▲□▶ ▲□▶ ▲目▶ ▲目▶ 目 のへぐ

$$\begin{split} \min \sum_{d \in D^{P}} c_{d} x_{d} \\ \sum_{d \in D^{P}} a_{dv} x_{d} &= 1 \qquad \forall v \in V \\ \sum_{d \in D^{P}} e_{dv} x_{d} &= 0 \qquad \forall v \in V \\ \sum_{d \in D^{P}} l_{dv} x_{d} \geq 0 \qquad \forall v \in V \\ \sum_{d \in D^{P}} w_{dv} x_{d} \geq 0 \qquad \forall v \in V \\ x_{d} \in \{0,1\} \ \forall d \in D^{P} \end{split}$$

Partial duty formulation

Dollevoet, Spliet

Crew scheduling problem

MILP formulations

Duty formulatio

Partial duty formulation

Numerical comparison

/alid inequalities

Main idea

Relaxation

Outlook

(ロト (個) (E) (E) (E) (9)

Partial duty formulation - Pricing problem

Partial duty formulation

Dollevoet, Spliet

Crew scheduling problem

MILP formulations

Duty formulatio

Partial duty formulation

Numerical comparison

Valid inequalities

Main idea

Relaxation

Outlook

- Shortest Path Problem(s)
- Polynomially solvable

Numerical comparison

Partial duty formulation

Dollevoet, Spliet

Crew scheduling problem

MILP formulations

Duty formulation Partial duty formulation

Numerical comparison

Valid inequalities

Main idea

Relaxation

Outlook

| | Duty formulation | | Partial duty formulation | |
|-------|------------------|----------|--------------------------|----------|
| Tasks | LP bound | Time (s) | LP bound | Time (s) |
| 216 | 54549.3 | 58 | 53542.2 | 2 |
| 259 | 61300.4 | 389 | 59965.1 | 4 |
| 310 | 67352.1 | 5618 | 66079.6 | 5 |
| 372 | 78307.2 | 40878 | 76652.9 | 10 |

Dollevoet, Spliet

Crew scheduling problem

MILP formulations

Duty formulation Partial duty formulation Numerical comparison

Valid inequalities

Main idea

Relaxation

Outlook

Valid inequalities

Main idea - Why is the LP bound worse?

Suppose:

- Maximum duty length is 12 hours
- Maximum work duration is 8 hours

Partial duty formulation

Dollevoet, Spliet

Crew scheduling problem

MILP formulations

Duty formulation Partial duty formulation Numerical comparison

Valid inequalities

Main idea Relaxation

Outlook

・ロト・西ト・田・・田・・日・ シック

Main idea - Why is the LP bound worse?

Suppose:

- Maximum duty length is 12 hours
- Maximum work duration is 8 hours
- Select with value 0.5 the partial duties:
 - Before the break, duty length 7, work duration 3
 - Before the break, duty length 5, work duration 5
- Select with value 1.0 the partial duty:
 - After the break, duty length 6, work duration 4

Partial duty formulation

Dollevoet, Spliet

Crew scheduling problem

MILP formulations

Duty formulation Partial duty formulation Numerical comparison

Valid inequalities

Main idea Relaxation

Outlook

Main idea - Valid inequalities

$\sum_{d \in D_{\geq}^{-}(v,l,w)} x_{d} \leq \sum_{d \in D_{\leq}^{+}(v,L-l,W-w)} x_{d} \quad \forall v, l, w$ $\sum_{d \in D_{\geq}^{-}(v,l,w)} x_{d} \leq \sum_{d \in D_{\geq}^{+}(v,L-l,W-w)} x_{d} \quad \forall v, l, w$

Partial duty formulation

Dollevoet, Spliet

Crew scheduling problem

MILP formulations

Duty formulation Partial duty formulation Numerical comparison

Valid inequalities

Main idea

Outlook

Relaxation

Relax the duty length constraint.

Partial duty formulation

Dollevoet, Spliet

Crew scheduling problem

MILP formulations

Duty formulation Partial duty formulation Numerical comparison

Valid inequalitie

Main idea

Relaxation

Outlook

◆□ ▶ ◆昼 ▶ ◆ 臣 ▶ ◆ 臣 ● ● ● ●

Relaxation

Relax the duty length constraint.

Theorem

For the relaxed problem, the LP bound of the partial duty formulation with the valid inequalities is equal to the LP bound of the duty formulation. Partial duty formulation

Dollevoet, Spliet

Crew scheduling problem

MILP formulations

Duty formulation Partial duty formulation Numerical comparison

Valid inequalities

Main idea

Relaxation

Outlook

Relaxation

Relax the duty length constraint.

Theorem

For the relaxed problem, the LP bound of the partial duty formulation with the valid inequalities is equal to the LP bound of the duty formulation.

Separation can be done in polynomial time.

Partial duty formulation

Dollevoet, Spliet

Crew scheduling problem

MILP formulations

Duty formulation Partial duty formulation Numerical comparison

Valid inequalities

Main idea

Relaxation

Outlook

◆□▶ ◆□▶ ◆□▶ ◆□▶ □ のQ@

Relaxation - Preliminary experiment



Partial duty formulation

Dollevoet, Spliet

Crew scheduling problem

MILP formulations

Duty formulation Partial duty formulation Numerical comparison

Valid inequalities

Main idea

Relaxation

Outlook

▲□▶ ▲圖▶ ▲臣▶ ▲臣▶ 臣 のへで

Dollevoet, Spliet

Crew scheduling problem

MILP formulations

Duty formulation Partial duty formulation Numerical comparison

Valid inequalitie

D.L. ...

Outlook

Outlook

Outlook

Partial duty formulation

Dollevoet, Spliet

Crew scheduling problem

MILP formulations

Duty formulation Partial duty formulation Numerical comparison

Valid inequalitie

Relaxation

Outlook

▲□▶ ▲□▶ ▲ 三▶ ▲ 三▶ 三三 - のへで

- Investigate the LP bound of the (not relaxed) partial duty formulation with valid inequalities.
- Develop branch-price-and-cut algorithm.