CONTRAINT AGGREGATION and INTEGRAL SIMPLEX for SET PARTITIONING PROBLEMS

François Soumis, Alpha-Saliou Barry,
Issmail Elhalloui, Frédéric Quesnel
GERAD

GLOBAL VIEW

- Integral Simplex:

  reach an optimal integer solution with a sequence of simplex pivots

  - For the SPP and GSPP the integer solutions are extreme points of the LP polyedron
- For the SPP
  - In the graph of extreme points connected by hedges, a non optimal integer solution has a beter adjacent integer solution (Tobin 1969)
  - In the graph of basis connected by simplex pivots a path of integer solutions of non-increasing cost permits to reach an optimal solution (Balas 1972)
  - Difficulty: an extreme point has a huge number of basis connected by degenerated pivots
  - **ISUD** finds a path of basis to reach optimality (pass throw degenerency)
- For the GSPP
  - The graph of integer extreme points can be non-connex
  - **GISUD** adds hedges in the polyedron to find a path of basis to reach optimality

DEGENERENCY for SPP and GSPP

p = 1 m-p = 0

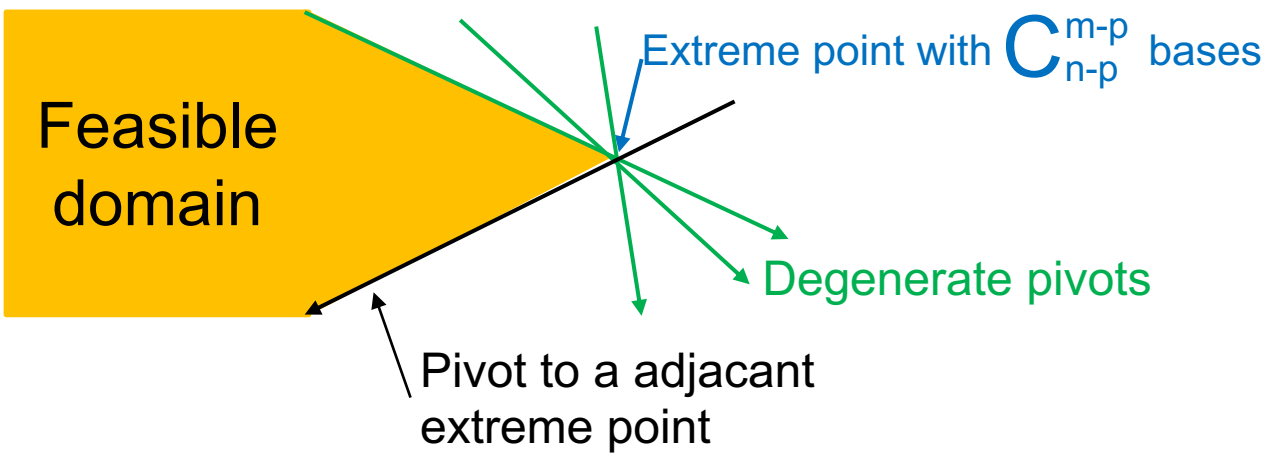
1 0 0 0	…	…
1 0 0 0	m basic	n-m non basic
1 0 0 0		
1 0 0 0		
0 1 0 0		
0 1 0 0	…	…
0 1 0 0		
0 0 1 0		
0 0 1 0		
0 0 1 0		
0 0 0 1		
0 0 0 1	…	…

$=$

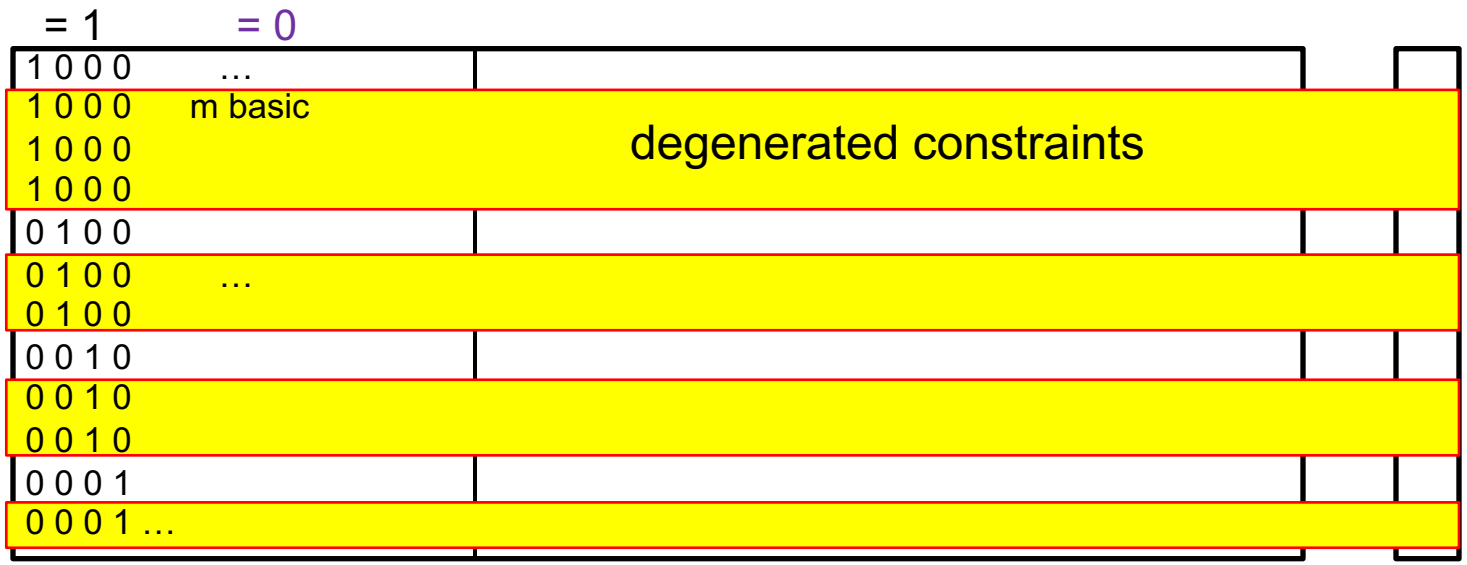
1

or

b_i



DEGENERENCY for SPP

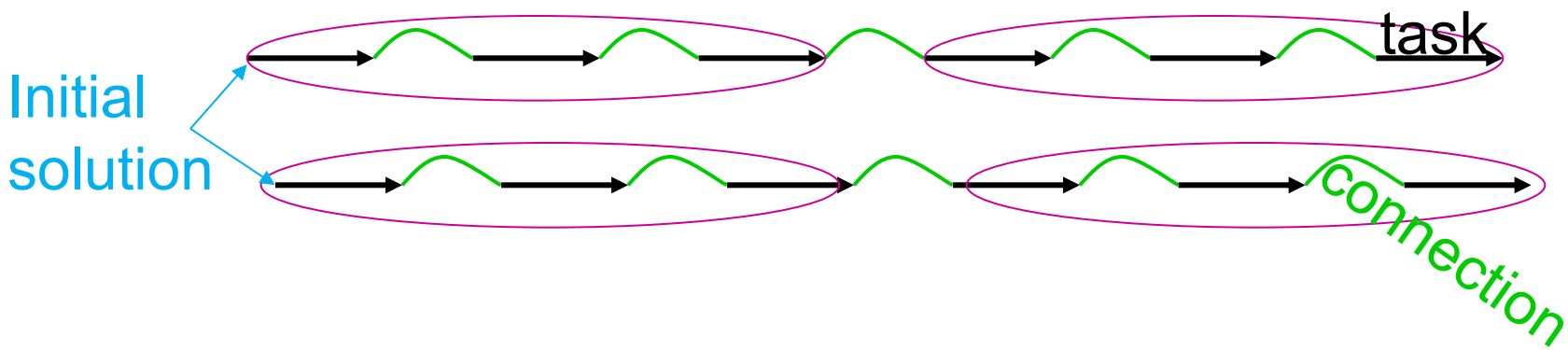


A column (path) cover some consecutives tasks

The task aggregation exploit it

TASK AGGREGATION FOR SPP

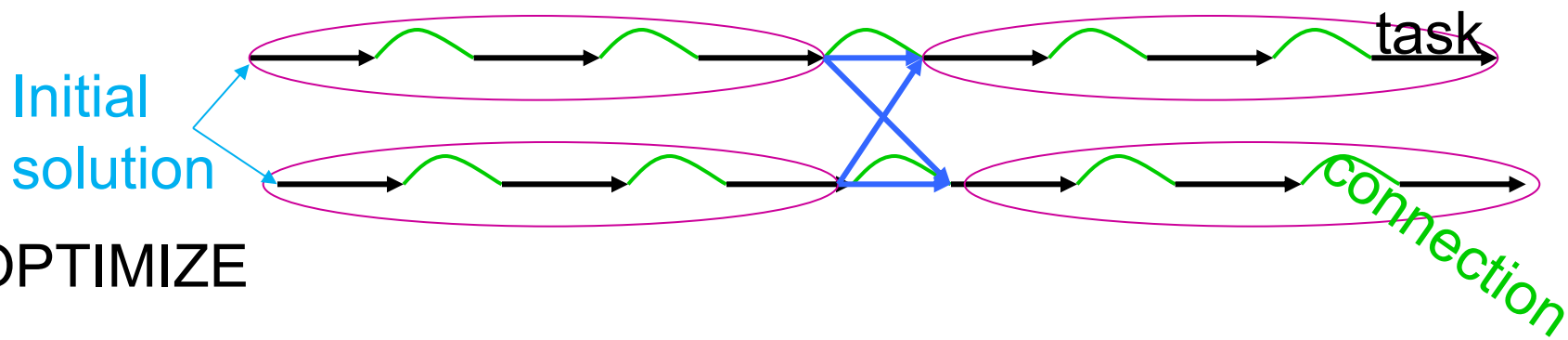
- AGGREGATE CONSECUTIVE TASKS IN CLUSTER



- CLUSTERS CAN COME FROM ANY INITIAL SOLUTION
  - Crew follow aircrafts or bus
  - Any heuristic, machine learning
  - Solution to reoptimize

TASK AGGREGATION FOR SPP

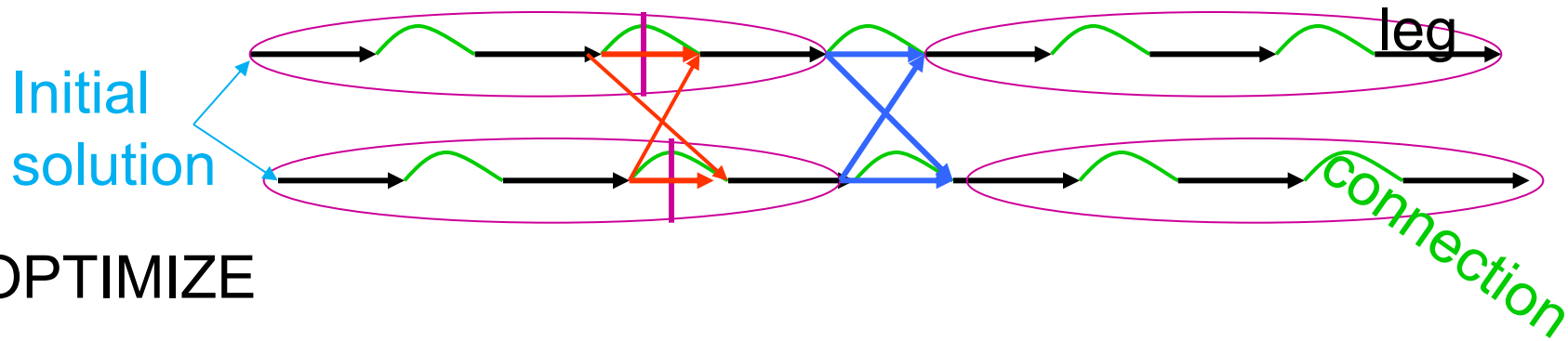
- AGGREGATE TASKS IN CLUSTERS



- OPTIMIZE
  - FAST OPT. ON CLUSTERS Blue var. only
    - Smaller problem (one constraint per cluster)

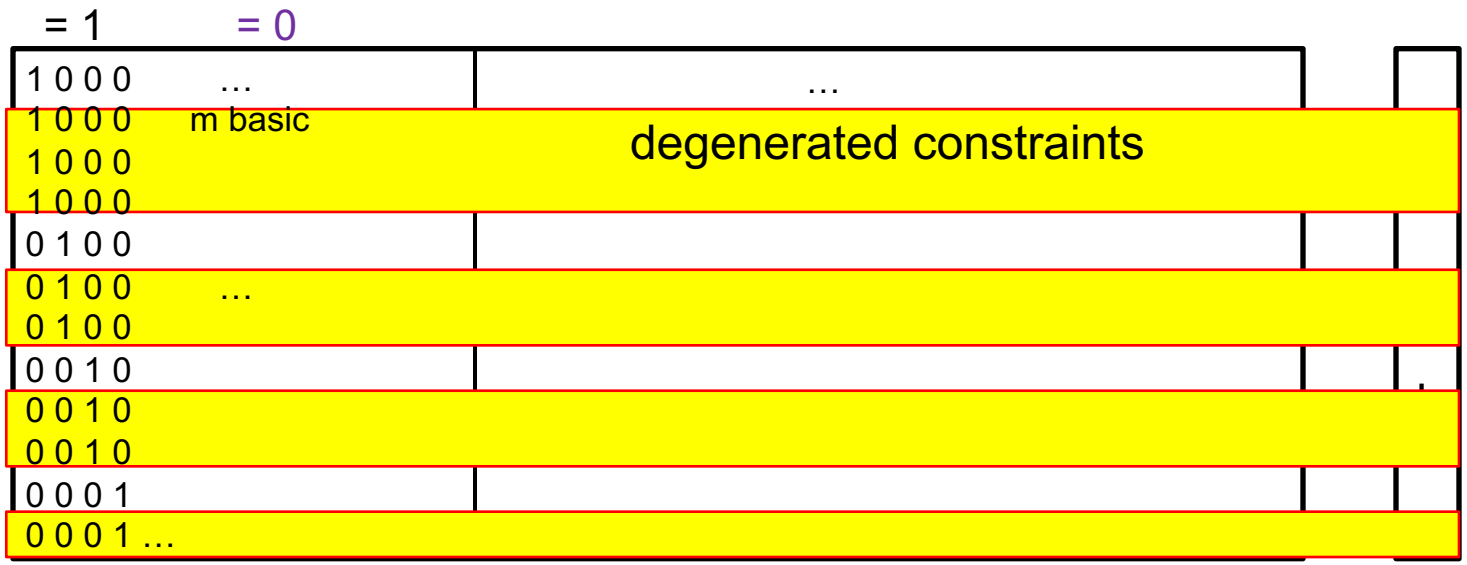
TASK AGREGATION
FOR SET COVERING PROBLEMS

- AGGREGATE LEGS IN CLUSTERS



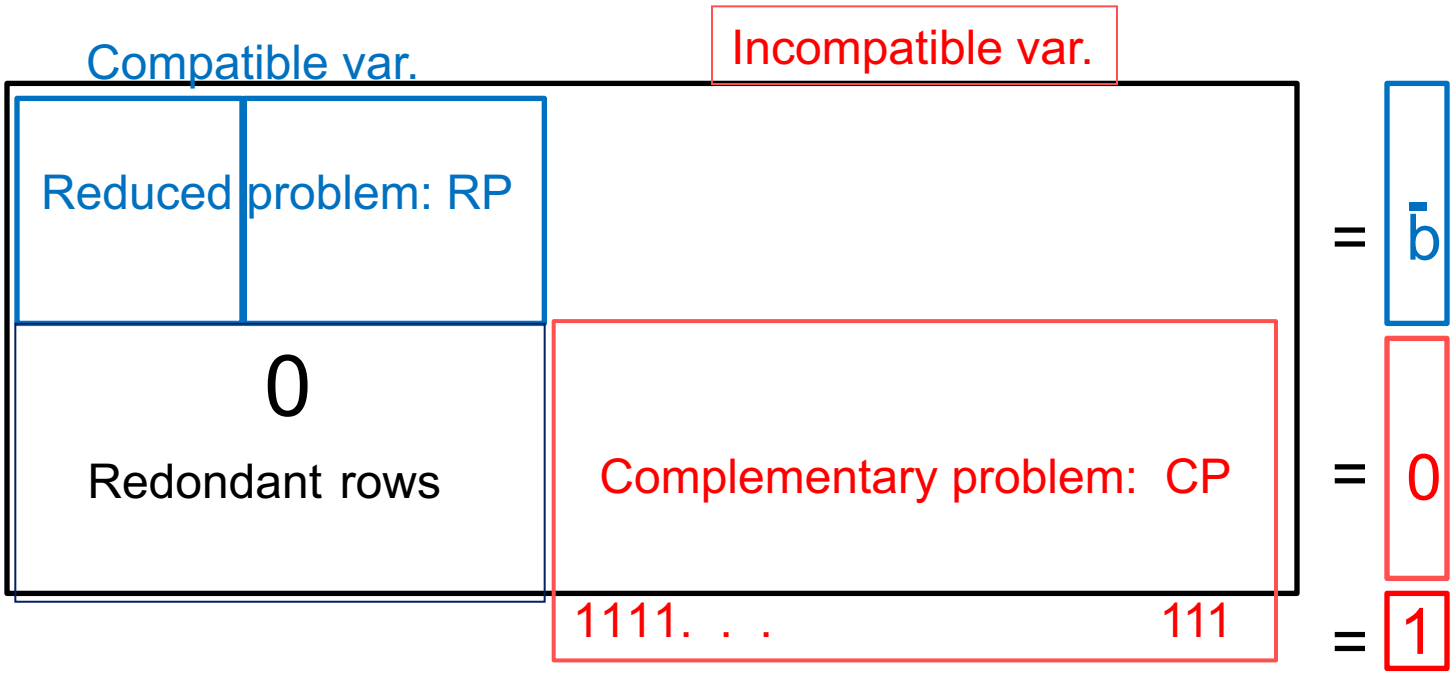
- OPTIMIZE
  - FAST OPT. ON CLUSTERS **Blue var. only**
    - Smaller problem (one constraint per cluster)
  - MODIFY CLUSTERING TO REACH OPTIMALITY
    - Add some red var. (selected using reduced cost ???)

DEGENERENCY for SPP



We substract the first row of a cluster to the other rows and move them in the lower part of the matrix

REDUCED AND COMPLEMENTARY PROBLEMS



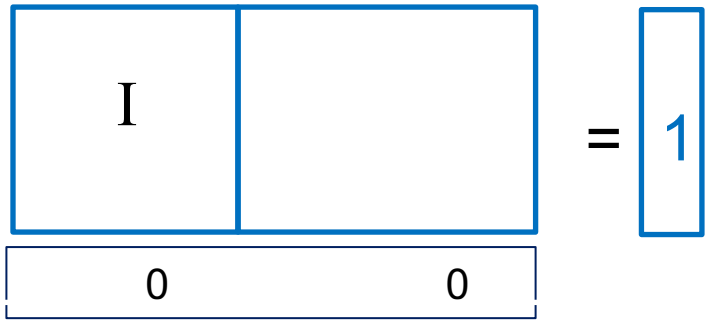
P=set of non degenerated rows

Z=set of degenerated rows

Compatible var.: $\bar{A}_j^Z = 0$ Incompatible var.: $\bar{A}_j^Z \neq 0$

IMPROVING INTEGER SOLUTION WITH THE REDUCED PROBLEM

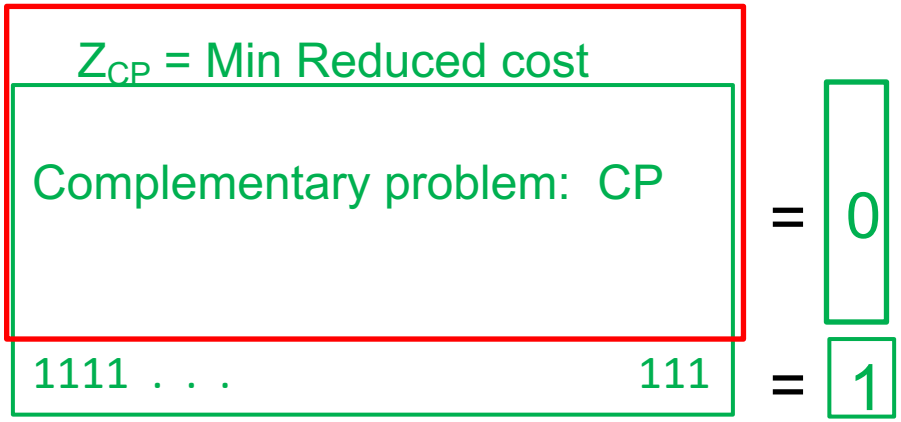
Reduced problem: RP



A pivot on a compatible column replaces one or some columns in the basis

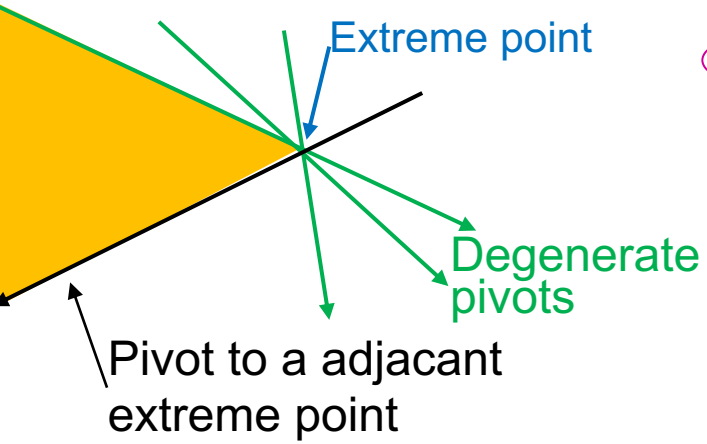
- A pivot on a negative reduced cost variable
  - Is a non degenerated pivot with $a_{i,j} = 1$ and $b_i = 1$
  - Produces an improved integer solution
- Degenerated constraints are removed before the following pivot

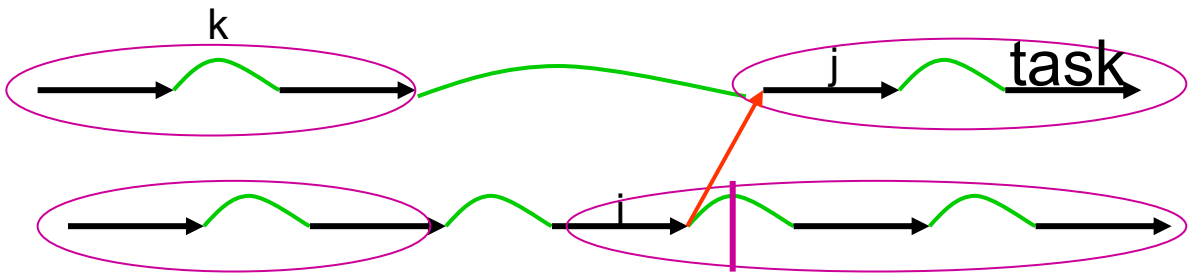
RESULTS FROM THE COMPLEMENTARY PROBLEM



- $S = \{j \mid x^*_j \neq 0\}$ is minimal, (no proper subset)
- $W = \sum_{j \in S} \bar{A}_j$ is compatible with RP
- If $Z_{CP} < 0$, a pivot on the surrogate column w improves the RP solution (negative reduce cost, non-degenerated)
- If $Z_{CP} < 0$, a sequence of pivots on S improve the RP solution

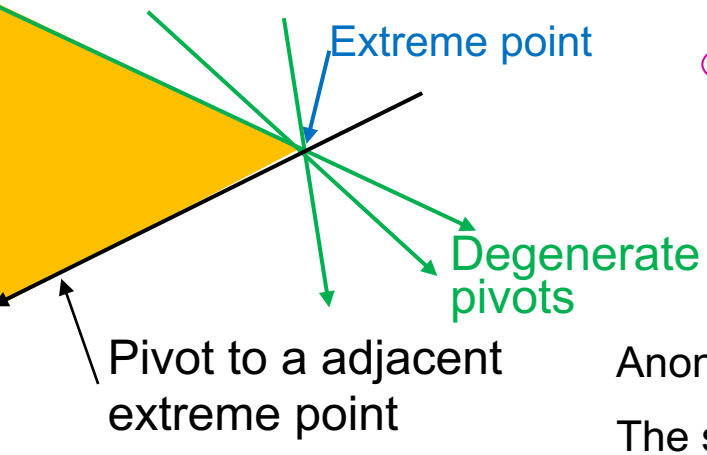
PIVOT ON AN INCOMPATIBLE VARIABLE

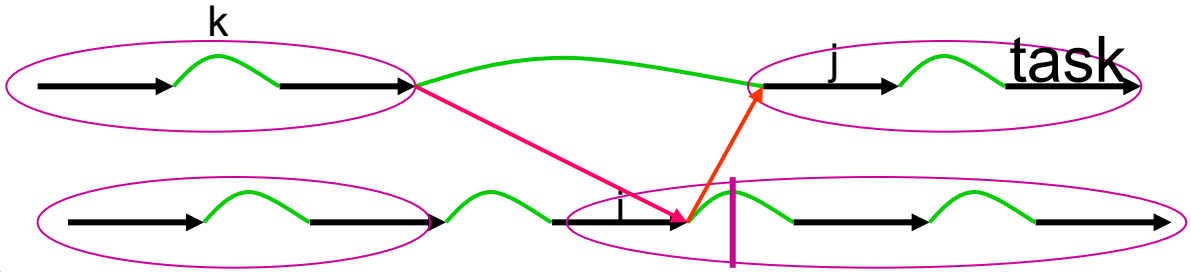




A pivot entering the column with +1 on row j and -1 on row i produces a degenerated pivot on row i

PIVOTS ON A SET S of INCOMPATIBLE VARIABLES



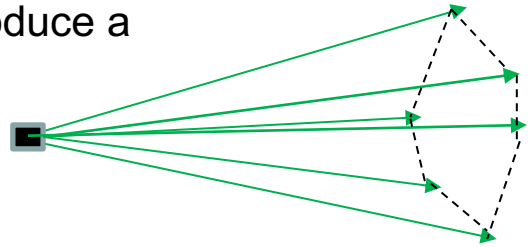


Anon-degenerated pivot

The sets S produced by the complementary problem contains such group of columns
pivot entering the column with +1 on row j and -1 on row i produces a degenerated pivot on row i

Entering, after a column with -1 on row k and +1 on row i produce a

The cone of the complementary problem

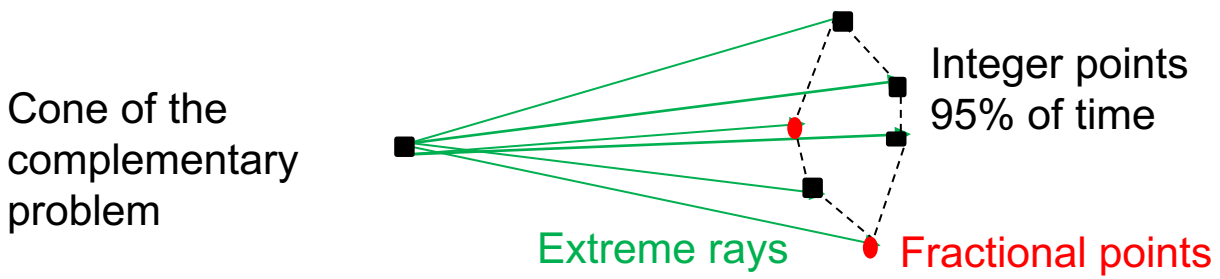


Multi-phase: phase K permit columns breaking at most k clusters

Extreme rays
Improving direction in RP, non degenerated

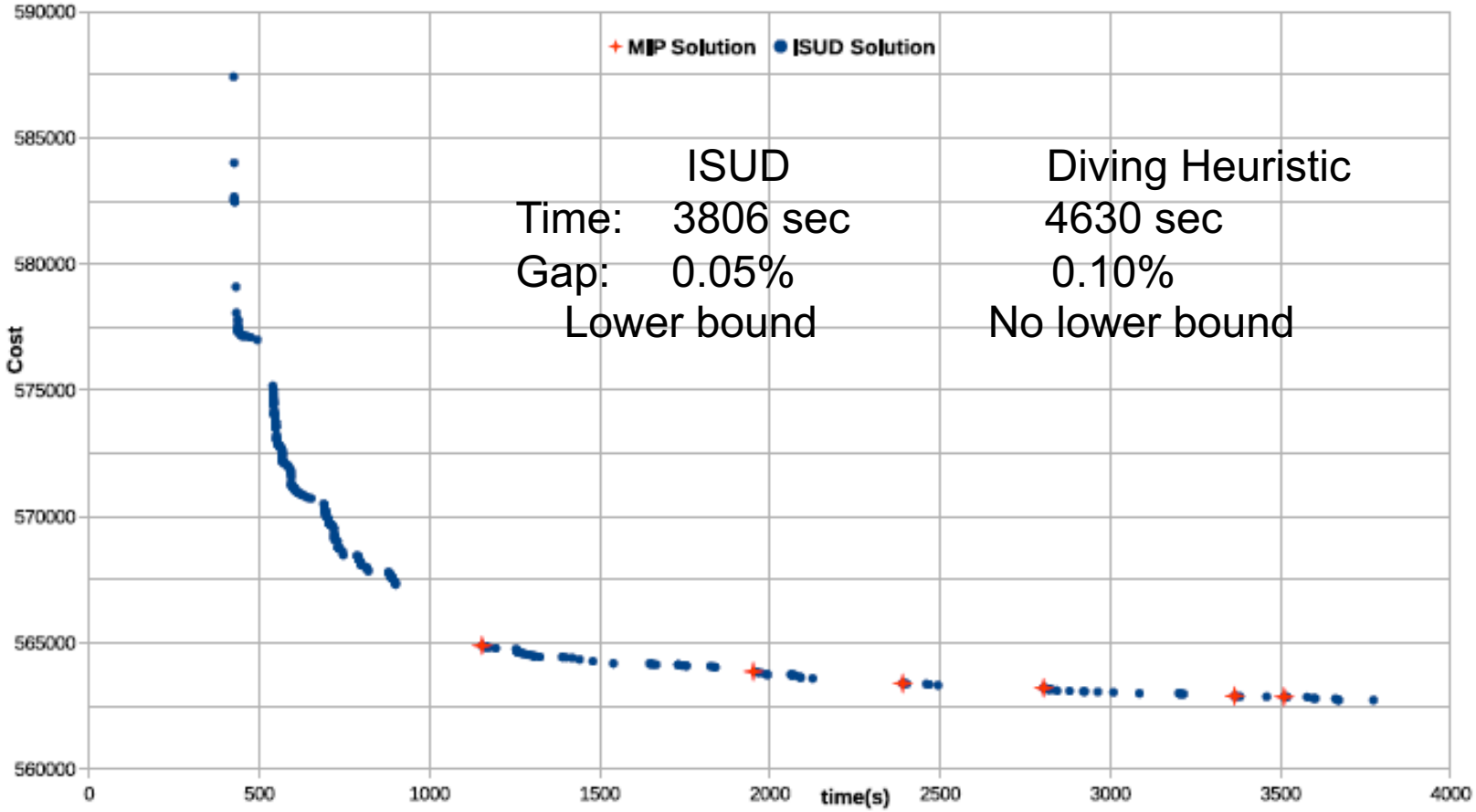
IMPROVING INTEGER SOLUTION WITH THE COMPLEMENTARY PROBLEM

- If $Z_{CP} < 0$ and S in column-disjoint (def:$(A_i.A_j)=0, i, j \in S$)
  - $w = \sum_{j \in S} A_j$ is binary
  - From x^1 integer solution of RP and Ꝑ, a sequence of pivots on S produces x^2 an improved integer solution



- IP on positives columns in the base $\cup$ S

WEEKLY CREW PAIRING, 1740 FLIGHTS with Column Generation, no initial information



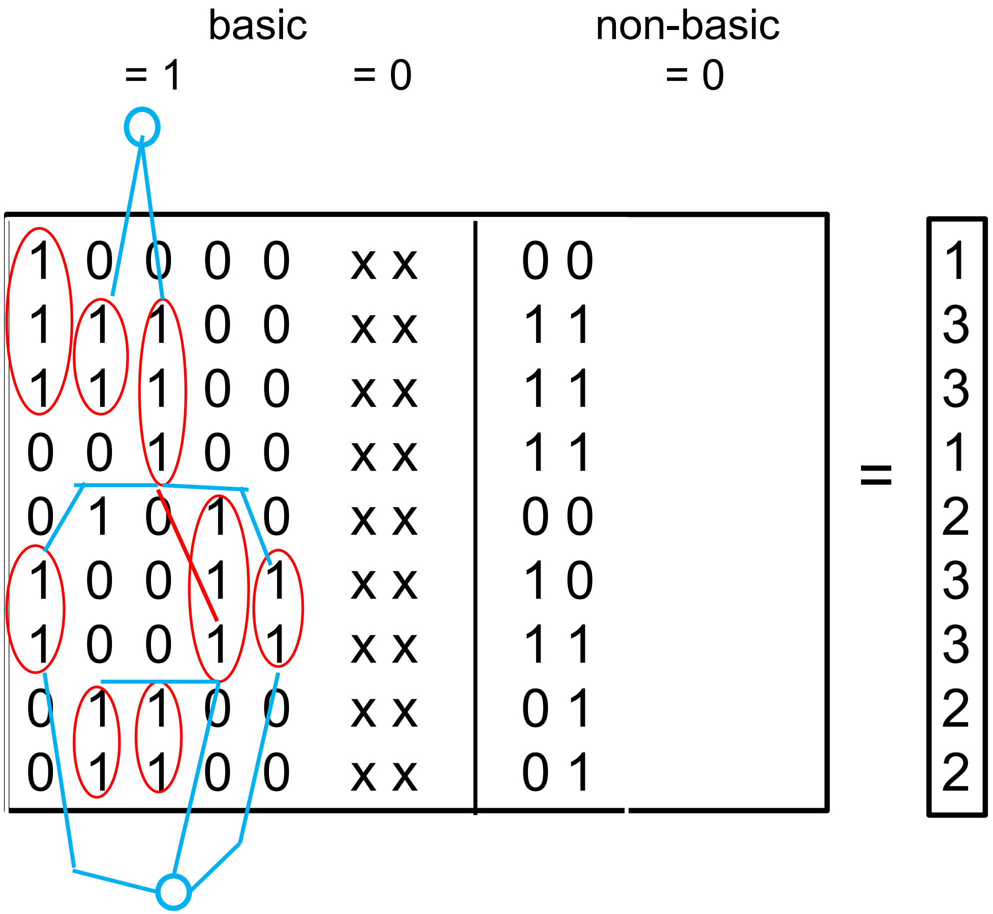
INTEGRAL SIMPLEX for GSPP

- Objective: reach an optimal integer solution for GSPP
  with a sequence of simplex pivots

- IDEAS in **GISUD**
  - Difficulty: an extreme point has a huge number of basis connected by degenerated pivots
  - Finds a path of basis to reach optimality (pass throw degenerency)
  - Develp some adaptations of ISUD ideas

  - The graph of integer extreme points can be non-connex
  - Adds hedges in the polyedron to find a path of basis to reach optimality

ADAPTATION of ISUD to PASS THROW DEGENERANCY



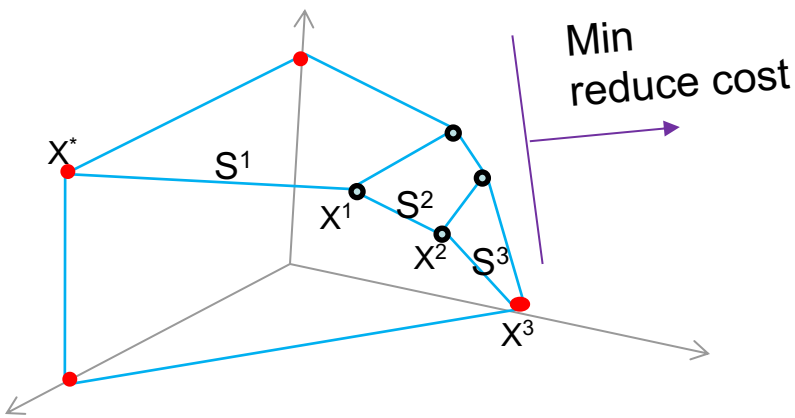
$$
\left[\begin{array}{ccccccc|cc}
1 & 0 & 0 & 0 & 0 & x & x & 0 & 0 \\
1 & 1 & 1 & 0 & 0 & x & x & 1 & 1 \\
1 & 1 & 1 & 0 & 0 & x & x & 1 & 1 \\
0 & 0 & 1 & 0 & 0 & x & x & 1 & 1 \\
0 & 1 & 0 & 1 & 0 & x & x & 0 & 0 \\
1 & 0 & 0 & 1 & 1 & x & x & 1 & 0 \\
1 & 0 & 0 & 1 & 1 & x & x & 1 & 1 \\
0 & 1 & 1 & 0 & 0 & x & x & 0 & 1 \\
0 & 1 & 1 & 0 & 0 & x & x & 0 & 1
\end{array}\right]
=
\left[\begin{array}{c}
1 \\ 3 \\ 3 \\ 1 \\ 2 \\ 3 \\ 3 \\ 2 \\ 2
\end{array}\right]
$$

- The basis in not `I`
- Does not define a partition of tasks
- Non disjoint clusters
- Def. compatible variable:
  enter in the basis at integrality
- Exiting variables = path of clusters
- Many paths: combinatorial?
- No, solve by dynamic programming
  - Min reduce cost
  - Ressource cont.: # red arcs K
- K = 0 → Compatible variable
- K = 1, 2, 3,… Multi-phases

CP to go at an ADJACANT POINT

- Same CP as ISUD
- If the solution is integer (column disjoints) integrate it in the RP. The integer solution of RP is improved
- If not integer we move at a point at distance 2, 3, …
  (see next slide)
- If no integer point is reached, use ZOOM

MOVE at a POINT at DISTANCE at 2,3, …

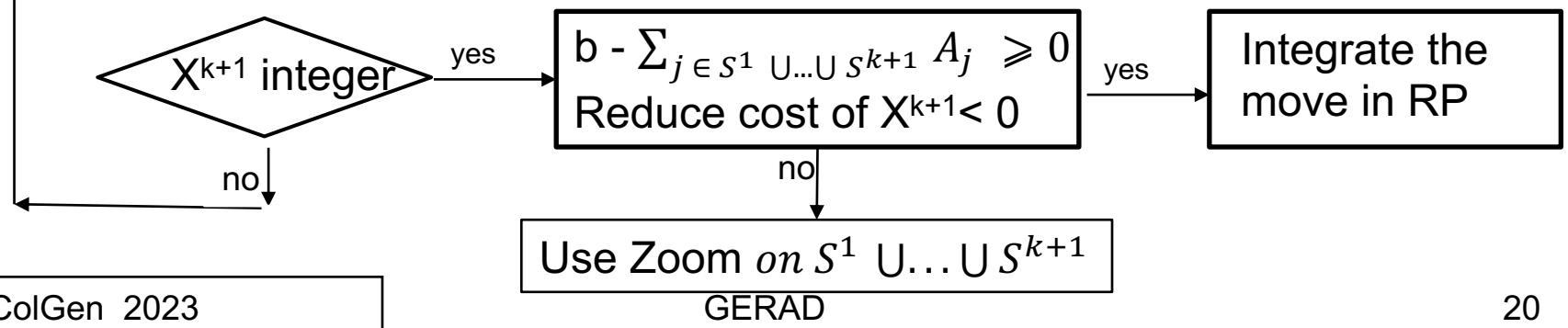


- Integer solution
- Fractional solution

1. When X^k is not integer
2. Solve the next CP with a cone rooted at X^k

   2.1 Remove S^k from CP and obtain S^{k+1} and X^{k+1}

   2.2



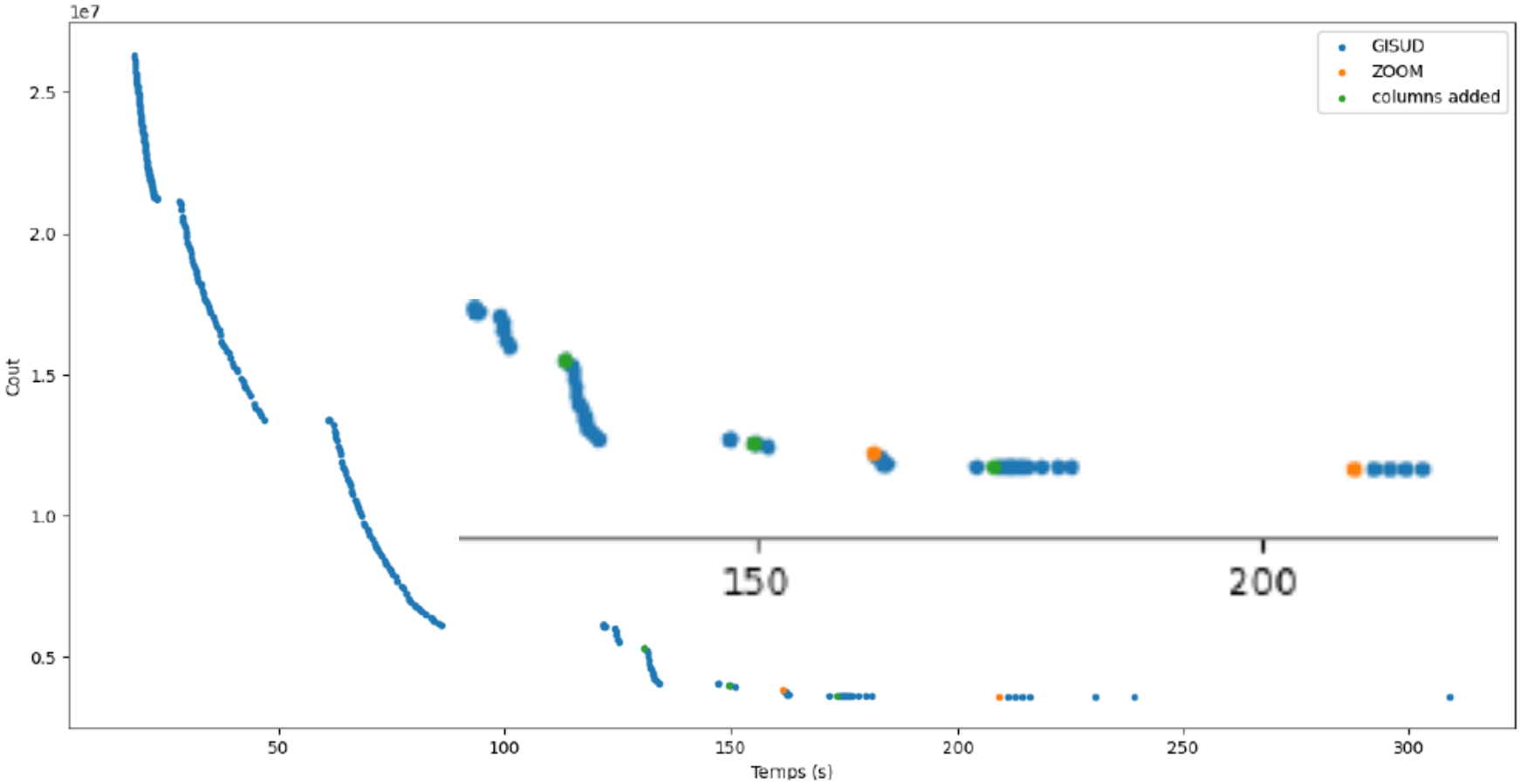
RESULTS with GISUD

- Crew pairing problems: 2175 flights, 955 416 pairings
- 1434 $b_i = 1$, 689 $b_i = 2$, 70 $b_i = 3$
- Initial solution obtained by perturbing 40% of an optimal solution

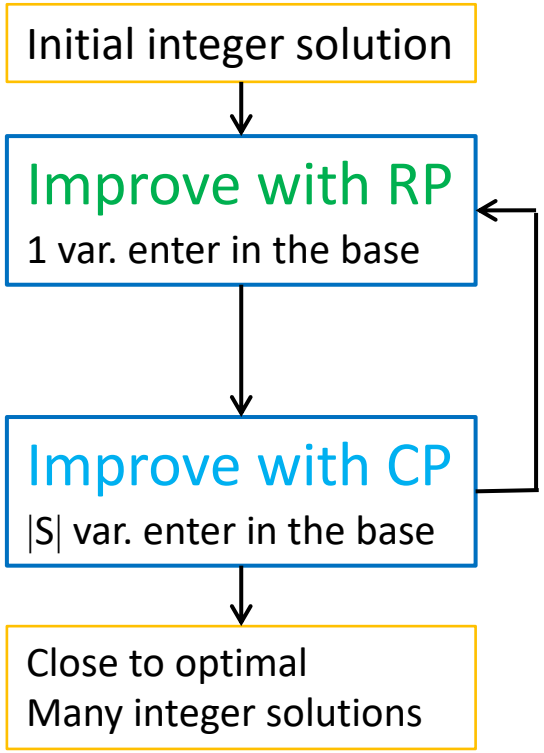
	CPLEX		GISUD						
	Gap	Time sec.	Gap	Time sec.	Integer solutions	columns added	% succes	distance	Zoom
	0,09	1881	0,15	502	259	2	50	2	6
	0,64	1871	0,14	479	269	4	75	2	4
	0,08	2068	0,13	346	276	3	100	2	2
	0,13	2155	0,15	369	295	1	100	2	3
	0,07	1881	0,15	408	302	2	100	2	2
	0,08	1835	0,12	230	270	3	66	2	2
Mean	0,18	1949	0,14	389	279	2,50	81,8	2	3,17

- GISUD produces a lot of integer solutions, 5 times faster than CPLEX.

SEQUENCE of INTEGER SOLUTIONS



THE GISUD ALGORITHM



- Degenerated constraints are removed when they appear
- An improved integer solution is obtained after |S| pivots in RP
- Multi-phase pricing in CP
- Add variables or branching when necessary to obtain integer solutions

- Future work: test with larger bi
- Integrate with column generation

REFERENCE LIST

REDUCING THE NUMBER OF CONSTRAINTS

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APPLICATIONS

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