Branch-price-and-cut algorithm for the multi-commodity two-echelon vehicle routing problem with time windows

Tayeb Mhamedi
Polytechnique Montréal and GERAD, Canada

Under the supervision of: Guy Desaulniers, Marilène Cherkesly

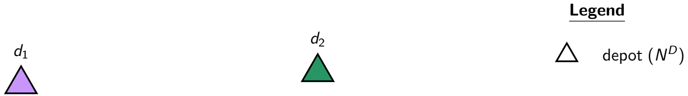
Column Generation 2023
Montréal, Québec

May 16-19, 2023

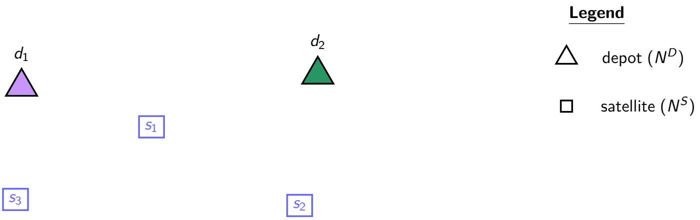
Outline

Outline

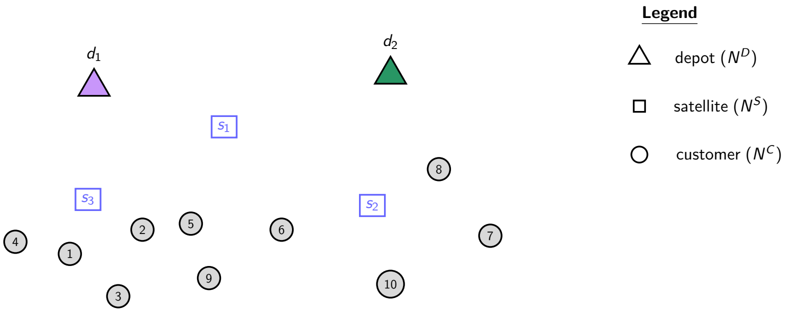
Multi-commodity two-echelon vehicle routing problem with time windows (MC-2E-VRPTW)



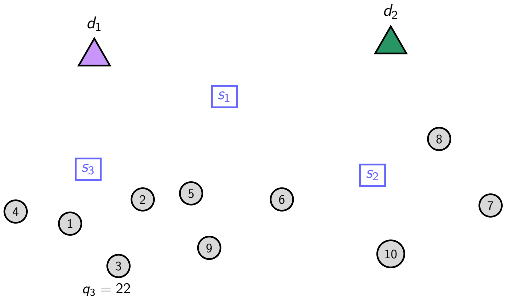
Multi-commodity two-echelon vehicle routing problem with time windows (MC-2E-VRPTW)



Multi-commodity two-echelon vehicle routing problem with time windows (MC-2E-VRPTW)



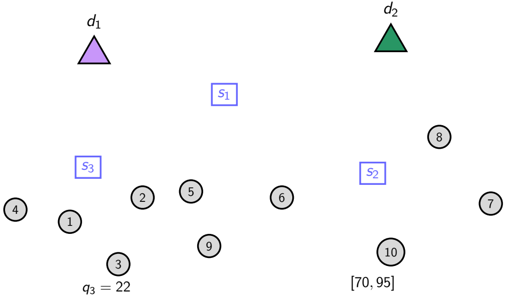
Multi-commodity two-echelon vehicle routing problem with time windows (MC-2E-VRPTW)



Legend

- △ depot (N^D)
- □ satellite (N^S)
- ○ customer (N^C)
- q_i customer demand

Multi-commodity two-echelon vehicle routing problem with time windows (MC-2E-VRPTW)



Legend


depot (N^D)

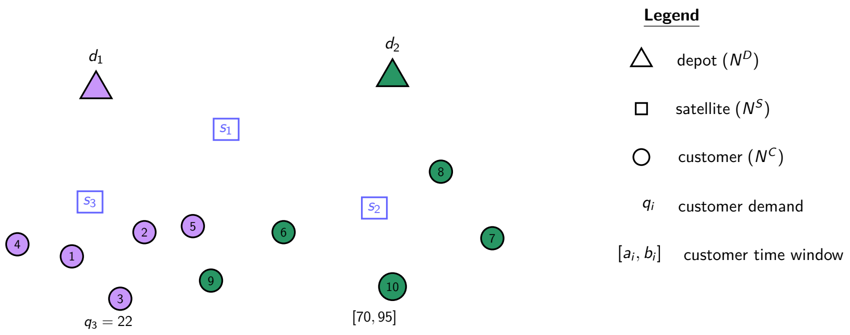
satellite (N^S)

customer (N^C)

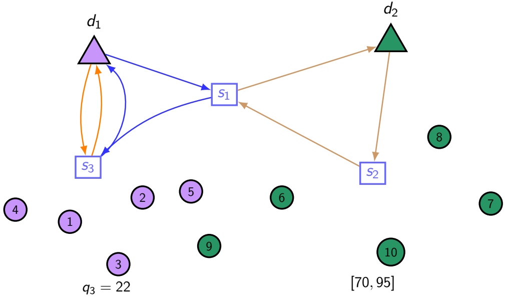
q_i customer demand

$[a_i, b_i]$ customer time window

Multi-commodity two-echelon vehicle routing problem with time windows (MC-2E-VRPTW)



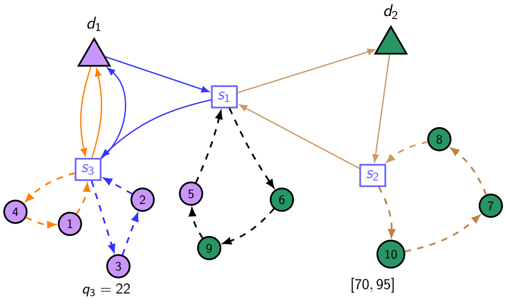
Multi-commodity two-echelon vehicle routing problem with time windows (MC-2E-VRPTW)



Legend

△	depot (N^D)
□	satellite (N^S)
○	customer (N^C)
q_i	customer demand
$[a_i, b_i]$	customer time window
→	first-echelon route

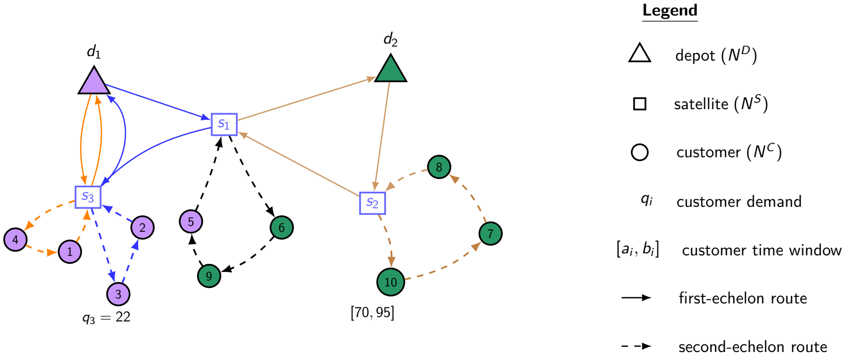
Multi-commodity two-echelon vehicle routing problem with time windows (MC-2E-VRPTW)



Legend

- depot (N^D)
- satellite (N^S)
- customer (N^C)
- q_i customer demand
- $[a_i, b_i]$ customer time window
- first-echelon route
- second-echelon route

Multi-commodity two-echelon vehicle routing problem with time windows (MC-2E-VRPTW)



- Minimize total routing costs
- First-echelon route constraints: vehicle capacity (Q^1)
- Second-echelon route constraints: vehicle capacity (Q^2) & time windows
- Every customer's demand is available at a **specific** depot and is fed by a **single** first-echelon vehicle

Literature Review

- Surveys: Cuda et al. (2015); Guastaroba et al. (2016); Sluijk et al. (2022)
- Exact algorithms for 2E-VRPTW
  - Dellaert et al. (2019)
  - Mhamedi et al. (2022)
  - Marques et al. (2022) (2E-VRPTW-FC)
- Exact algorithms for the MC-2E-VRPTW : Dellaert et al. (2021)
  - Two-path formulation (MC2E-2P): solution method based on a branch-and-price approach
  - Instances with up to 5 satellites and 100 customers are solved to optimality

Literature Review

- Surveys: Cuda et al. (2015); Guastaroba et al. (2016); Sluijk et al. (2022)
- Exact algorithms for 2E-VRPTW
  - Dellaert et al. (2019)
  - Mhamedi et al. (2022)
  - Marques et al. (2022) (2E-VRPTW-FC)
- Exact algorithms for the MC-2E-VRPTW : Dellaert et al. (2021)
  - Two-path formulation (MC2E-2P): solution method based on a branch-and-price approach
  - Instances with up to 5 satellites and 100 customers are solved to optimality

Our goal

Develop an efficient branch-price-and-cut algorithm for multi-commodity 2E-VRPTW

Outline

Notation

First echelon

- $\mathcal{M}$: Set of all first-echelon routes
  - c_m: Cost of route m
  - $S(m)$: Set of satellites visited by route m
- Q^1: First-echelon vehicle capacity
- x_m: Binary variable equal to 1 if route m is chosen, 0 otherwise.

Notation

Second echelon

- $\mathcal{L}$: Set of all feasible second-echelon routes
  - c_l: Cost of route l
  - $\mathcal{P}_l$: Set of supply-patterns that can be assigned to route l
  - a_{il}: Binary parameter equal to 1 if route l services customer i
  - Q_l^{mp}: Total demand serviced by route l that was provided by first-echelon route m when supply-pattern p is assigned to l
- Q^2: Second-echelon vehicle capacity
- y_{lp}: Binary variable equal to 1 if second-echelon route l with supply-pattern $p \in \mathcal{P}_l$ is selected, 0 otherwise.

Route-based formulation

$$\min \quad \sum_{m \in \mathcal{M}} c_m x_m + \sum_{l \in \mathcal{L}} \sum_{p \in \mathcal{P}_l} c_l y_{lp} \tag{1}$$

$$\text{s.t.} \quad \sum_{l \in \mathcal{L}} \sum_{p \in \mathcal{P}_l} a_{il} y_{lp} = 1, \qquad \forall i \in N^C, \tag{2}$$

$$\sum_{l \in \mathcal{L}} \sum_{p \in \mathcal{P}_l} Q_l^{mp} y_{lp} \leq Q^1 x_m, \qquad \forall m \in \mathcal{M}, \tag{3}$$

$$x_m \in \{0,1\}, \qquad \forall m \in \mathcal{M}, \tag{4}$$

$$y_{lp} \in \{0,1\}, \qquad \forall l \in \mathcal{L}, p \in \mathcal{P}_l. \tag{5}$$

- First-echelon routes are enumerated a priori

Route-based formulation

$$
\begin{aligned}
\min \quad & \sum_{m\in\mathcal{M}} c_m x_m + \sum_{l\in\mathcal{L}} \sum_{p\in\mathcal{P}_l} c_l y_{lp} && && (1)\\
\text{s.t.} \quad & \sum_{l\in\mathcal{L}} \sum_{p\in\mathcal{P}_l} a_{il} y_{lp} = 1, && \forall i \in N^C, && (2)\\
& \sum_{l\in\mathcal{L}} \sum_{p\in\mathcal{P}_l} Q_l^{mp} y_{lp} \le Q^1 x_m, && \forall m \in \mathcal{M}, && (3)\\
& x_m \in \{0,1\}, && \forall m \in \mathcal{M}, && (4)\\
& y_{lp} \in \{0,1\}, && \forall l \in \mathcal{L}, p \in \mathcal{P}_l. && (5)
\end{aligned}
$$

- First-echelon routes are enumerated a priori
- A very large number of second-echelon routes : apply BPC.

Outline

Pricing problems

Reduced cost of second-echelon route variable y_{lp}:

$$\bar{c}_{lp} = c_l - \sum_{i \in N^C} a_{il}\sigma_i - \sum_{i \in N^C} \sum_{m \in \mathcal{M}_{d_i}} \delta_{im}^{p}(q_i \pi_m)$$

where σ_i, π_m are the dual variables of constraints (2), and (3).

PP per satellite s

- Modeled on a network G^s
- Find least-reduced-cost second-echelon route associated with s
- Variant of an elementary shortest path problem with resource constraints (on time and load)
- Challenge: Handling the subset of selected π_m dual values (at most one per depot d)

Pricing problems

Reduced cost of second-echelon route variable y_{lp}:

$$\bar{c}_{lp} = c_l - \sum_{i \in N^C} a_{il}\sigma_i - \sum_{i \in N^C} \sum_{m \in \mathcal{M}_{d_i}} \delta^p_{im}(q_i \pi_m)$$

where σ_i, π_m are the dual variables of constraints (2), and (3).

PP per satellite s

- Modeled on a network G^s
- Find least-reduced-cost second-echelon route associated with s
- Variant of an elementary shortest path problem with resource constraints (on time and load)
- Challenge: Handling the subset of selected π_m dual values (at most one per depot d)

Labeling algorithm

Label E represents a partial second-echelon route $R(E)$ assigned to a supply-pattern

- $\eta(E)$: Last visited node on path $R(E)$
- $\bar{c}(E)$: Reduced cost
- $t(E)$: Earliest service start time at node $\eta(E)$
- $q(E)$: Total load
- $\mathcal{U}(E)$: Set of unreachable nodes
- $\theta(E)$: Time service starts at the satellite associated to $R(E)$

Labeling algorithm

Label E represents a partial second-echelon route $R(E)$ assigned to a supply-pattern

- $\eta(E)$: Last visited node on path $R(E)$
- $\bar{c}(E)$: Reduced cost
- $t(E)$: Earliest service start time at node $\eta(E)$
- $q(E)$: Total load
- $\mathcal{U}(E)$: Set of unreachable nodes
- $\theta(E)$: Time service starts at the satellite associated to $R(E)$

Component $\theta(E)$ is required to retrieve information regarding the best available π_m values (per depot d, $m \in \mathcal{M}_{ds}$) when extending a path

Labeling algorithm

Specificities

- Initialization
  - One label E^k for every arrival time $t_k \in T_s$
- Label extension
  - Copy time service start component
  - Reduced cost is updated using information derived from $\theta(E)$
- Dominance rule A label E_1 dominates a label E_2 when
  - $\eta(E_1) = \eta(E_2)$
  - $t(E_1) \leq t(E_2)$, $q(E_1) \leq q(E_2)$, $\mathcal{U}(E_1) \subseteq \mathcal{U}(E_2)$
  - $\bar{c}(E_1) \leq \bar{c}(E_2) + (Q^2 - q(E_2)) \times \Delta\big(\theta(E_1), \theta(E_2)\big)$

$$\Delta\big(\theta(E_1), \theta(E_2)\big) = \begin{cases} 0, & \text{if } \theta(E_1) \geq \theta(E_2) \\ \min\limits_{d \in \mathcal{D}_{\theta(E_1)}} \left\{ \pi_{m_{(\theta(E_1),d)}} - \pi_{m_{(\theta(E_2),d)}} \right\}, & \text{otherwise} \end{cases}$$

Additional algorithmic components

Dual inequalities

Transfer inequalities (Mhamedi et al., 2022) are used to speed-up CG convergence

Valid inequalities

- Rounded capacity cuts , Laporte and Nobert (1983)
- Subset-row inequalities , Jepsen et al. (2008)
- (Lifted) visited satellites inequalities, Marques et al. (2020)

Hierarchical branching

1- Branch on whether or not commodities associated with a depot are allowed to be supplied through a given satellite

2- Total flow on a back-and-forth route

3- Number of vehicles

4- Use of a first-echelon route

5- Branch on servicing a customer from a specific satellite

6- Flow on an arc

7- Branch on servicing a customer using a given first-echelon route

Outline

MC-2E-VRPTW instances

- Generated from instances for the 2E-VRPTW (Dellaert et al., 2019)
- Number of customers $|N^C| \in \{30, 50, 100\}$
- 2 groups of instances per $|N^C|$ value
  - 2 depots, 3 satellites
  - 3 depots, 5 satellites
- $Q^1 = 200$, $Q^2 = 50$
- Each customer is randomly assigned to a depot to introduce commodities
- 3-hour time limit

Comparison with Dellaert et al. (2021)

			Dellaert et al. †		Our BPC algorithm ‡		BMSU
$\|N^D\|$	$\|N^S\|$	$\|N^C\|$	T (s)	# Opt	T (s)	# Opt	factor
2	3	30	79.8	20/20	2.2	20/20	36.2
2	3	50	2235	4/5	5.1	5/5	18.2
3	5	50	2794.2	5/5	530.9	5/5	5.3
2	3	100	10800	0/5	1514.6	5/5	7.1
3	5	100	9223.8	1/5	10800	0/5	-

† Tests ran on a computer equipped with an Intel Core i7-4770 processor
‡ Tests ran on a computer equipped with an Intel Core i7-8700 processor
T (s) Average time in seconds over the solved instances
BMSU: Biased minimal speed-up factor (bias induced by computers)

Sensitivity analysis

Omit one component at a time (subset of 10 instances solved by our full BPC algorithm)

	Instances with $\lvert N^C \rvert = 50$			Instances with $\lvert N^C \rvert = 100$		
Family	# Opt	T (s)	ΔT (%)	# Opt	T (s)	ΔT (%)
DDOIs	3/5	1711.1	847.5	5/5	2252.2	48.7
RCIs	5/5	1299.2	144.7	5/5	1553.5	2.6
SRIs	5/5	964.2	81.7	5/5	3802.4	151.1
VSIs	3/5	1941.9	342.1	5/5	1106.9	-26.9
CSBr∗	4/5	2243.2	392.3	5/5	1311.8	-13.9

∗ CSBr: commodity-supply-through-satellite branching

Outline

Conclusions

- Efficient branch-price-and-cut algorithm for the MC-2E-VRPTW
  - Labeling algorithm allowing the aggregation of pricing problems per satellite
- Outperforms existing state-of-the-art algorithm

Conclusions

- Efficient branch-price-and-cut algorithm for the MC-2E-VRPTW
  - Labeling algorithm allowing the aggregation of pricing problems per satellite
- Outperforms existing state-of-the-art algorithm

Future works

- Study extensions where reverse merchandise flows are introduced
- Investigate the effect of considering different synchronization schemes

Thank you !

tayeb.mhamedi@polymtl.ca

Cuda, R., Guastaroba, G., and Speranza, M. (2015). A survey on two-echelon routing problems. Computers & Operations Research, 55:185–199.

Dellaert, N., Dashty Saridarq, F., Van Woensel, T., and Crainic, T. G. (2019). Branch-and-price–based algorithms for the two-echelon vehicle routing problem with time windows. Transportation Science, 53(2):463–479.

Dellaert, N., Van Woensel, T., Crainic, T. G., and Saridarq, F. D. (2021). A multi-commodity two-echelon capacitated vehicle routing problem with time windows: Model formulations and solution approach. Computers & Operations Research, 127:105154.

Guastaroba, G., Speranza, M. G., and Vigo, D. (2016). Intermediate facilities in freight transportation planning: A survey. Transportation Science, 50(3):763–789.

Jepsen, M., Petersen, B., Spoorendonk, S., and Pisinger, D. (2008). Subset-row inequalities applied to the vehicle-routing problem with time windows. Operations Research, 56(2):497–511.

Laporte, G. and Nobert, Y. (1983). A branch and bound algorithm for the capacitated vehicle routing problem. Operations-Research-Spektrum, 5(2):77–85.

Marques, G., Sadykov, R., Deschamps, J.-C., and Dupas, R. (2020). An improved branch-cut-and-price algorithm for the two-echelon capacitated vehicle routing problem. Computers & Operations Research, 114:104833.

Marques, G., Sadykov, R., Dupas, R., and Deschamps, J.-C. (2022). A branch-cut-and-price approach for the single-trip and multi-trip two-echelon vehicle routing problem with time windows. Transportation Science.

Mhamedi, T., Andersson, H., Cherkesly, M., and Desaulniers, G. (2022). A branch-price-and-cut algorithm for the two-echelon vehicle routing problem with time windows. Transportation Science, 56(1):245–264.

Sluijk, N., Florio, A. M., Kinable, J., Dellaert, N., and Van Woensel, T. (2022). Two-echelon vehicle routing problems: A literature review. European Journal of Operational Research.