Branch-Price-and-Cut-Based Solution of Order Batching Problems

Julia Wahlen, Timo Gschwind

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Introduction

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Outline

1. Problem Description and Formulation
2. Pricing Subproblem
3. Cutting Planes
4. Branching Scheme
5. BPC-Based Heuristics
6. Computational Results

Problem Description

The **Order Batching Problem (OBP)** is defined by

- Warehouse layout
- Storage locations of all items
- Routing strategy
- Customer orders $o \in O$ each comprising q_o individual items
- Pickers with picking capacity Q (capacity consumption of each item is assumed to be 1)

Problem Description

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- Pickers with picking capacity Q (capacity consumption of each item is assumed to be 1)

Task

Find a set of picking batches such that

- each order is assigned to exactly one batch,
- all batches satisfy the capacity restriction, and
- the total traveled distance by the pickers is minimal.

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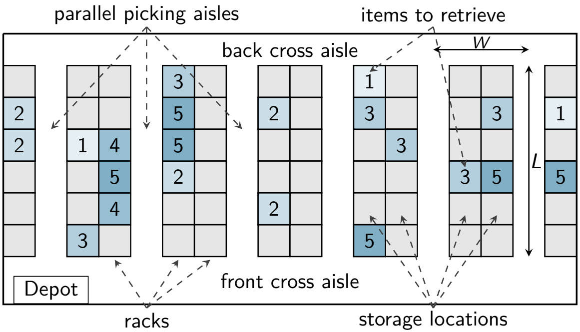
Warehouse Layout

The **standard layout** considers a rectangular warehouse with

- parallel aisles of equal length and width
- cross aisles in the front and back
- common depot located in front of the leftmost aisle



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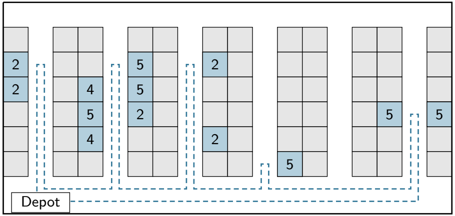
Routing Strategy

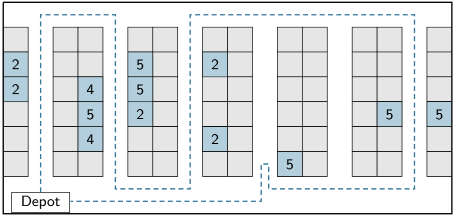
Cost of a batch (=distance traveled) depends on the given routing strategy.

In practice: use of heuristics with simple patterns due to non-intuitive structure of optimal routes.

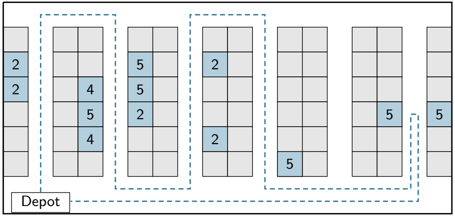
Return distance: 68



Optimal distance: 60



Traversal (S-Shape) distance: 64



Related Literature (Exact Approaches)

Article	Routing strategies	Approach
Gademann and van de Velde (2005)	optimal	CG/B&P
Öncan (2015)	return, traversal, midpoint	MIP solver
Muter and Öncan (2015)	return, traversal, midpoint	CG/cuts/MIP solver
Valle *et al.* (2016, 2017)	optimal	B&C
Bahçeci and Öncan (2021)	composite, largest gap, mixed, optimal	MIP solver
Our approach	**return, traversal, midpoint, largest gap, combined, optimal**	**CG/BPC**

Problem Formulation

Master program:

Ω set of feasible batches

a_{ob} binary parameter equal to 1 if batch b includes order o, 0 otherwise

c_b cost of batch b ($\rightarrow$ distance function depending on the routing strategy)

λ_b binary variable equal to 1 if batch b is selected, 0 otherwise

$$\begin{aligned} \min \quad & \sum_{b \in \Omega} c_b \lambda_b & & \\ \text{s.t.} \quad & \sum_{b \in \Omega} a_{ob} \lambda_b = 1 \qquad (\rightarrow \pi_o) & & \forall o \in O \\ & \lambda_b \in \{0, 1\} & & \forall b \in \Omega \end{aligned}$$

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Problem Formulation

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- Ω set of feasible batches
- a_{ob} binary parameter equal to 1 if batch b includes order o, 0 otherwise
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$$
\begin{aligned}
\min \quad & \sum_{b \in \Omega} c_b \lambda_b \\
\text{s.t.} \quad & \sum_{b \in \Omega} a_{ob} \lambda_b = 1 \qquad (\rightarrow \pi_o) && \forall o \in O \\
& \lambda_b \in \{0,1\} && \forall b \in \Omega
\end{aligned}
$$

Pricing problem:

- Find negative reduced-cost batch or prove that none exists
- Reduced cost of batch $b \in \Omega$ is $\tilde{c}_b = c_b - \sum_{o \in b} \pi_o$

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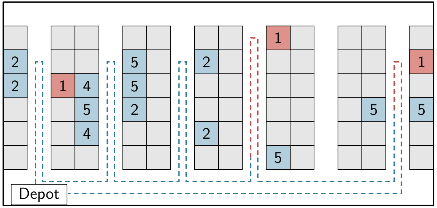
Special Characteristic of OBP

Key difficulty: The cost c_b of a batch b is non-separable in the orders $o \in b$.

- Example (return strategy):



Additional cost for adding order 1
to batch $b_1 = \{2,4,5\}$: $2 \cdot 5 + 2 \cdot 2 = 14$

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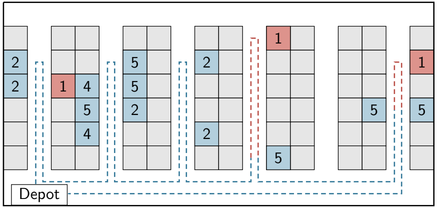

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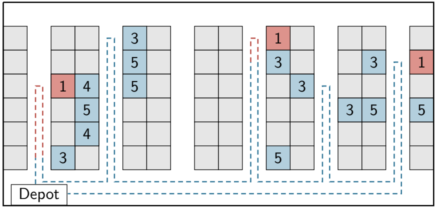


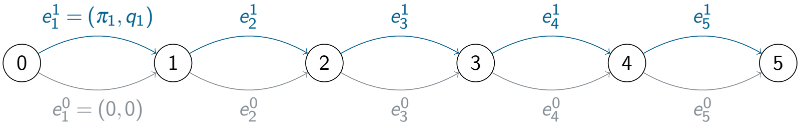
Additional cost for adding order 1 to batch $b_2 = \{3,4,5\}$: $2 \cdot 3 + 2 \cdot 1 = 8$

Outline

CG Pricing Problem

The pricing problem is modeled as a **Shortest Path Problem with Resource Constraints**:

- Dummy source 0
- One **vertex for each order** $o \in \{1, \dots, n\}$
- **Two parallel arcs** between vertices $o-1$ and o:
  - e_o^1 include order o
  - e_o^0 do not include order o

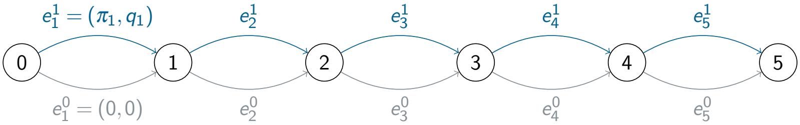


Task

Find a $0-n$-path with minimal reduced cost such that the picker capacity Q is not exceeded.

Pricing Problem Solution

The pricing problem is solved with a **dynamic-programming labeling algorithm:**



- Each partial path is represented by a **label** $L = (vertex(L), orders(L), load(L), rcost(L))$

- Extension along arc e_o^0

  $vertex(L') = o$

  $orders(L') = orders(L)$

  $load(L') = load(L)$

  $rcost(L') = rcost(L)$

- Extension along arc e_o^1

  $vertex(L') = o$

  $orders(L') = orders(L) \cup o$

  $load(L') = load(L) + q_o$

  $rcost(L') = c_{orders(L')} - \sum_{o \in orders(L')} \pi_o$

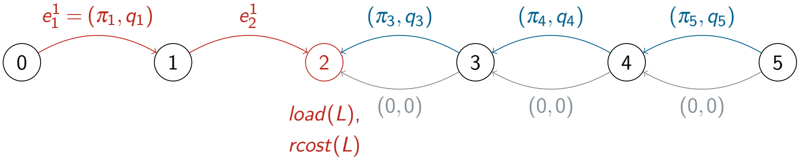
- **Recall:** $rcost(L)$ is not separable in the orders $o \in orders(L)$

  $\rightarrow$ Recalculation necessary for each inclusion of an order

  $\rightarrow$ Standard dominance rule on the resources is not valid

Bounding in Labeling

- Example: label L at vertex 2, after the orders 1 and 2 have been included



- **Completion bound**: What is the maximum profit (sum of duals) that can be achieved with capacity feasible extensions? $\rightarrow$ **Knapsack Problem** (KP)
- Bounding whenever a label's reduced cost exceeds the corresponding completion bound

Task

Solve a single binary KP with profits π_o, weights q_o and capacity Q on the same graph in backward direction.

Note: The distance function has to be monotone, i.e., $b_1 \subseteq b_2 \Rightarrow c_{b_1} \leq c_{b_2}$.

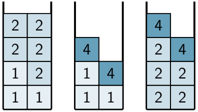
Outline

Subset-Row Cuts

A **Subset-Row Cut** for subset $U \subset O$ with $|U| = 3$ is given by $\sum_{b \in \Omega} \left\lfloor \frac{1}{2} \sum_{o \in U} a_{ob} \right\rfloor \lambda_b \leq 1.$

- Example: $U = \{1, 2, 4\}$

  $\rightarrow$ LHS $= 1.5 > 1$ $\lightning$



$\lambda_1 = 0.5$ $\lambda_2 = 0.5$ $\lambda_3 = 0.5$

- **Impact on pricing problem:** if batch b comprises two or more orders of U, the dual price σ_U of U has to be subtracted from the reduced cost $\rightarrow \tilde{c}_b = c_b - \sum_{o \in b} \pi_o - \sum_{U : |U \cup b| \geq 2} \sigma_U$
- **Impact on completion bounds:** negative dual prices $\sigma_U < 0 \rightarrow$ no modifications needed

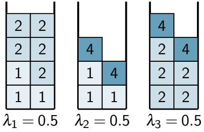
Capacity Cuts

A **Capacity Cut** for any subset $S \subset O$ can be stated as $\sum_{b \in \Omega_S} \lambda_b \geq \kappa(S)$,

where $\kappa(S)$ is the minimum number of batches needed to pick all orders in S and $\Omega_S \subseteq \Omega$ is the subset of batches comprising at least one order from S.

- Example: $Q = 8,\ S = \{1, 2, 4\}$

  $\rightarrow$ LHS $= 1.5 < 2 = \kappa(S)$ ↯



- **Impact on pricing problem:** if batch b comprises at least one order of S, the dual price ρ_S of S has to be subtracted from the reduced cost $\rightarrow \tilde{c}_b = c_b - \sum_{o \in b} \pi_o - \sum_{S : b \in \Omega_S} \rho_S$
- **Impact on completion bounds:** positive dual prices $\rho_S > 0$ have to be included whenever at least one order of S is packed $\rightarrow$ determination by solving extended binary KP

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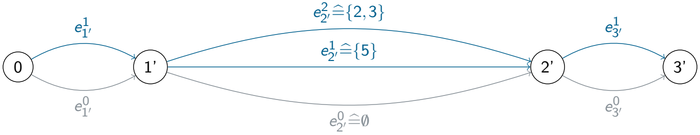
Outline

Branching Decisions

1. Branching on the **number of pickers**
2. **Ryan-and-Foster** branching
   - Branching on whether two orders $o_1, o_2 \in O$ are in the same or different batch
   - **Impact on pricing problem**: decide on groups of orders simultaneously
   - Example: orders 2 and 3 in same batch, and orders 2 and 5 in different batches:



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BPC-Based Heuristics

Observation: The LP-relaxation of the master program can be solved very quickly.

Idea

Solve (extended) root node with CG and apply heuristic procedure to obtain integer solutions.

Set-covering heuristic

- Solve RMP as MIP considering only columns generated up to the (extended) root node
- No branching required

Depth-first heuristic

- Change node selection in BPC procedure to depth-first
- Hard time limit guarantees a quick termination

Outline

Computational Setup

Pricing

- Orders are sorted non-increasingly by relative profit $\frac{\pi_o}{q_o}$ in the linear graph
- Heuristic pricing
  - Any feasible partial path defines a feasible batch
  - Stop labeling when there are k negative reduced-cost labels at a vertex

Cut separation

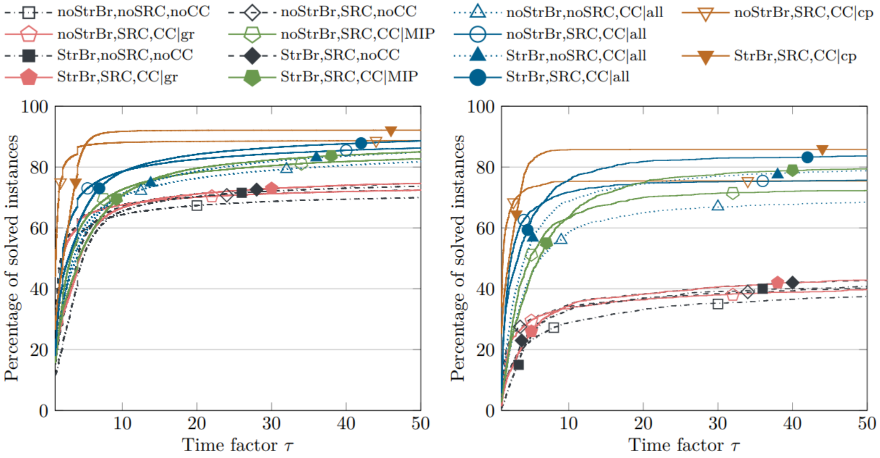
- Subset-row cuts: by enumeration
- Capacity cuts: connected-component-based heuristic
  - Sequentially remove subsets of orders from a candidate set
  - Seems to predominantly identify violated cuts with large sets

Strong branching

- Candidate set of order pairs $o_1, o_2 \in b$ in a batch with value λ_b closest to 0.5
- Rough evaluation of the two child nodes by solving only the corresponding RMP

Evaluation of Algorithmic Components

- Performance profiles for Henn and Wäscher (2012) instances (left) and Muter and Öncan (2015) instances (right)



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Results of Exact BPC

Benchmark of Muter and Öncan (2015):

- $Q \in \{24, 36, 48\}$, $n \in \{20, 30, \ldots, 100\}$, $q_o \in \{2, \ldots, 10\}$
- 10 instances per class
- Warehouse layout: 10 aisles with 2×10 storage locations each

Comparison of **number of optimal solutions** and **time** (time limit `M&Ö`: 1+1 h, `own`: 1 h):

		Traversal				Return				Midpoint			
		M&Ö		own		M&Ö		own		M&Ö		own	
Q	inst	opt	$t[s]$	opt	$t[s]$	opt	$t[s]$	opt	$t[s]$	opt	$t[s]$	opt	$t[s]$
24	90	42	1970.7	88	228.5	50	1660.0	90	118.5	56	1441.5	89	102.1
36	90	10	3202.4	72	1053.7	17	2934.0	83	769.5	14	3047.0	88	531.3
48	90	3	3481.7	48	1923.4	4	3443.7	64	1349.9	5	3407.4	67	1287.9
Total	270	55	2884.9	**208**	**1068.5**	71	2679.2	**237**	**746.0**	75	2632.0	**244**	**640.4**

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Results of BPC-Based Heuristics

Large-scale benchmark of Žulj *et al.* (2018):

- (Q, n)-pairs: $(6, 200), (6, 300), ..., (6, 600), (9, 200), (12, 200), (15, 200)$, $q_o \in \{1, \ldots, 5\}$
- 10 instances per class
- Warehouse layout: 10 aisles with 2×45 storage locations each

Comparison of **gap** and **time** for traversal strategy (time limit BPC-based heuristics: 2 min):

		Žulj		Set-Covering		Depth-First	
Q	n	gap	$t[s]$	gap	$t[s]$	gap	$t[s]$
6	200	0.99	221.2	0.05	2.3	0.08	108.1
	300	1.18	747.9	0.03	8.0	0.11	108.1
	400	1.60	1737.9	0.03	18.6	0.23	109.8
	500	1.84	3388.3	0.02	40.2	0.48	120.0
	600	1.91	5616.7	0.02	53.7	1.04	120.0
9	200	2.45	248.6	0.16	28.3	1.48	120.0
12	200	4.49	289.1	1.49	119.6	1.73	120.0
15	200	6.01	313.2	3.41	119.6	3.68	120.0
Total		2.56	1570.4	**0.65**	**48.8**	1.10	115.8

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Summary and Outlook

Summary:

- Full-fledged BPC algorithm and two BPC-based heuristics for the standard OBP and routing strategies return, traversal, midpoint, largest gap, combined and optimal
- Promising computational results:
  - BPC significantly outperforms state-of-the-art exact approach
  - BPC-based heuristics significantly outperform state-of-the-art heuristic approach

Outlook:

- Consider
  - different warehouse layouts including additional aspects or routing strategies
  - integrated optimization problems such as the joint planning of order batching, picker routing, and sequencing
  - other problems with similar structural characteristics
- Focus on techniques to raise the dual bounds and to identify high-quality solutions more effectively within the BPC

Thank you!

Julia Wahlen, Timo Gschwind
RPTU Kaiserslautern-Landau

Mail: {julia.wahlen,gschwind}@rptu.de

References

Bahçeci, U. and Öncan, T. (2021). An evaluation of several combinations of routing and storage location assignment policies for the order batching problem. *International Journal of Production Research*, **0**(0), 1–20.

Gademann, N. and van de Velde, S. (2005). Order batching to minimize total travel time in a parallel-aisle warehouse. *IIE Transactions*, **37**(1), 63–75.

Henn, S. and Wäscher, G. (2012). Tabu search heuristics for the order batching problem in manual order picking systems. *European Journal of Operational Research*, **222**(3), 484–494.

Muter, İ.. and Öncan, T. (2015). An exact solution approach for the order batching problem. *IIE Transactions*, **47**(7), 728–738.

Öncan, T. (2015). MILP formulations and an iterated local search algorithm with tabu thresholding for the order batching problem. *European Journal of Operational Research*, **243**(1), 142–155.

Valle, C. A., Beasley, J. E., and da Cunha, A. S. (2016). Modelling and solving the joint order batching and picker routing problem in inventories. In *Lecture Notes in Computer Science*, pages 81–97. Springer International Publishing.

Valle, C. A., Beasley, J. E., and Salles da Cunha, A. (2017). Optimally solving the joint order batching and picker routing problem. *European Journal of Operational Research*, **262**(3), 817–834.

Žulj, I., Kramer, S., and Schneider, M. (2018). A hybrid of adaptive large neighborhood search and tabu search for the order-batching problem. *European Journal of Operational Research*, **264**(2), 653–664.

Results of BPC for H&W

Benchmark of Henn and Wäscher (2012):

- $Q \in \{30, 45, 60, 75\}$, $n \in \{20, 30, \ldots, 100\}$, $q_o \in \{5, \ldots, 25\}$
- 40 instances per class
- Warehouse layout: 10 aisles, 45 pick locations on both sides each
- Two scenarios: class-based demands / uniformly distributed demands

Comparison of number of optimal solutions and time (time limit: 1 h):

		Traversal		Return		Midpoint		Largest gap		Combined		Optimal	
Q	inst	opt	$t[s]$	opt	$t[s]$	opt	$t[s]$	opt	$t[s]$	opt	$t[s]$	opt	$t[s]$
30	1440	1420	55.6	1440	0.1	1440	0.1	1440	0.1	1438	5.2	1440	1.6
45	1440	1352	307.8	1419	98.0	1438	20.7	1434	39.3	1419	104.1	1424	78.9
60	1440	1020	1203.4	1356	380.8	1384	276.2	1374	348.2	1355	428.5	1346	456.0
75	1440	720	1960.2	1247	820.8	1262	732.6	1228	848.8	1249	798.0	1204	923.9
Total	5760	4512	881.8	5462	325.0	5524	257.4	5476	309.1	5461	333.9	5414	365.1

Detailed Results of BPC

- Instances of M&Ö and H&W
- Routing strategies traversal and optimal

		Traversal								Optimal							
Q	inst	opt	$t[s]$	t^{LP}	gap	gap^{RF}	nodes	CC	SRC	opt	$t[s]$	t^{LP}	gap	gap^{RF}	nodes	CC	SRC
M&Ö instances																	
24	90	88	228.5	0.2	0.98	0.31	2346	21	42	88	131.3	0.3	0.80	0.23	1070	12	36
36	90	72	1053.7	1.2	2.12	0.54	2633	41	91	79	802.0	2.7	1.74	0.40	1716	38	69
48	90	48	1923.4	5.3	2.98	0.68	1750	55	113	65	1451.2	16.6	2.74	0.66	558	67	99
Total	270	208	1068.5	2.2	2.03	0.51	2243	39	82	232	794.8	6.5	1.76	0.43	1114	39	68
H&W instances																	
30	1440	1420	55.6	0.0	0.30	0.03	533	8	1	1440	1.6	0.0	0.22	0.01	45	2	1
45	1440	1352	307.8	0.1	0.57	0.23	7103	21	27	1424	78.9	0.2	0.40	0.16	1396	10	21
60	1440	1020	1203.4	0.4	0.96	0.34	20562	21	54	1346	456.0	0.7	0.77	0.29	3147	22	40
75	1440	720	1960.2	1.0	1.29	0.36	21534	28	80	1204	923.9	2.1	1.19	0.38	2569	37	57
Total	5760	4512	881.8	0.4	0.78	0.24	12433	19	41	5414	365.1	0.7	0.65	0.21	1789	18	30

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Heuristics Time Limits

- Hard time limits of 2, 3, and 5 minutes indicated with corresponding suffix
- Aggregated summary by benchmark set of `M&Ö`, `H&W` and `Žulj`

	Set-covering						Depth-first					
	SC-2		SC-3		SC-5		DF-2		DF-3		DF-5	
class	gap	$t[s]$	gap	$t[s]$	gap	$t[s]$	gap	$t[s]$	gap	$t[s]$	gap	$t[s]$
`M&Ö`	0.96	41.8	0.83	56.5	0.70	82.0	0.43	60.7	0.29	85.7	0.17	132.6
`H&W`	0.17	16.4	0.15	20.6	0.13	26.8	0.08	34.8	0.07	48.6	0.05	73.9
`Žulj`	0.48	52.1	0.41	68.9	0.36	98.6	0.84	118.1	0.63	177.0	0.51	294.7

Comparison of Routing Strategies

- Percentage increase in traveled distance compared to optimal routing

Q	Traversal	Return	Midpoint	Largest gap	Combined
M&Ö instances					
24	10.4%	32.8%	9.9%	5.8%	3.7%
36	7.1%	34.9%	13.5%	8.2%	2.5%
48	5.3%	36.5%	17.1%	10.8%	1.8%
Total	7.6%	34.7%	13.4%	8.2%	2.7%
H&W UDD instances					
30	17.4%	52.7%	15.4%	8.9%	7.2%
45	10.0%	53.9%	20.5%	12.5%	4.2%
60	7.5%	55.4%	24.6%	15.9%	2.9%
75	6.3%	56.9%	28.1%	19.0%	2.2%
Total	10.2%	54.7%	22.1%	14.0%	4.1%
H&W CBD instances					
30	19.2%	52.2%	9.4%	5.4%	8.7%
45	12.0%	52.4%	12.4%	7.2%	5.7%
60	9.1%	53.2%	15.2%	9.3%	4.1%
75	7.5%	54.1%	18.0%	11.3%	3.2%
Total	11.8%	53.0%	13.7%	8.3%	5.4%