Università
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Dantzig-Wolfe: from data-driven decomposition to parallel resolution

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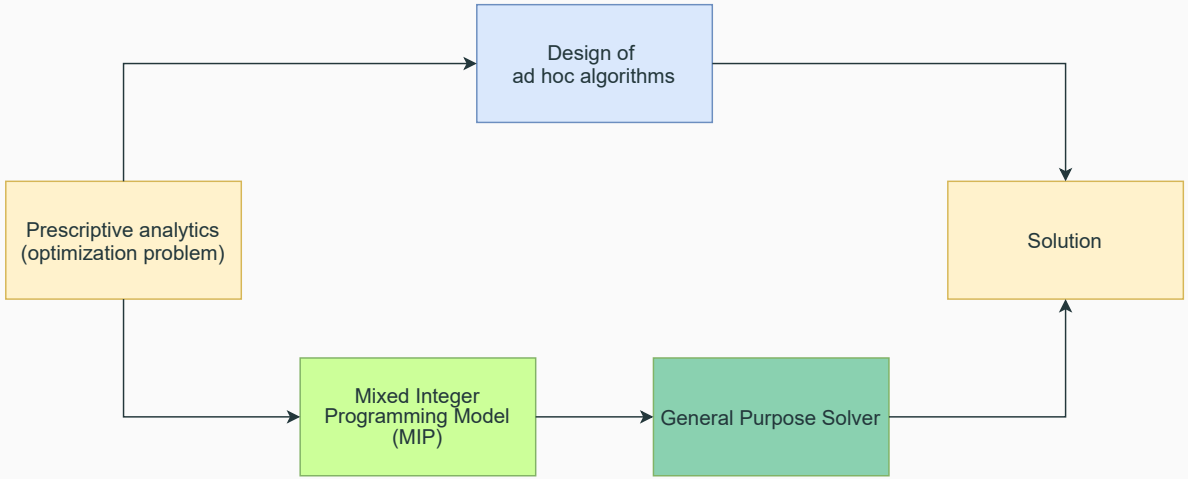
Column Generation Workshop 2023

Solving Prescriptive Analytics Optimization Problems



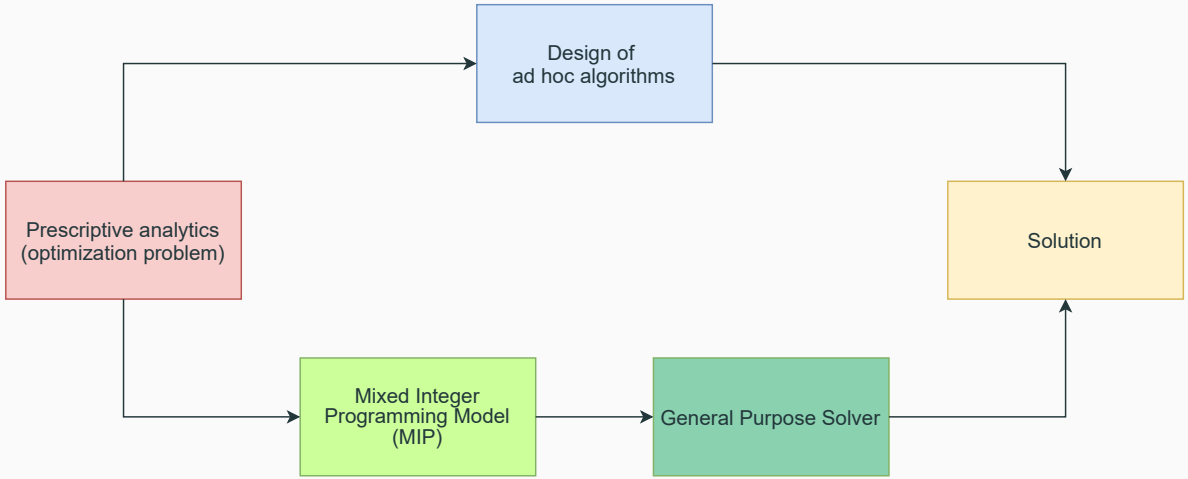
5	10.4	17.2	12	18.3	58	16	11

1	0	0	0	0	0	0	0	1
0	23.3	0	15	17	102	51	0	237
0	5.6	0	0	0	0	0	0	12.5
0	0	0	0	0	6.3	4.3	2.3	12.5
0	0	6.2	0	6.2	0	0	0	12.5
0	0	1	1	0	0	0	0	1
0	0	0	0	0	-1	1	0	1
7.4	0	0	3.1	0	-2.5	0	5.3	15

Use **branch-and-cut**

- problem complexity issues
- data size scalability issues

Solving Prescriptive Analytics Optimization Problems



5	10.4	17.2	12	18.3	58	16	11

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0	23.3	0	15	17	102	51	0	237
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0	0	0	0	0	6.3	4.3	2.3	12.5
0	0	6.2	0	6.2	0	0	0	12.5
0	0	1	1	0	0	0	0	1
0	0	0	0	0	-1	1	0	1
7.4	0	0	3.1	0	-2.5	0	5.3	15

1	0	0	0	0	0	0	0
0	5.6	0	0	0	0	0	0
0	0	6.2	0	6.2	0	0	0
0	0	1	1	0	0	0	0
0	0	0	0	0	-1	1	0
0	0	0	0	0	6.3	4.3	2.3
7.4	0	0	3.1	0	-2.5	0	5.3
0	23.3	0	15	17	102	51	0

1. to detect block structures by data driven decomposition methods
2. to exploit massive parallelism for optimization of large scale instances

Part I: Automatic Decomposition of MIPs

Looking for decomposition schemes:

- on the problem formulation (Frangioni et al, 2010 -)
- on the MIP instance (Wang Ralphs, 2013)

Detectors search suitable decomposition patterns of a given MIP instance

- by looking for previously known structures
- by exploiting *static* properties of that instance (Bergner Caprara Ceselli Furini Luebbecke Malaguti Traversi, 2015)
- by *communality* detection (Khaniyev Elhedhli Erenay, 2018)

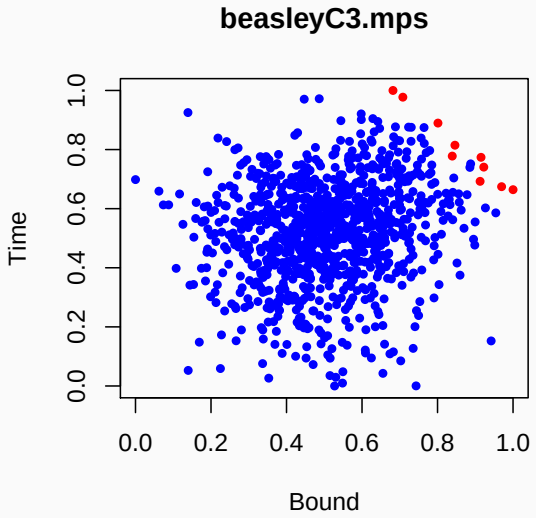
Generic branch-and-price-and-cut solvers:

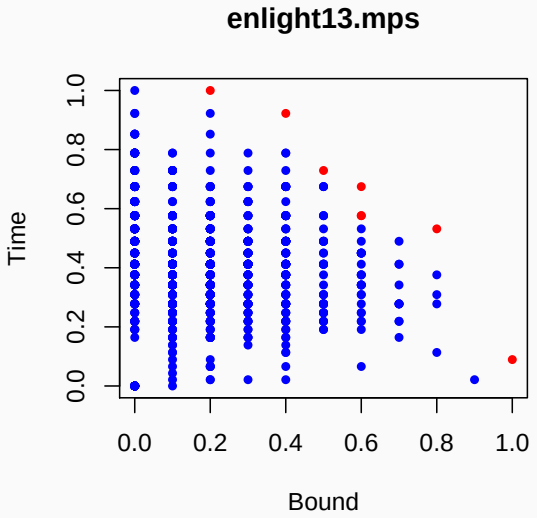
- GCG (Gamrath Luebbecke, 2010 -)
- DIP (Ralphs Galati, 2017 -)
- BapCod (Vanderbeck, 2017 -)
- Coluna.jl (Marquez Nesello Vanderbeck Pessoa Sadykov, 2023)

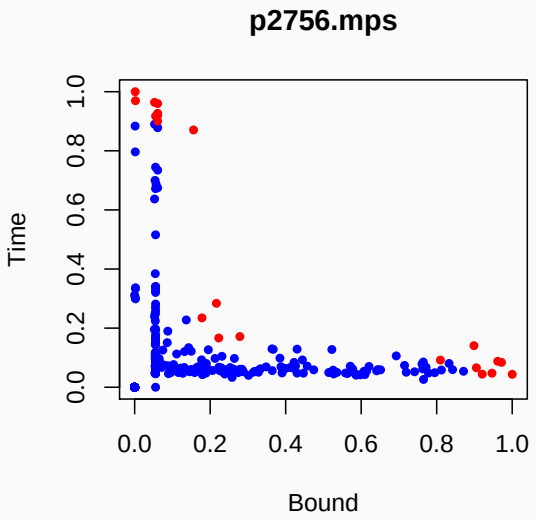
Machine learning to choose a decomposition algorithm (Kruber Luebbecke Parmentier, 2017)

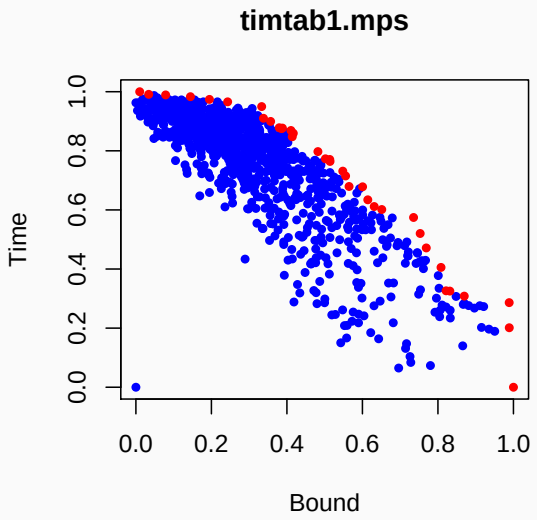
Promising Decompositions

Good decompositions: low running time, tight dual bound (Post-process)









Preliminary investigation

Research target
Investigate the link between static properties of MIP base instances and good decomposition patterns

Dataset A generation:

- 36 MIPLIB instances, generated 1000 random decompositions each
- Computed (117) static features (indep. on instance size)
- Optimized every decomposition, recorded time and bound

Experimental analysis of new decompositions from

- unknown decomp of *known* MIPs: base MIPs are part of Dataset A
- unknown decomp of *unknown* MIPs: new, unseen MIP instances

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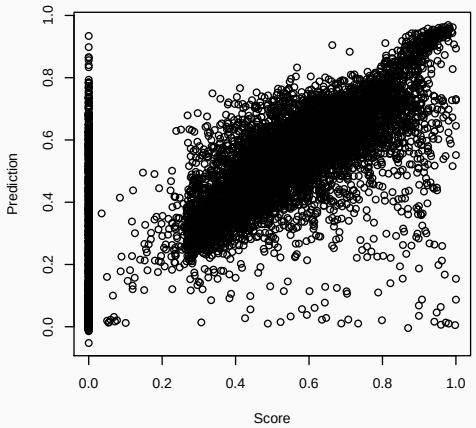
S. Basso, A. Ceselli, A. Tettamanzi "Random Sampling and Machine Learning to Understand Good Decompositions", Annals of Operations Research 284 (2018)

Mixing features: Regression for unknown problems

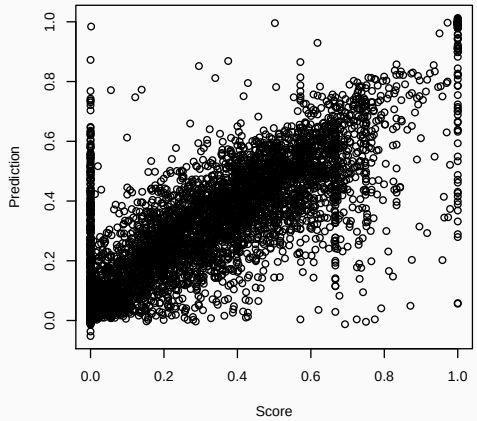
Independent Time and Bound regressors (XGboost). Preliminary results:

Known MIPs

Time

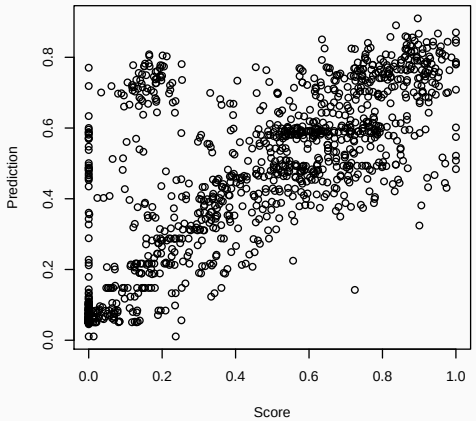


Bound

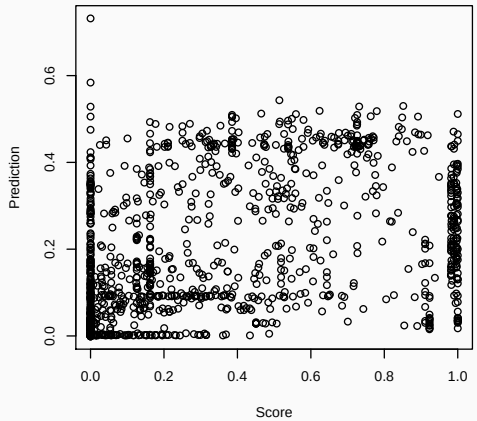


Unknown MIPs

Time



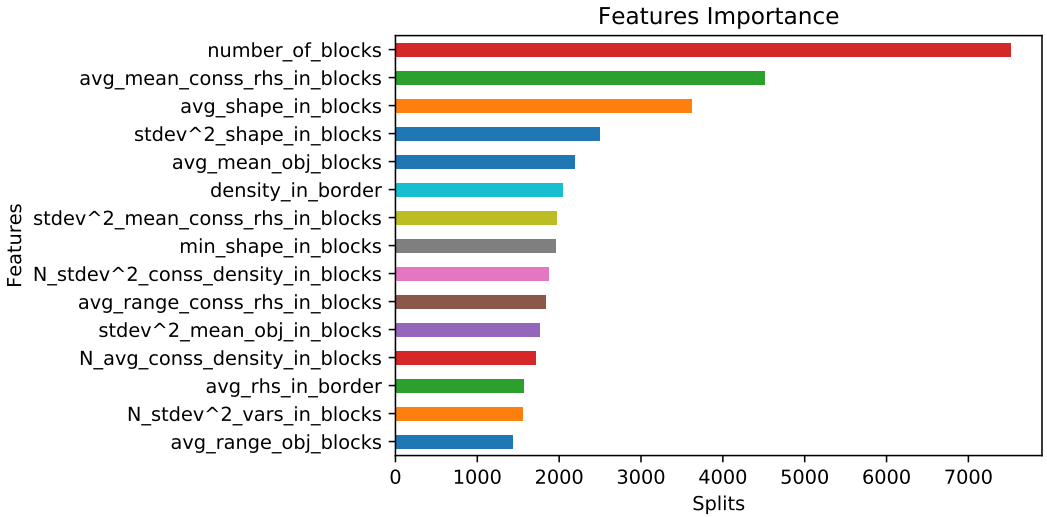
Bound

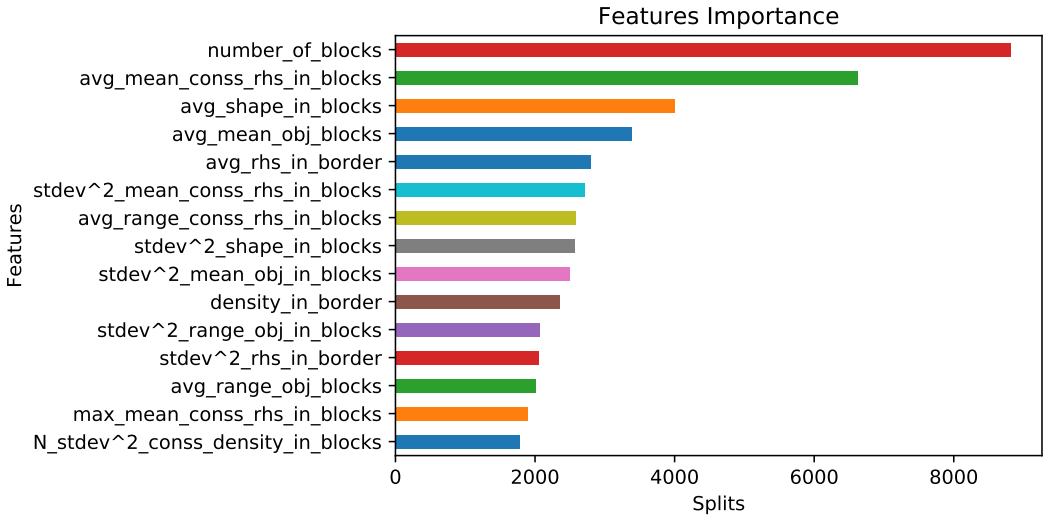


Time prediction always possible, Bound prediction out of reach

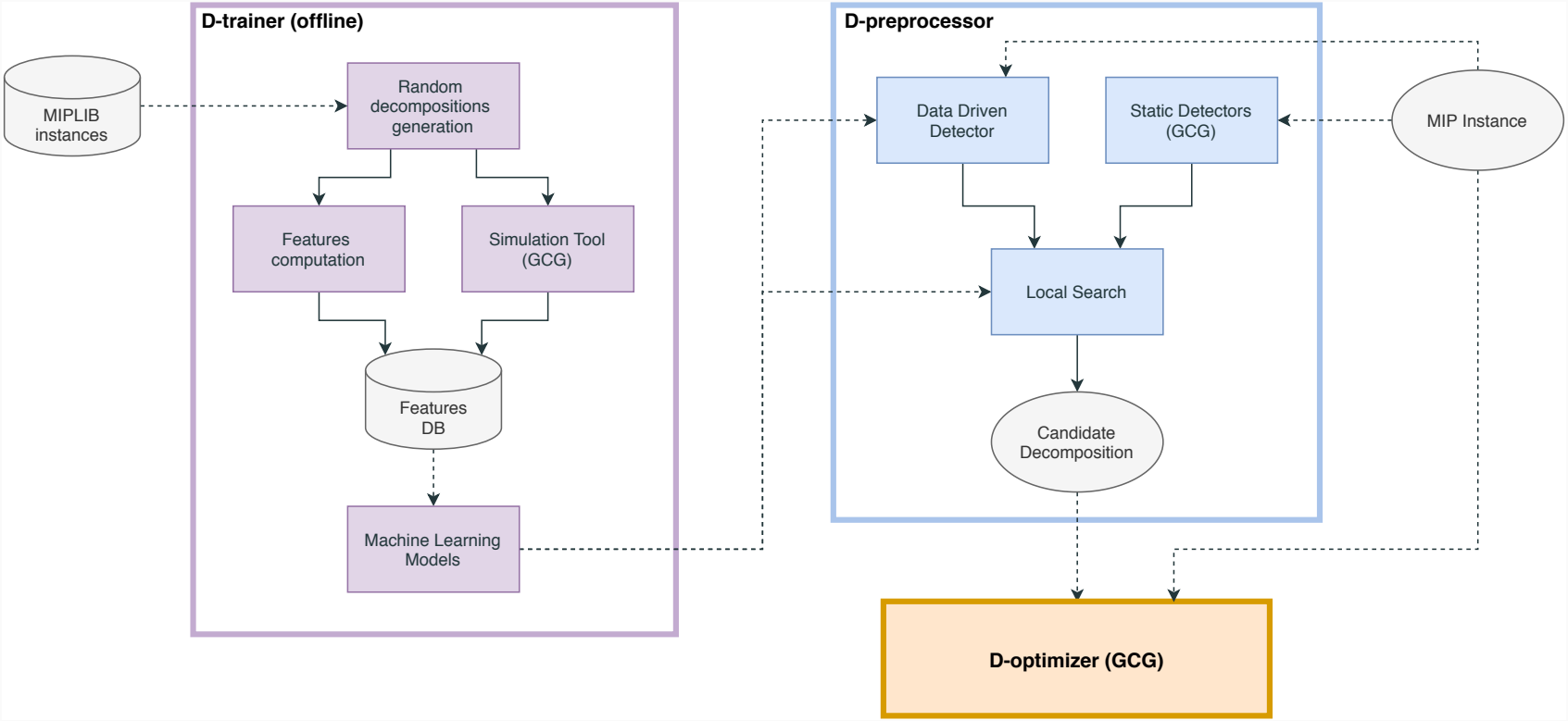
Feature importance

Time (top) and bound (bottom) regressors most important features:





A Data Driven Detection Framework

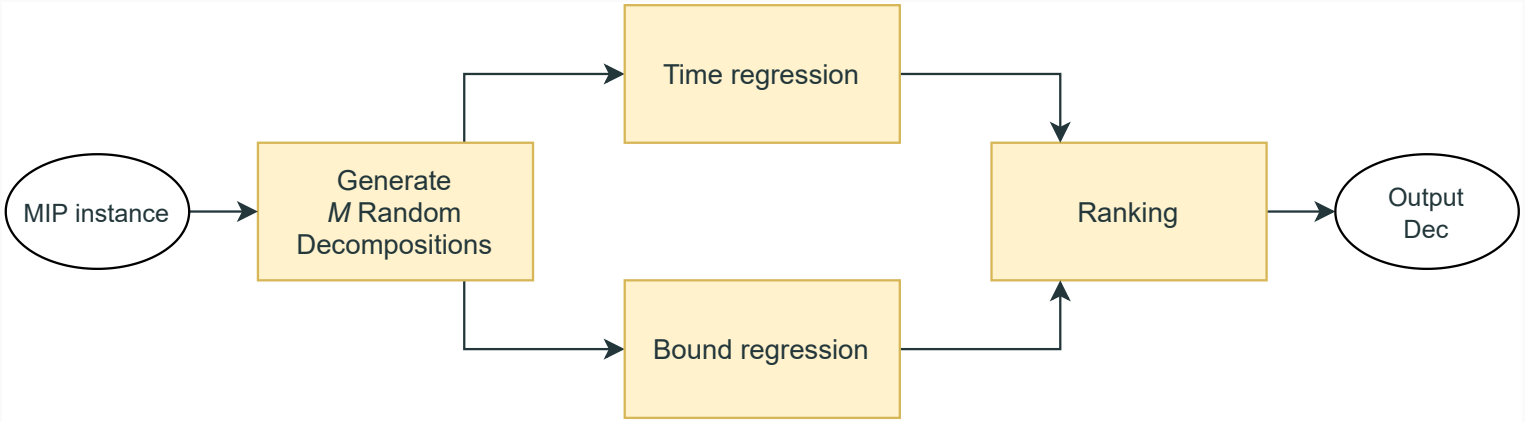


Three main components:

- D-trainer (Dataset A, xgboost regressors for Time and Bound)
- D-preprocessor
- D-optimizer (GCG)

A Data Driven Detector

Research target
Given a MIP instance, generate a suitable decomposition pattern



Ranking function D(i): Percentage of decompositions dominated by decomposition i

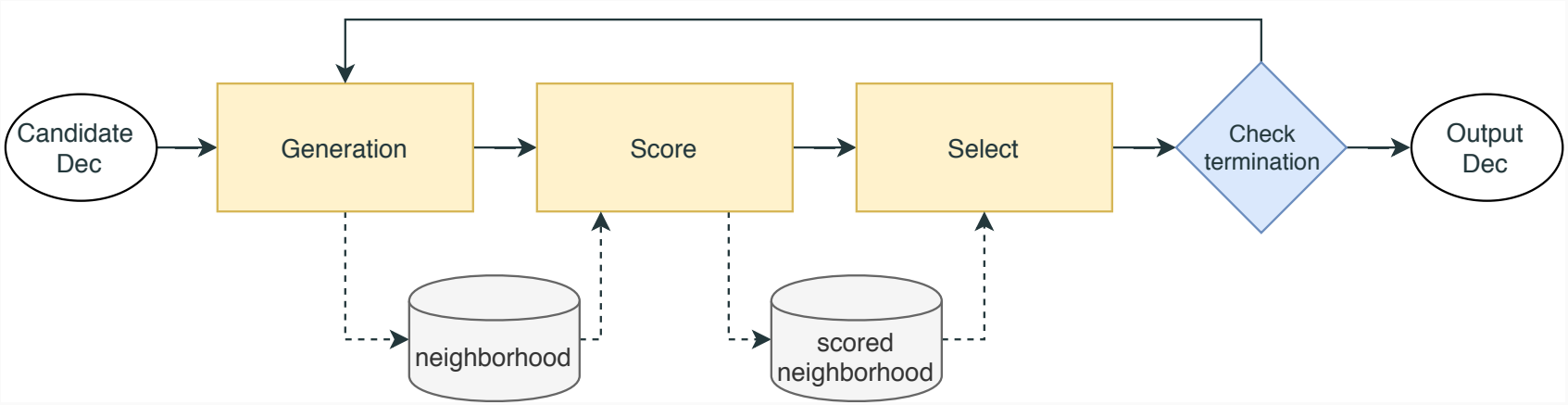
Given decompositions i and j, $i \to j$:

$$Time(i) \geq Time(j) \wedge Bound(i) > Bound(j), \quad \text{or}$$
$$Time(i) > Time(j) \wedge Bound(i) \geq Bound(j)$$

S. Basso, A. Ceselli "Computational evaluation of ranking models in an automatic decomposition framework", Proc. of EURO/ALIO 2018 (2018)

Local Search Algorithms

Research target
Given a base MIP instance, and a decomposition for it, improve it algorithmically



Starting decomposition:

static detectors or data driven detector

Neighbourhood:

set of decompositions that differ from candidate decomposition of one constraint

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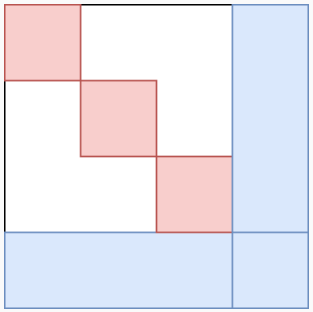
S. Basso, A. Ceselli "Computational Evaluation of Data Driven Local Search for MIP Decompositions", Proc. of ODS 2019 (2019)

Generation

Generation: Choose one constraint from the border, insert into one block



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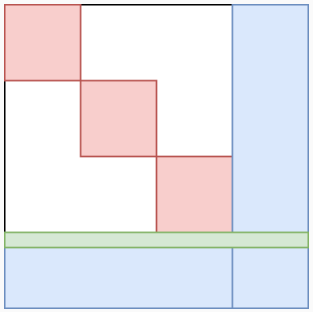

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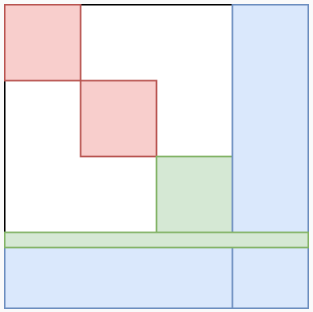

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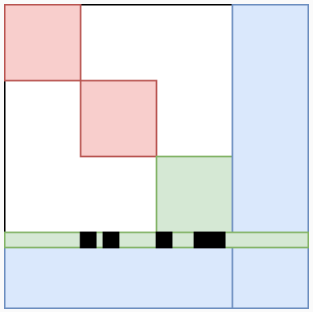

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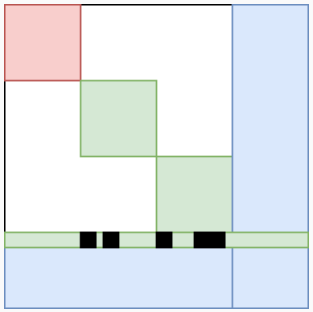

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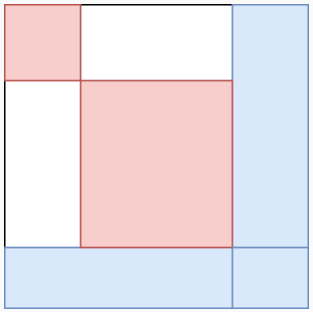

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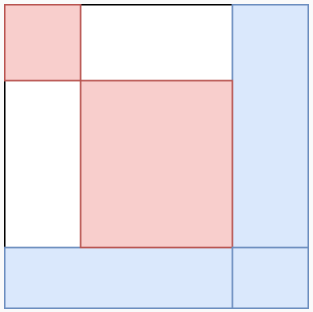


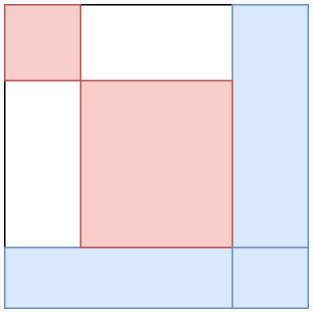
\+ bound cannot worsen

\- time prediction to avoid slow decompositions

Repeat for each constraint (border)/block combination

Generation

Generation: Choose one constraint from the border, insert into one block

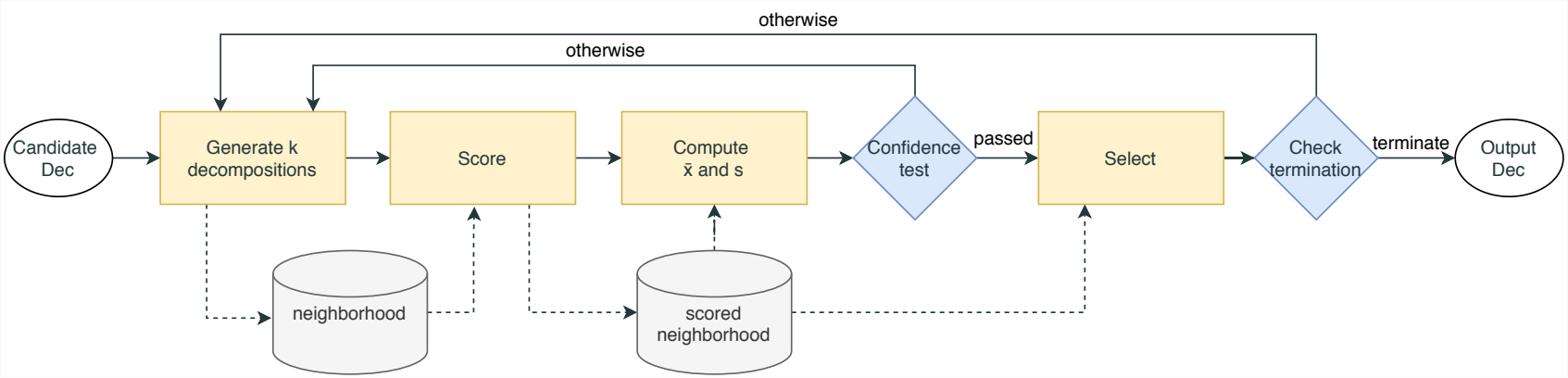


\+ bound cannot worsen

\- time prediction to avoid slow decompositions

Repeat for each constraint (border)/block combination

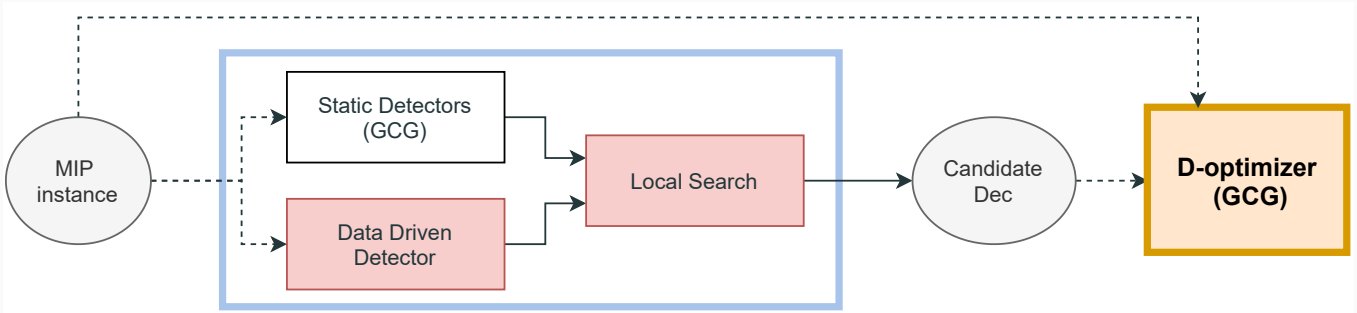
Faster computation: Sample a representative neighbourhood (**25x speedup**, little loss of quality)



Framework testing and Benchmark Configurations

Framework (D-preprocessor) configurations

D-preprocessor experimental setup



with sampling and orthogonal selection, termination: 85% convexification.

- C++11 custom library, Boost library, Python 3.6 scripts
- Intel i7-6700K CPU and 32GB RAM
- 30 unknown MIPLIB problems
- D-optimizer: GCG 3.0.1, 5 hours timelimit
- Benchmarks: GCG, SAS-Decomp

Root Node bounds comparison

DDW_{Sample}	SAS_{comm}		GCG		GCG_{Hmetis}		GCG_{Full}	
run 1	**DDW Best**	**10**	DDW Best	6	DDW Best	6	DDW Best	6
	Draw	4	**Draw**	**15**	**Draw**	**14**	**Draw**	**14**
	SAS Best	9	GCG Best	3	GCG Best	4	GCG Best	4
run 2	**DDW Best**	**10**	DDW Best	7	DDW Best	7	DDW Best	7
	Draw	5	**Draw**	**15**	**Draw**	**14**	**Draw**	**14**
	SAS Best	9	GCG Best	3	GCG Best	4	GCG Best	4
run 3	**DDW Best**	**10**	DDW Best	8	DDW Best	7	DDW Best	8
	Draw	5	**Draw**	**14**	**Draw**	**13**	**Draw**	**13**
	SAS Best	9	GCG Best	3	GCG Best	5	GCG Best	4

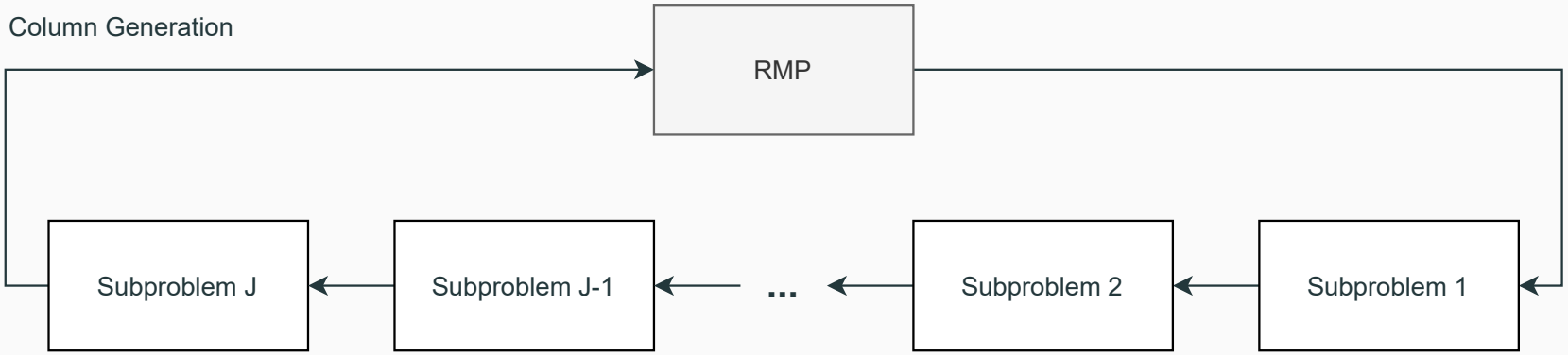
				DDW_{sample}		
	GCG	GCG_{Hmetis}	GCG_{Full}	run 1	run 2	run 3
Time [s]	905.17	416.92	847.79	344.08	347.41	420.92
Solver errors	1	1	0	6	5	5
Timeouts	14	13	14	6	5	7

SAS Decomp slightly faster, 6 timeouts

Part II: Concurrent Column Generation

Column Generation (CG)

Research target
Given a base MIP instance, and a decomposition for it, solve it as quickly as possible



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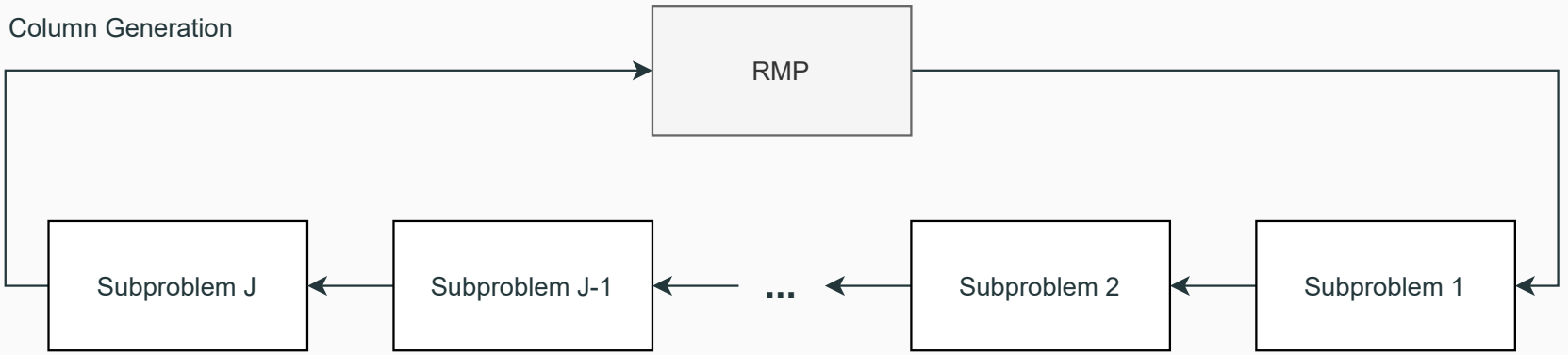

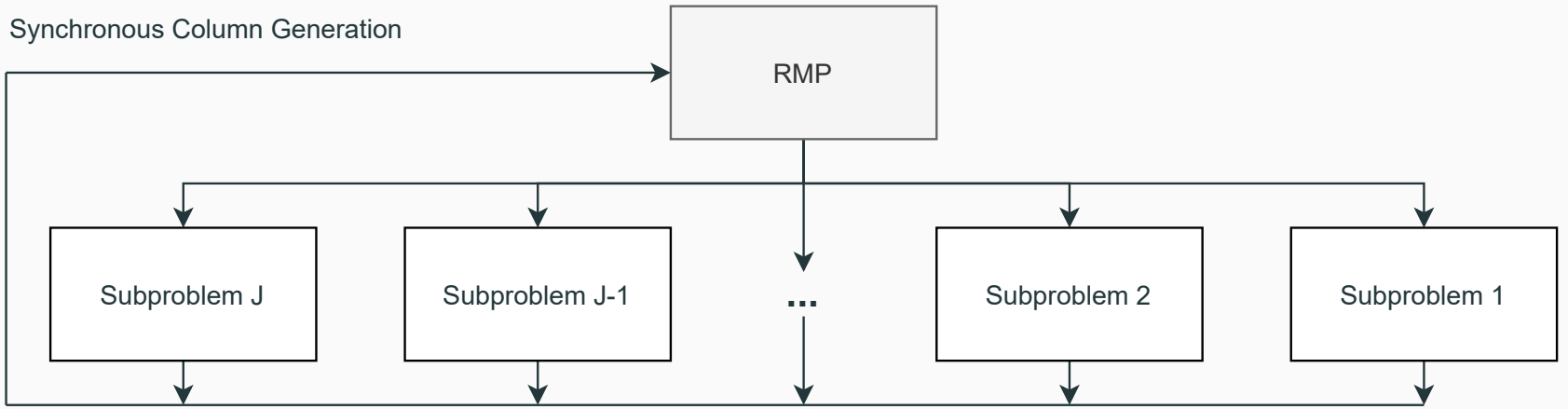

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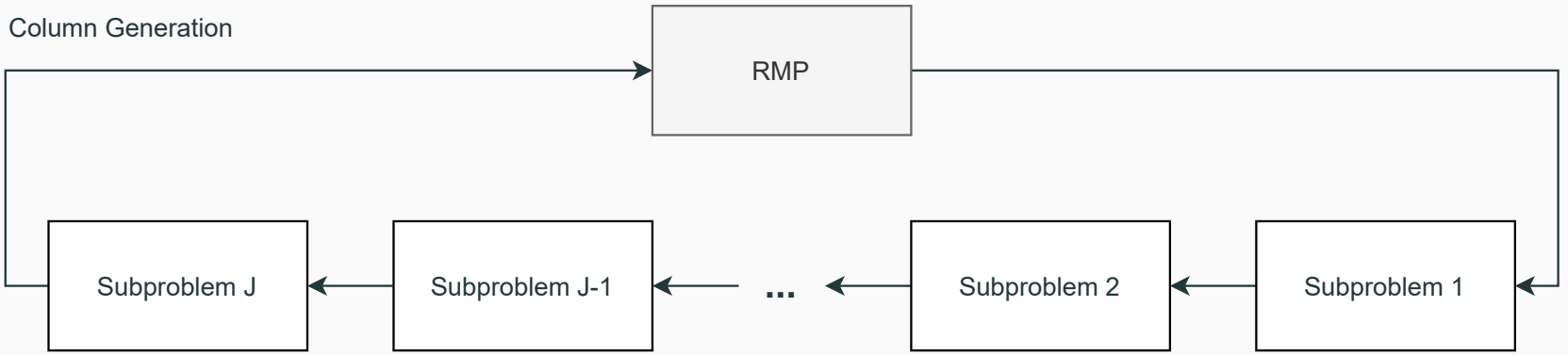

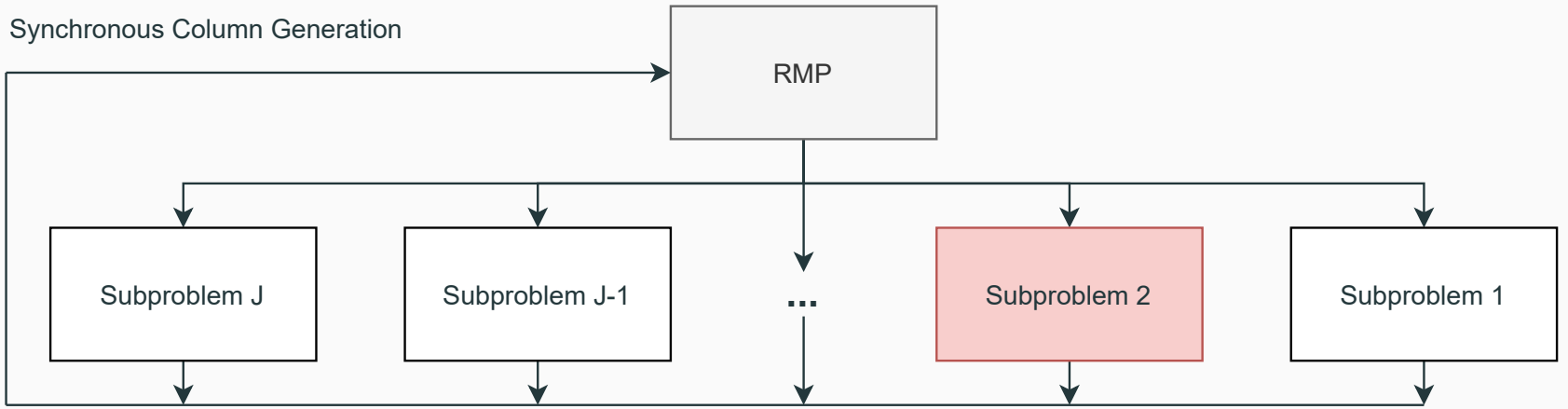

Column Generation (CG)

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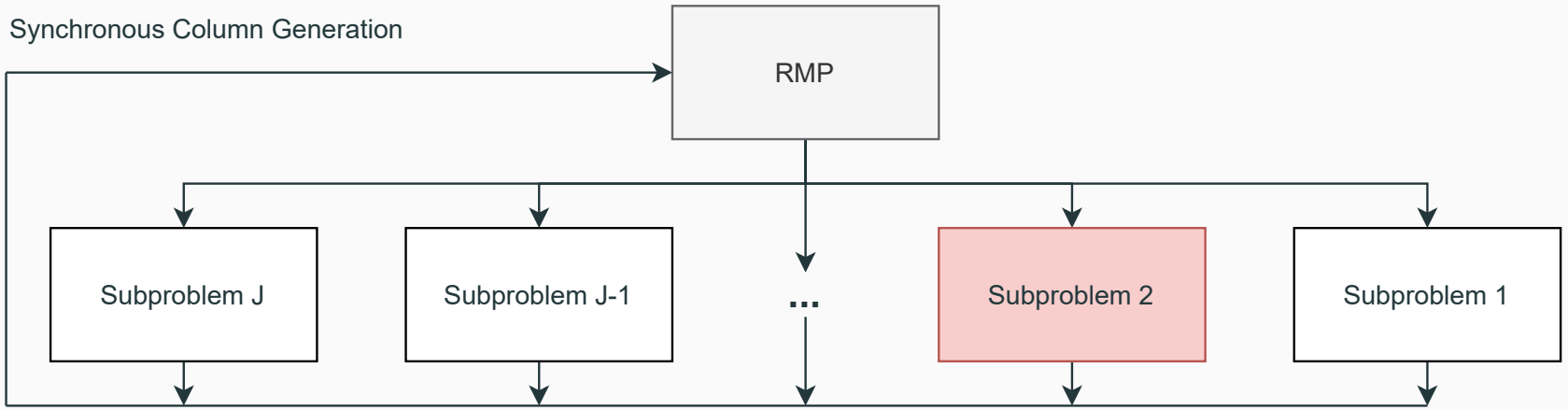

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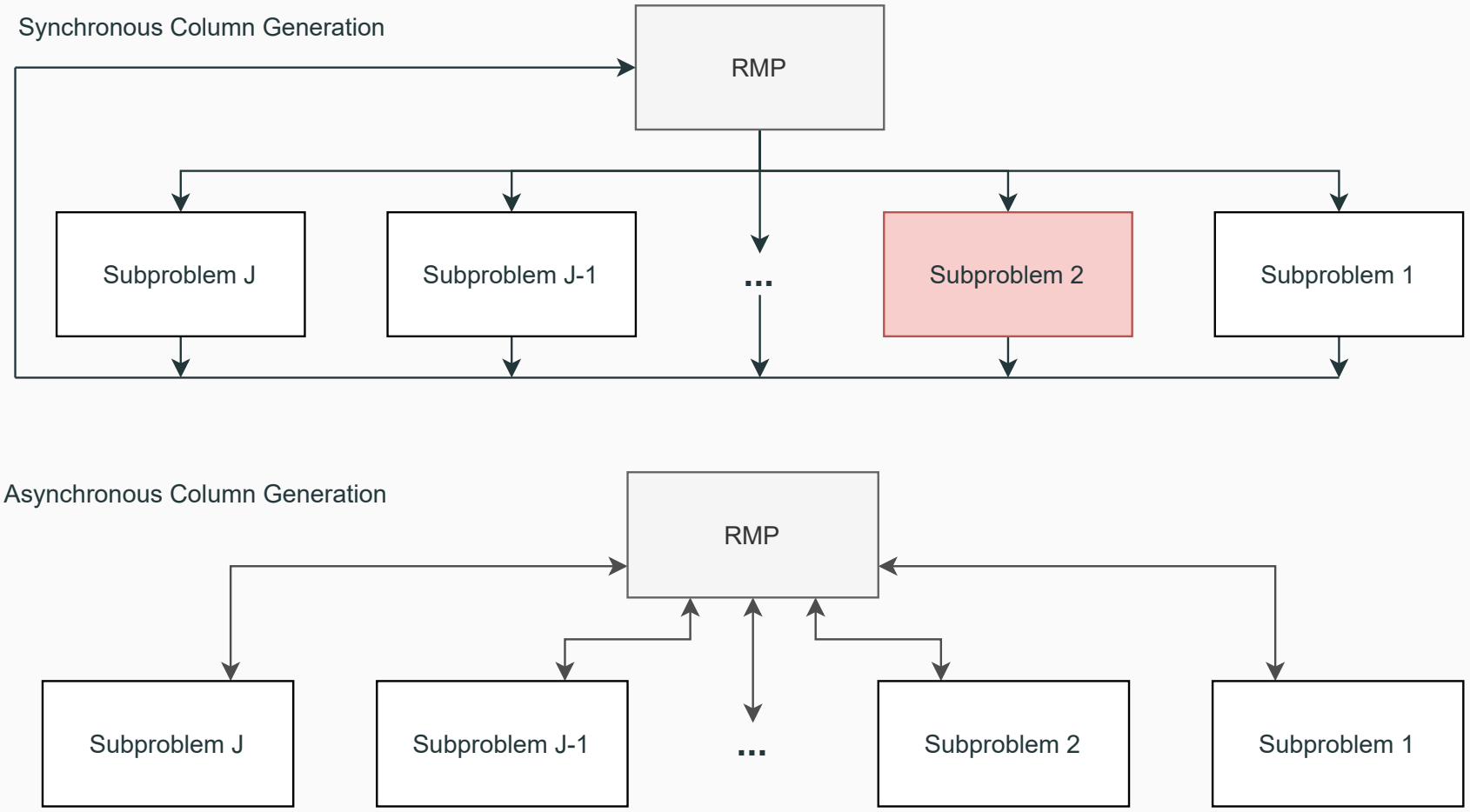
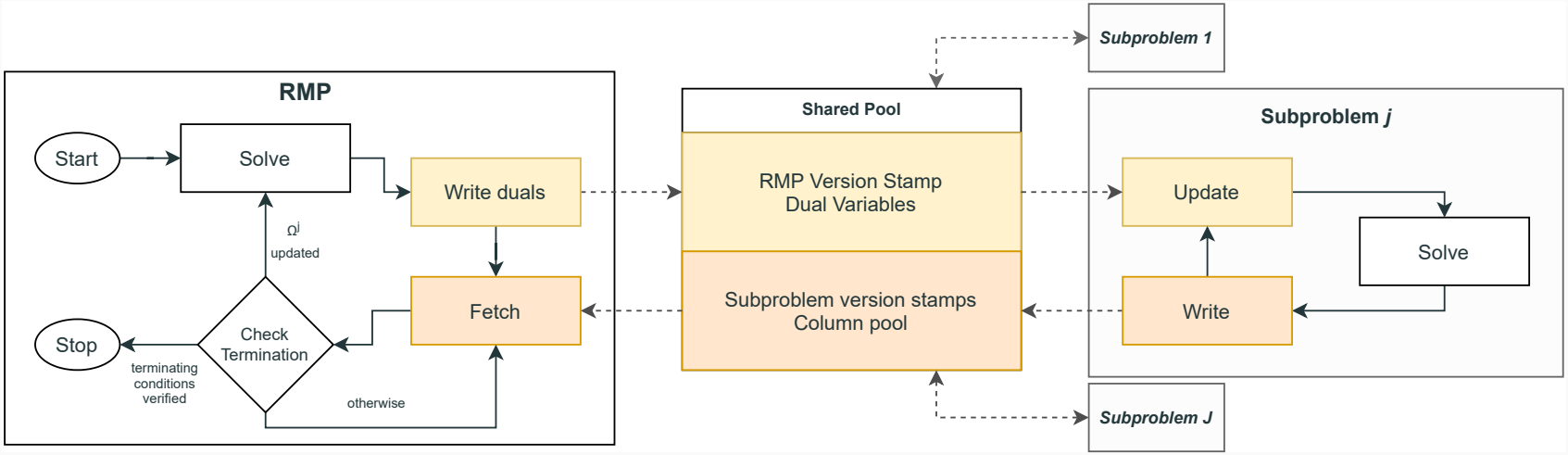


Asynchronous Column Generation (ACG)



- RMP and subproblem tasks run in parallel
- Communication through shared pool (exclusive access)
- RMP version stamps: Tasks might work with different sets of dual variables
- Generic implementation: tasks solved are solved with branch-and-cut

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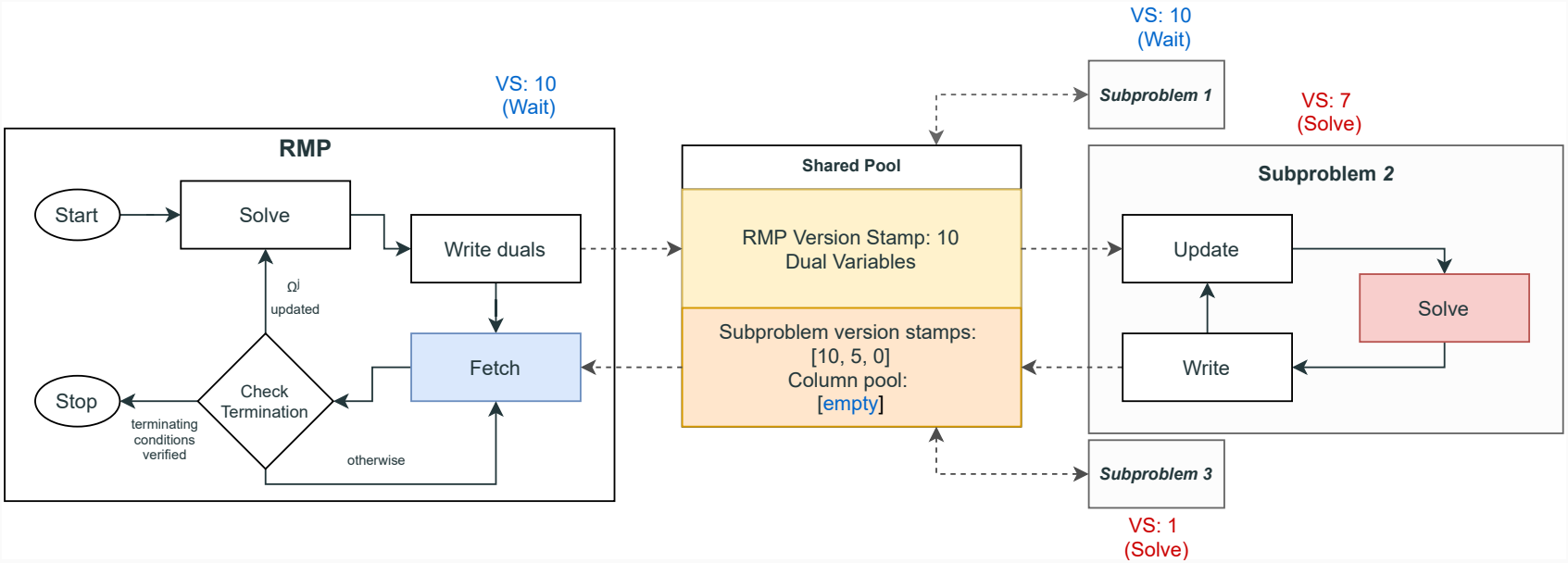
S. Basso, A. Ceselli "Asynchronous Column Generation", Proceedings of the Ninteenth Workshop on Algorithm Engineering and Experiments (ALENEX) (2017)

ACG example and terminating conditions



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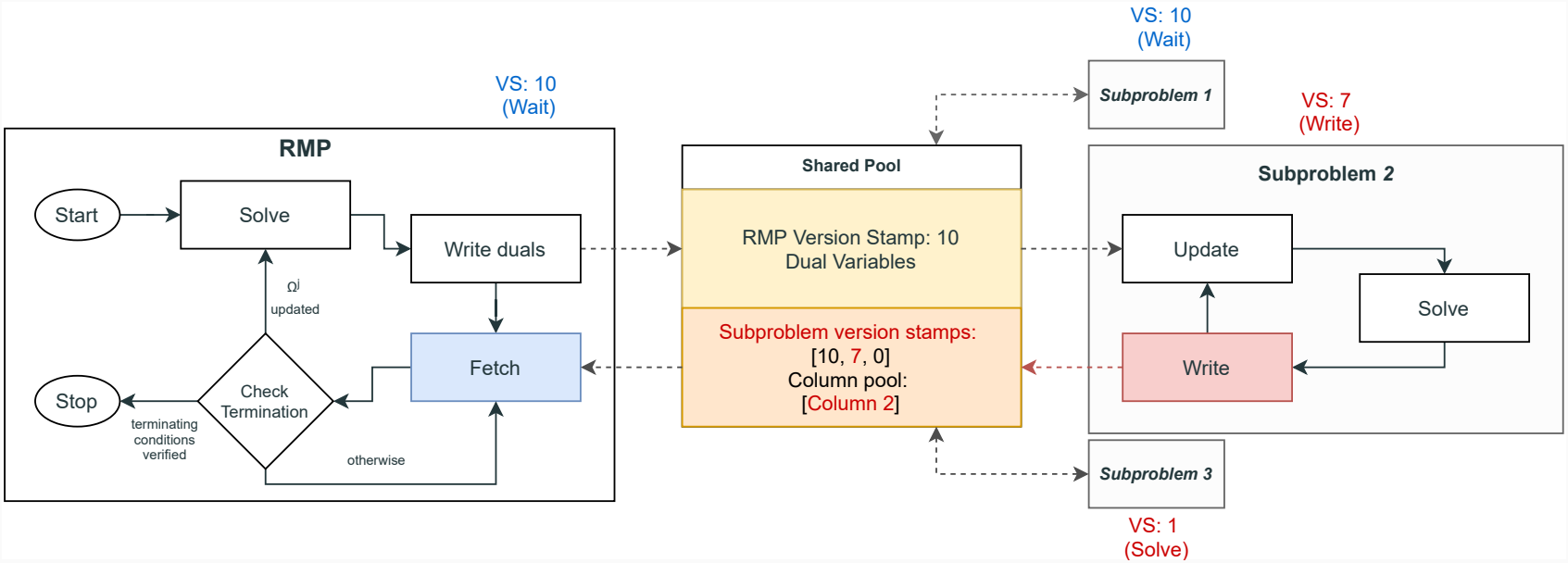

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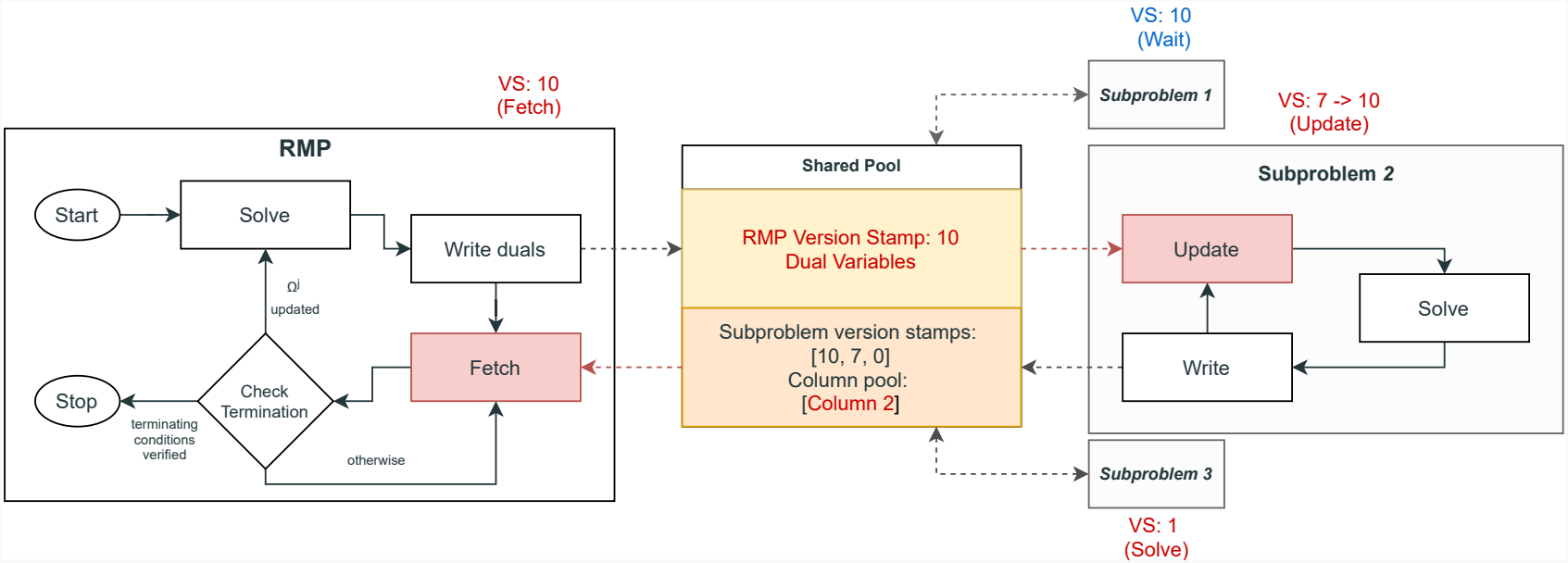

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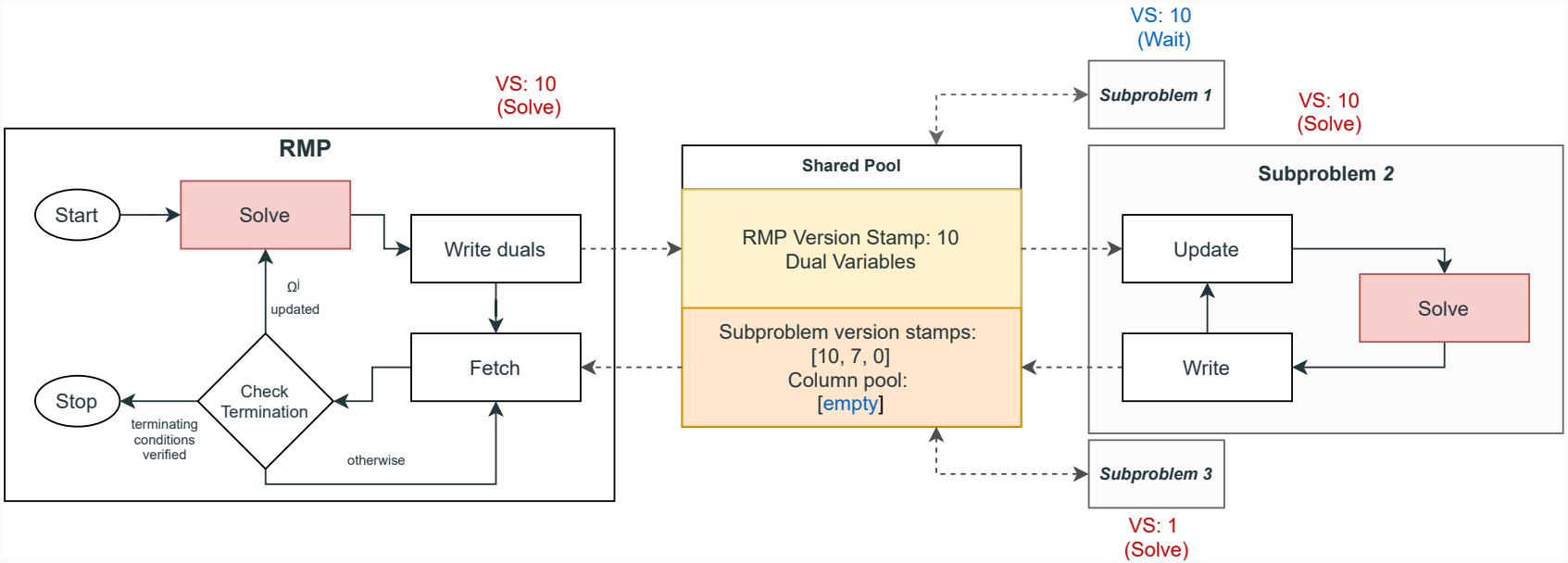
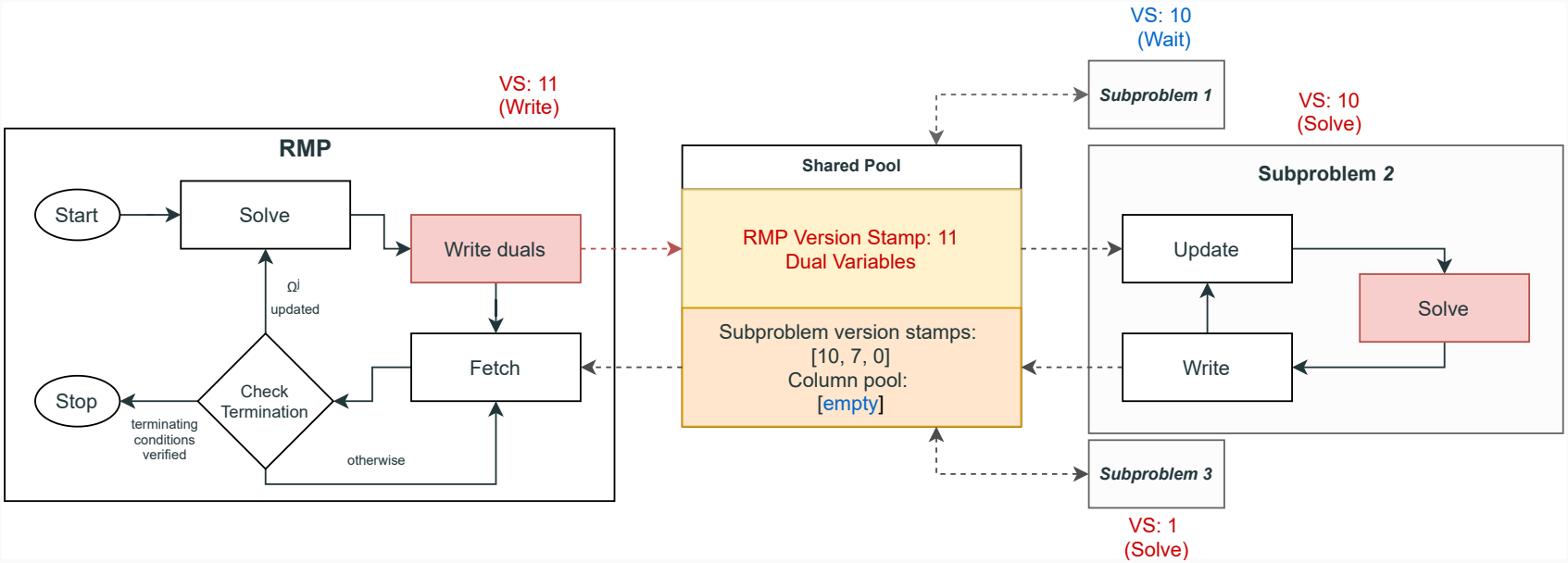
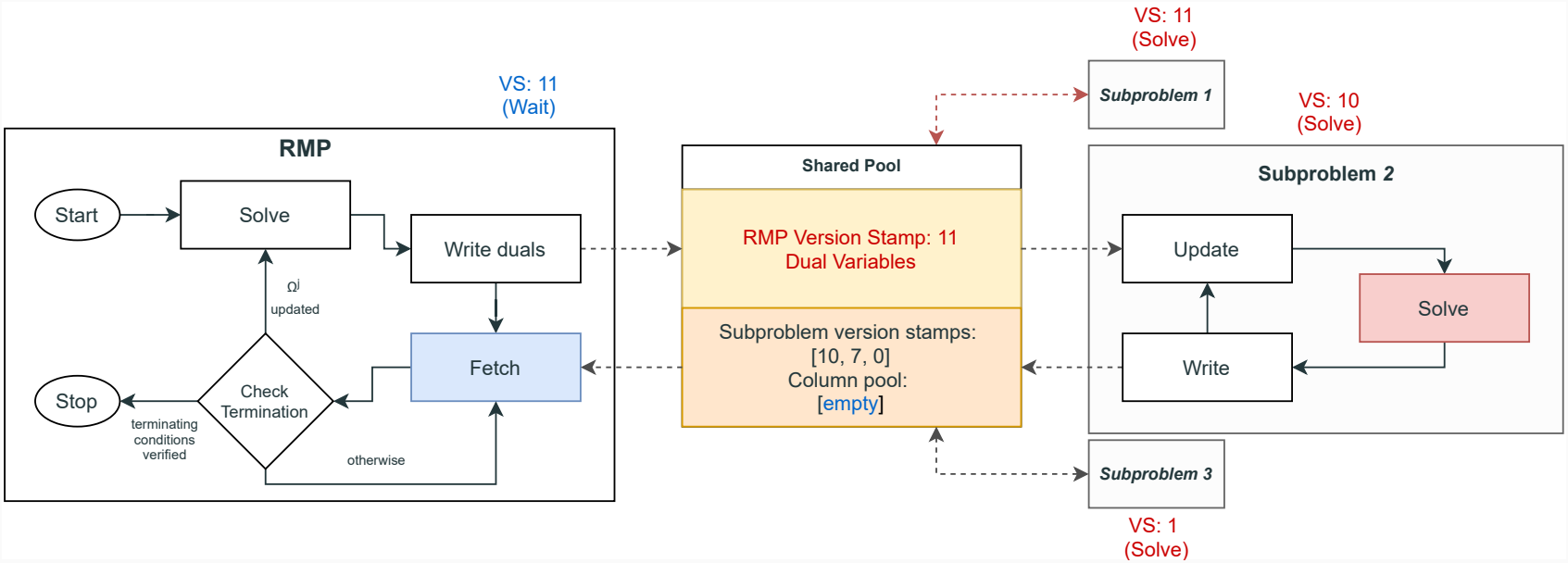


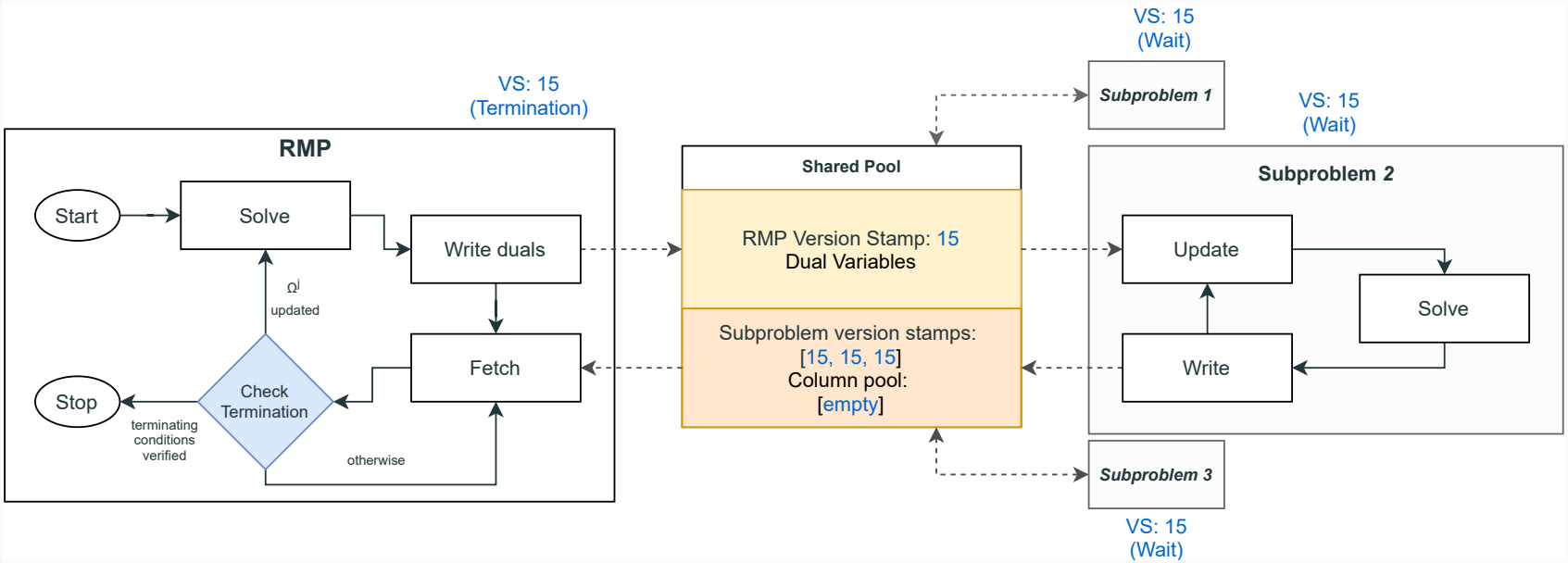
ACG example and terminating conditions



ACG example and terminating conditions



ACG example and terminating conditions

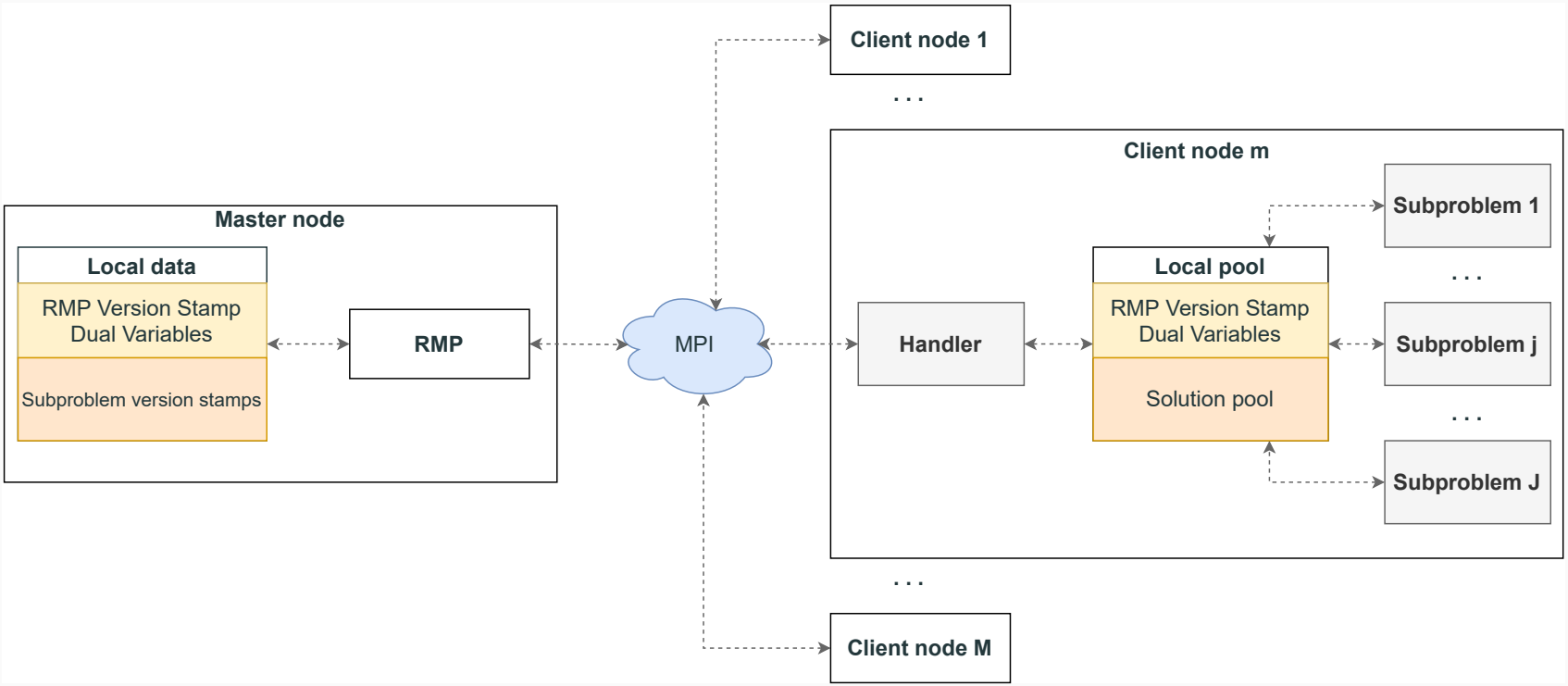


Proposition The following are sufficient and necessary conditions to guarantee convergence to the optimal solution

- each subproblem finished optimization with the latest RMP version stamp
- there are no new columns in the pool

Proof [...] □

Distributed Asynchronous Column Generation (DCG)



- RMP and Subproblem tasks run on different machines
- Communication through MPI protocol (Master-Client architecture)
- Local pool (exclusive access) on every node
- Additional handler task on every Client node

S. Basso, A. Ceselli "Distributed Asynchronous Column Generation", Computers & OR (2022)

Maximizing asynchronous computation

Experimentally, short optimization times maximize asynchronous computation:

- Dual variables are updated earlier
- RMP fetches new columns faster

Minimizing RMP optimization time:

- Rebalancing (from 10k to 20k solutions)
- RMP update policy (store only promising solutions for the current set of dual variables)

Minimizing Subproblems optimization time:

- Short timeouts during the `solve` step (from 1 to 60 seconds)

Test-bed Instances and Benchmark Algorithms

Configurations:

- CPLEX (version 12.6.3)
- Synchronous Column Generation (SCG)
- Asynchronous Column Generation (ACG)
- Distributed Asynchronous Column Generation (DCG)

Implementation: C++11 with Boost, OpenMP and OpenMPI libraries

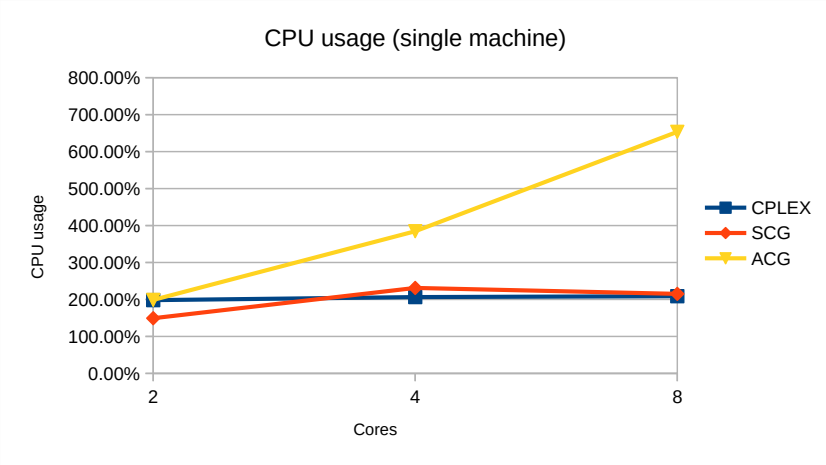
Experimental Setup: up to 4 machines with Intel i7-6700K CPU and 32GB RAM (Ubuntu 16.04)

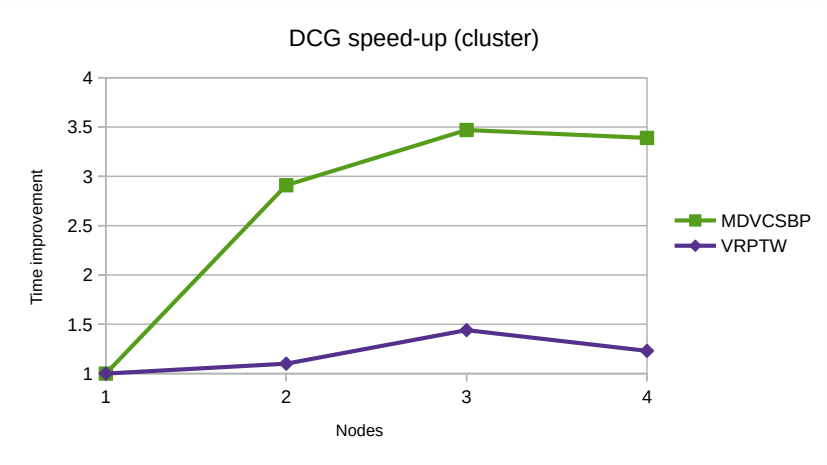
Test-bed instances (root node)

- 30 Large scale instances: Multi-dimensional Variable Size and Cost Bin-packing Problem (MDVCSBP) (250, 500, 750 subproblems)
- 21 "stress" test instances: Vehicle Routing Problem with Time Windows (VRPTW) (5 subproblems)
- 21 intermediate: Multi-Depot VRPTW (155 subproblems)

Scalability profiling

Single node (left) and multi node (right) scalability overview:

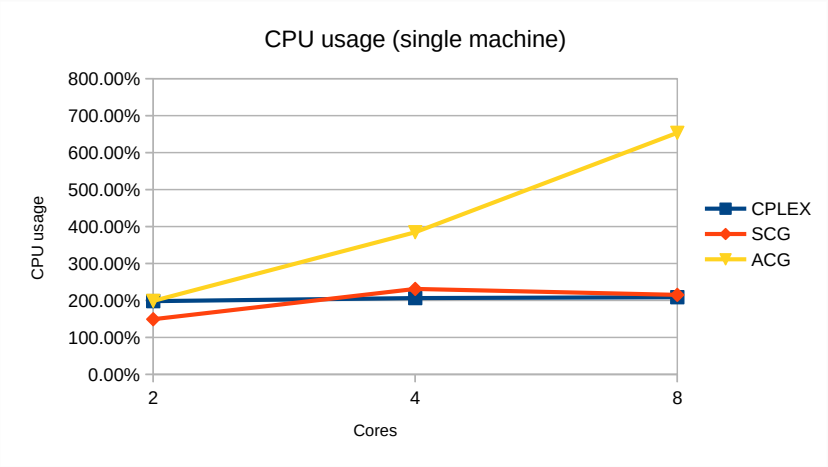


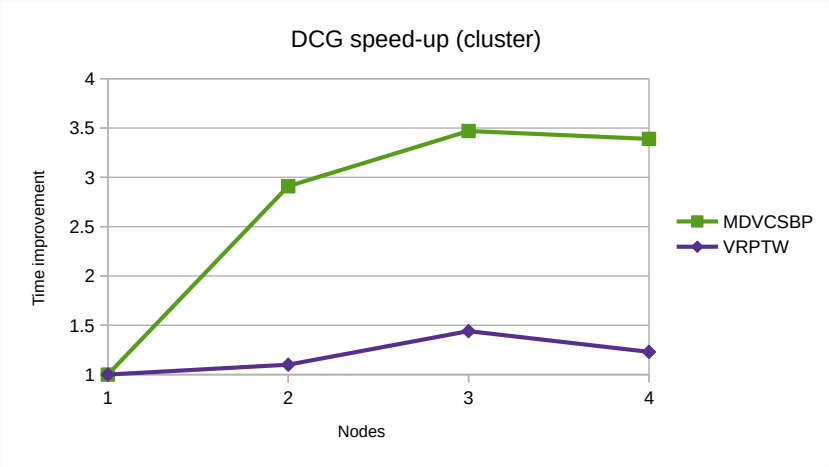


- Almost linear speed-ups for ACG on a single machine

Scalability profiling

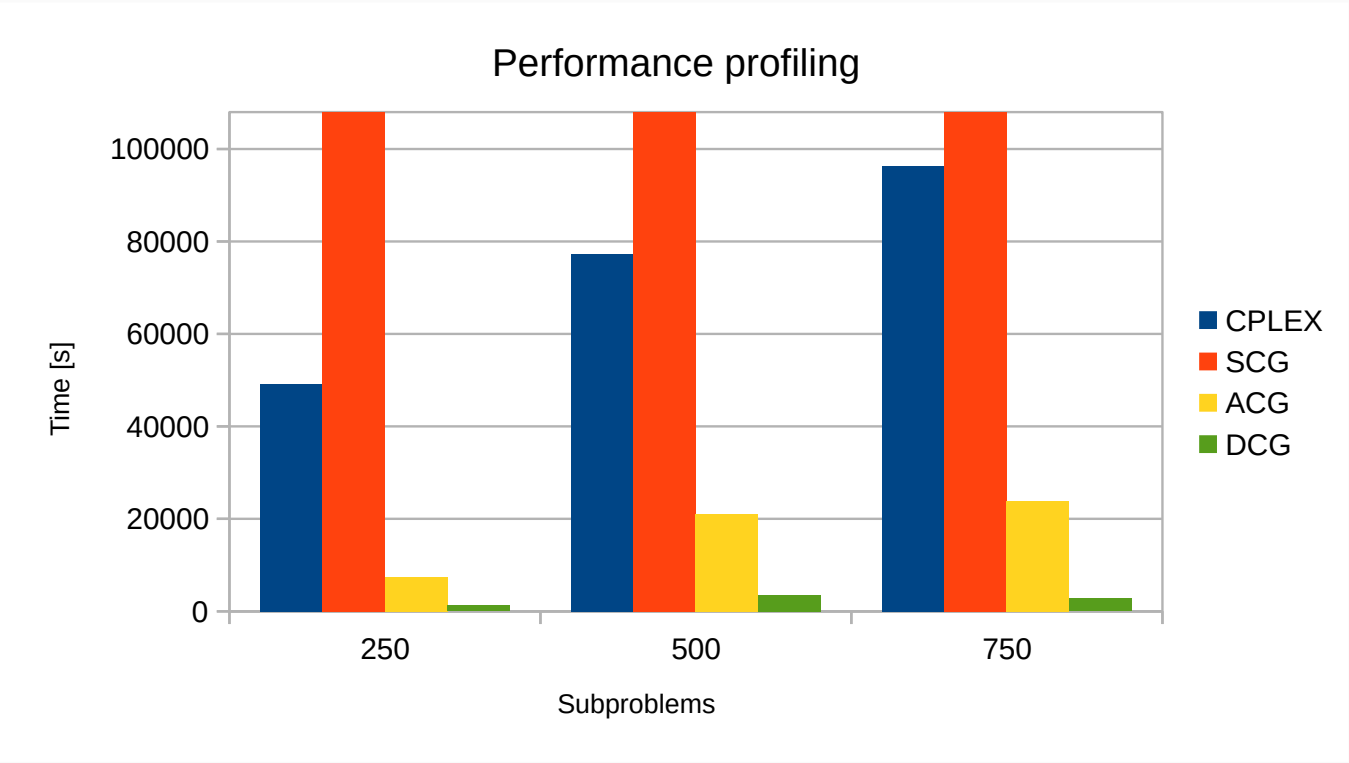
Single node (left) and multi node (right) scalability overview:





- Almost linear speed-ups for ACG on a single machine
- DCG scales well up to 3 machines on large scale instances
- Additional heuristic subproblems would improve performance

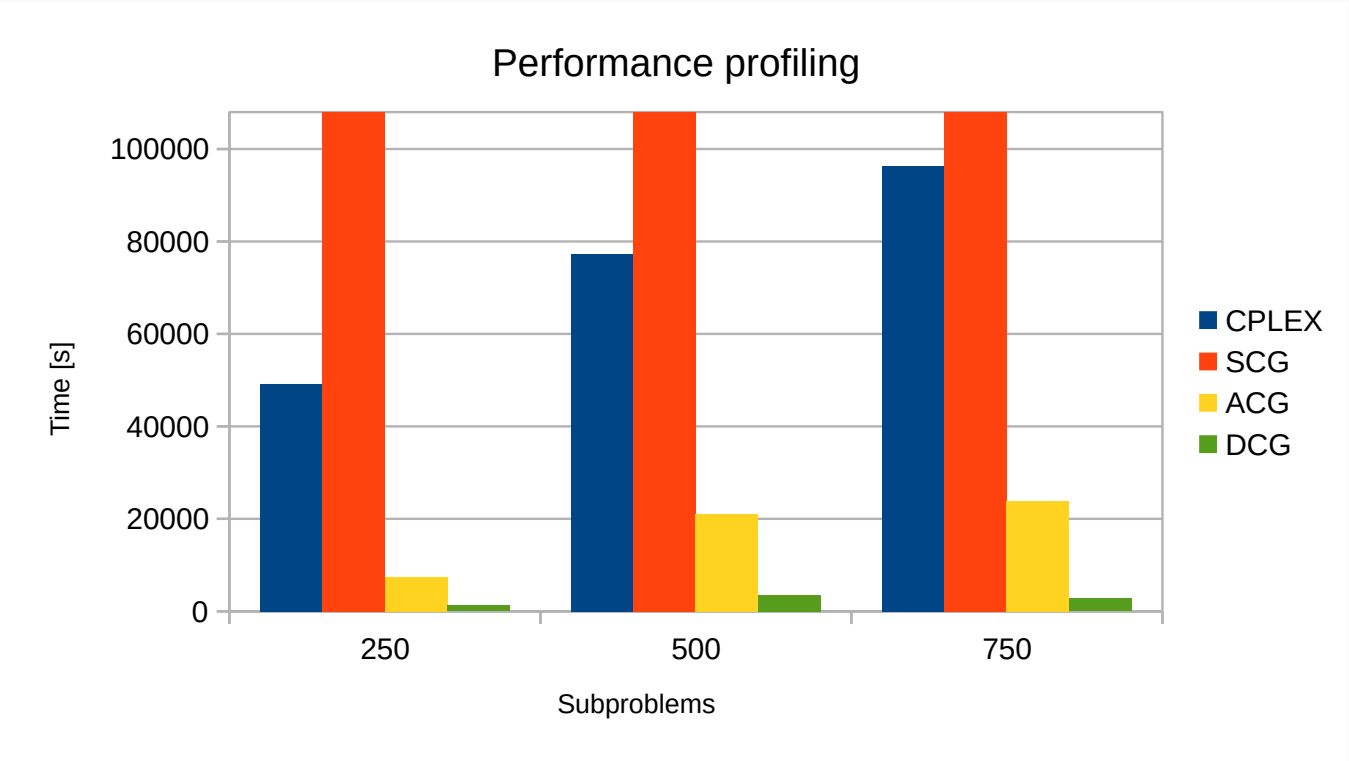
Overall results



30 Large scale instances (30h timeout):

- CPLEX hits 10 timeouts
- DCG is on average 87 times faster

Overall results



30 Large scale instances (30h timeout):

- CPLEX hits 10 timeouts
- DCG is on average 87 times faster

21 Stress test instances (10h timeout):

- CPLEX always out of memory (19 times) or timeout (2 times)
- DCG requires on average 14 minutes

Conclusions and Perspectives

Our data driven approach for the automatic decomposition of MIPs

- head-to-head with state-of-the-art detectors
- provides decompositions with a twisted flavor

Our parallel and distributed column generation approach

- performs one order of magnitude better than commercial solvers on large scale instances

There is still ground to cover for

- reaching commercial solvers on fully generic MIPs
- improving ML models and search algorithms
- using ML models as white boxes