Incorporating Holding Costs in Continuous-Time Service Network Design: New Model, Relaxation and Exact Algorithm

Roberto Baldacci

Engineering Management and Decision Sciences (EMDS), College of Science and Engineering (CSE),
Hamad Bin Khalifa University (HBKU), Doha, Qatar

rbaldacci@hbku.edu.qa

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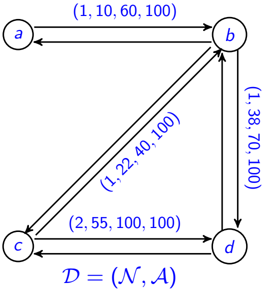
joint work with
Shengnan Shu, Zhou Xu
Department of Logistics and Maritime Studies
The Hong Kong Polytechnic University, Hong Kong

Outline I

SNDP: Problem Description

- Input
  - A physical network of terminals
  - Commodities with origins and destinations
- Design
  - Direct transportation services to open
  - Paths to transport commodities (unsplittable case)
  - Consolidations of commodities
- Constraints
  - Time windows
  - Capacities
- Objective: minimize the total cost (fixed and flow costs)



k	o^k	d^k	q^k	e^k	l^k
1	b	a	25	0	160
2	d	a	30	20	180
3	c	a	40	0	180

(c, f, τ, u) on arcs

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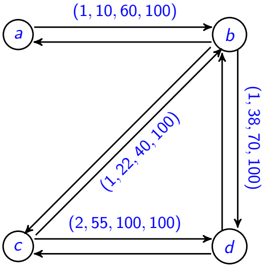

$\mathcal{D} = (\mathcal{N}, \mathcal{A})$

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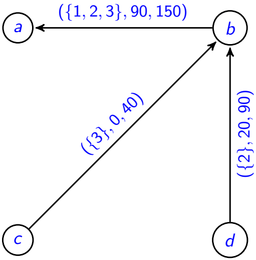

SNDP solution

$(\{commodities\}, dep.time, arr.time)$

Fixed cost: 70 (=22 (c, b) + 38 (d, b) + 10 (b, a))

Flow cost: 165

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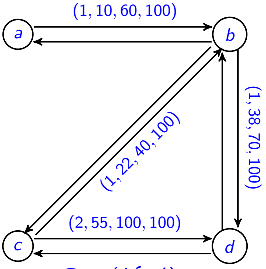

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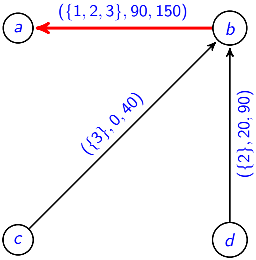

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Fixed cost: 70 (=22 (c, b) + 38 (d, b) + 10 (b, a))

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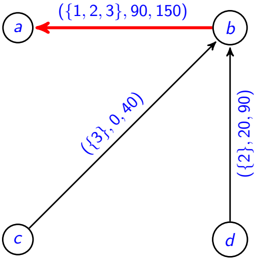
The three commodities are *consolidated* on arc (b, a)

SNDP with Holding Costs

- In-Transit Holding Costs
  - Caused by transportation
  - Can be included in flow costs
- In-Storage Holding Costs
  - Caused by consolidation
  - E.g. Commodities 1 (b-a) and 3 (c-a) wait at terminal b for 90 and 50 minutes to be consolidated with commodity 2 (d-a)
- Optimal design without incorporating holding cost can go arbitrarily worse



SNDP opt. sol.

({commodities}, dep.time, arr.time))

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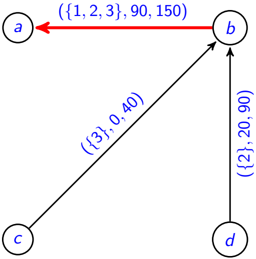

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Hold.cost. of 0.01 at term. *b*
0.01*(25*90+40*50)=42.5

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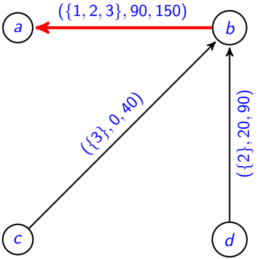

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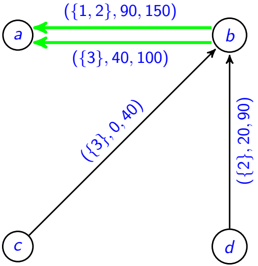

With hold. costs

({commodities}, dep.time, arr.time))

Fixed cost:
80 (=22 (c, a) + 38 $(d, b) + 2 \times 10(b, a)$)

Flow cost: 165

Holding cost:
0.01*(25*40)=10

277.5 vs 255, 8% savings in total cost

Models for the SNDP

- Wide applications: transportation, telecommunication, logistics, and production-distribution systems [Crainic, 2000; Wieberneit, 2008]
- Time-dependent compact models that use continuous variables to model time $\Rightarrow$ weak linear programming (LP) relaxations
- Time-indexed models [Andersen et al., 2009b,a; Pedersen et al., 2009]:
  - Discretization level: Δ
  - Time-expanded network: $\mathcal{D}_{\mathcal{T}}^{\Delta} = (\mathcal{N}_{\mathcal{T}}^{\Delta}, \mathcal{H}_{\mathcal{T}}^{\Delta} \cup \mathcal{A}_{\mathcal{T}}^{\Delta})$ (*holding* $\mathcal{H}_{\mathcal{T}}^{\Delta}$, *service* $\mathcal{A}_{\mathcal{T}}^{\Delta}$)
  - Variables indexed by time: $x_{ij}^{kt\bar{t}}$ be 0-1 variable equal to 1 if commodity $k \in \mathcal{K}$ is routed along arc $(i,j) \in \mathcal{A}$ departing from i at time t and arriving at j at time $\bar{t}$, 0 otherwise
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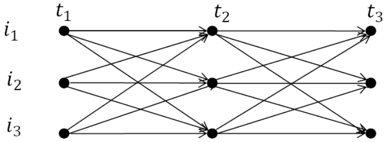

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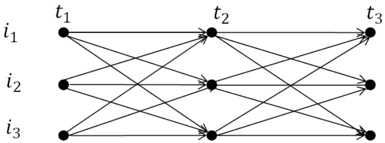

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Continuous-Time Service Network Design (CTSNDP)

- When $\Delta \Rightarrow 0$, time-indexed (TI) model size $\Rightarrow$ infinity, but solution $\Rightarrow$ optimal in continuous-time
- Boland et al. [2017] tackled the CTSNDP
  - Existence of a finite time-expanded network whose time-indexed model solution is continuous-time optimal

    $\Rightarrow$ The size of the resulting TI model may be prohibitively
  - Propose a Dynamic Discretization Discovery (DDD) algorithm
    $\Rightarrow$ optimal continuous-time solution obtained by considering a *small portion* of the complete TI model
- Follow-up works [Marshall et al., 2020; Hewitt, 2019; Medina et al., 2019; Vu et al., 2020] on CTSNDP, its variations, and other continuous time transportation optimization problems
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Existence of a Complete TI model for the CTSNDP

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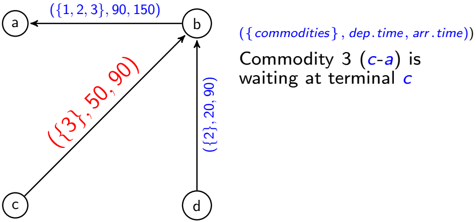

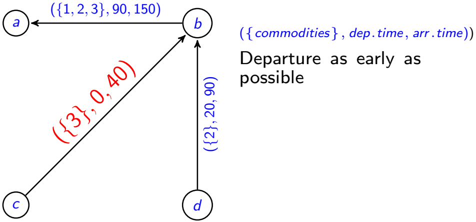

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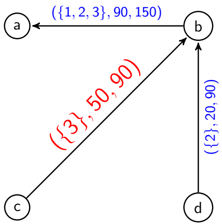
If terminal b has positive holding cost, then:

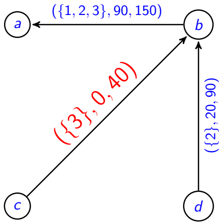


$(\{\,commodities\,\}\,,\,dep.time\,,\,arr.time))$

Commodity 3 (c-a) is waiting at terminal c

Commodity 3 is not waiting at terminal b

$\Rightarrow$ Holding cost of commodity 3 $=0$



$(\{\,commodities\,\}\,,\,dep.time\,,\,arr.time))$

Departure as early as possible

Commodity 3 is waiting at terminal 2

$\Rightarrow$ Holding cost of commodity 3 > 0

Continuous-Time Service Network Design with Holding Costs (CTSNDP-HC): Challenges and Our Results

- Prove the existence of a complete TI model with $\Delta = 1$ for CTSNDP-HC
  - This is needed to guarantee the convergence of the DDD algorithm to be developed
- Develop a DDD algorithm for CTSNDP-HC
  - New cuts
  - New and more effective upper bound heuristic
  - New and effective refinement strategy
- Demonstrate the effectiveness of the new DDD algorithm and the benefits that can be gained by taking into account holding costs

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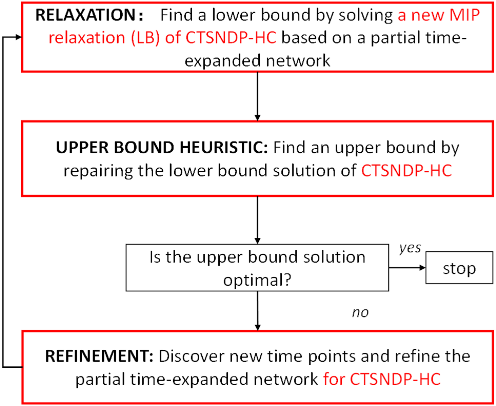
Dynamic Discretization Discovery Algorithm for CTSNDP-HC

A ~~column~~ point-generation based approach



- Proposed by Boland et al. [2017]
- The algorithm converges to optimal and stops in a finite number of iterations

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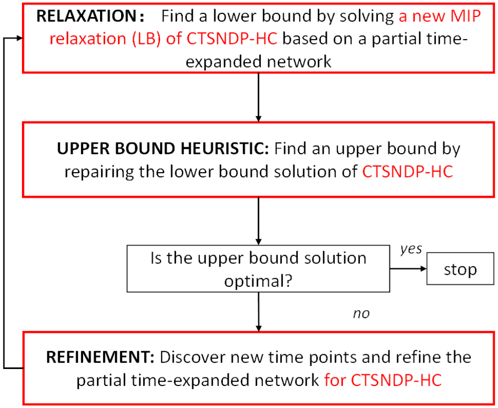

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Time-Index model for the SNDP with Holding Costs

- Formulation SND-HC($\mathcal{D}_{\mathcal{T}}^{\Delta}$):

$$z(\mathcal{D}_{\mathcal{T}}^{\Delta}) = \min \sum_{((i,t),(j,\bar{t}))\in\mathcal{A}_{\mathcal{T}}^{\Delta}} f_{ij} y_{ij}^{t\bar{t}} + \sum_{k\in\mathcal{K}} \sum_{((i,t),(j,\bar{t}))\in\mathcal{A}_{\mathcal{T}}^{\Delta}} (c_{ij}^{k} q^{k}) x_{ij}^{kt\bar{t}} + \sum_{k\in\mathcal{K}} \sum_{i\in\mathcal{N}} (h_i^k q^k) w_i^k$$

$$\sum_{((i,t),(j,\bar{t}))\in\mathcal{A}_{\mathcal{T}}^{\Delta}\cup\mathcal{H}_{\mathcal{T}}^{\Delta}} x_{ij}^{kt\bar{t}} - \sum_{((j,\bar{t}),(i,t))\in\mathcal{A}_{\mathcal{T}}^{\Delta}\cup\mathcal{H}_{\mathcal{T}}^{\Delta}} x_{ji}^{k\bar{t}t} = \begin{cases} 1 & (i,t) = (o^k, e^k), \\ -1 & (i,t) = (d^k, l^k), \forall k \in \mathcal{K}, (i,t) \in \mathcal{N}_{\mathcal{T}}^{\Delta}, \\ 0 & \text{otherwise}, \end{cases}$$

$$\sum_{k\in\mathcal{K}} q^k x_{ij}^{kt\bar{t}} \le u_{ij} y_{ij}^{t\bar{t}}, \quad \forall\ ((i,t),(j,\bar{t})) \in \mathcal{A}_{\mathcal{T}}^{\Delta},$$

$$w_i^k = \begin{cases} \displaystyle\sum_{((i,t),(j,\bar{t}))\in\mathcal{A}_{\mathcal{T}}^{\Delta}} t\, x_{ij}^{kt\bar{t}} - e^k, & i = o^k, \\ \displaystyle l^k - \sum_{((j,\bar{t}),(i,t))\in\mathcal{A}_{\mathcal{T}}^{\Delta}} t\, x_{ji}^{k\bar{t}t}, & i = d^k, \qquad \forall\ i \in \mathcal{N}, \forall\ k \in \mathcal{K}, \\ \displaystyle\sum_{((i,t),(j,\bar{t}))\in\mathcal{A}_{\mathcal{T}}^{\Delta}} t\, x_{ij}^{kt\bar{t}} - \sum_{((j,\bar{t}),(i,t))\in\mathcal{A}_{\mathcal{T}}^{\Delta}} t\, x_{ji}^{k\bar{t}t}, & \text{otherwise}, \end{cases}$$

$$x_{ij}^{kt\bar{t}} \in \{0,1\}, \quad \forall((i,t),(j,\bar{t})) \in \mathcal{A}_{\mathcal{T}}^{\Delta} \cup \mathcal{H}_{\mathcal{T}}^{\Delta}, k \in \mathcal{K},$$

$$y_{ij}^{t\bar{t}} \in \mathbb{N}_{\ge 0}, \quad \forall((i,t),(j,\bar{t})) \in \mathcal{A}_{\mathcal{T}}^{\Delta},$$

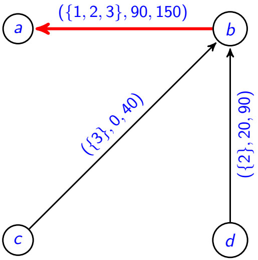
$$w_i^k \ge 0, \quad \forall i \in \mathcal{N}, k \in \mathcal{K}.$$

Existence of a Complete TI Model

- Define a flat solution as $\mathcal{S} = (\mathcal{P}, \mathcal{C})$
  - $\mathcal{P}$: paths for commodities
  - $\mathcal{C}$: consolidation plans for arcs of paths
- Continuous-time solution: $(\mathcal{P}, \mathcal{C})$ + services' departure times
- Given $\mathcal{S} = (\mathcal{P}, \mathcal{C})$, optimal service' departure times can be solved by an LP model
- The LP model is totally unimodular

  $\Rightarrow$ optimal departure times are integers (with integers data)

  $\Rightarrow$ $\Delta = 1$ is sufficient



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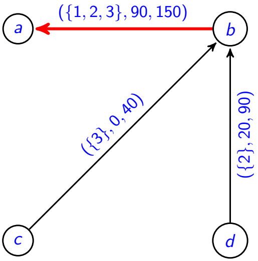

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Deriving the Relaxation

- Fully time-expanded network $\mathcal{D}_{\hat{\mathcal{T}}}$
- Consider a Partially time-expanded network $\mathcal{D}_{\mathcal{T}} = (\mathcal{N}_{\mathcal{T}}, \mathcal{H}_{\mathcal{T}} \cup \mathcal{A}_{\mathcal{T}})$
  $\Rightarrow$ derive lower bounds on both transportation costs and holding costs
- Associate to each feasible path in the fully time-expanded network a feasible path in the partially time-expanded network

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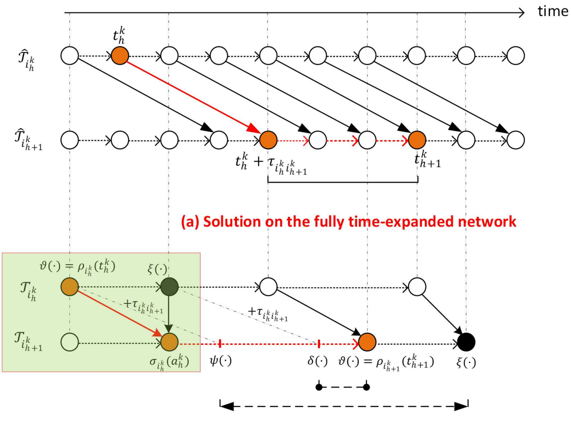
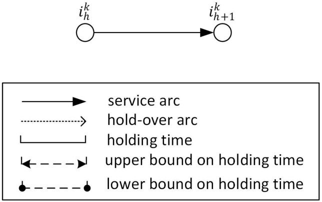


(a) Solution on the fully time-expanded network

(b) Mapped solution on the partially time-expanded network



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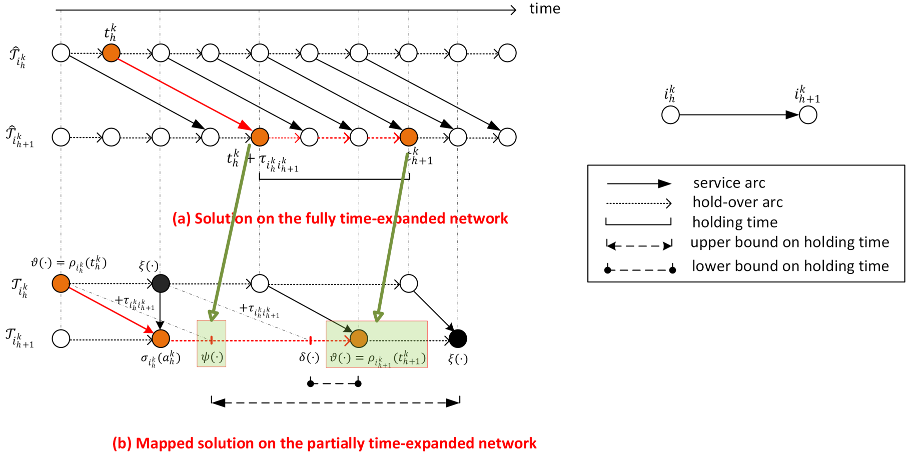

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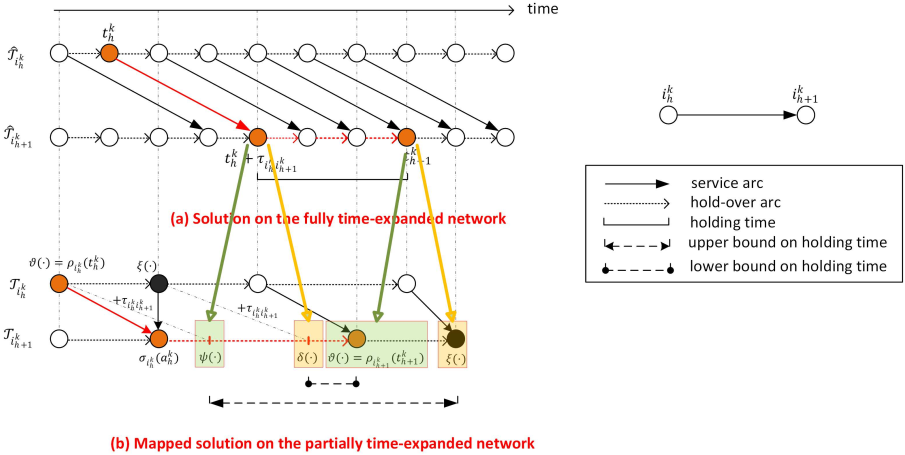

New Relaxation for the CTSNDP-HC

- Partially time-expanded network $\mathcal{D}_{\mathcal{T}} = (\mathcal{N}_{\mathcal{T}}, \mathcal{H}_{\mathcal{T}} \cup \mathcal{A}_{\mathcal{T}})$
- Relaxation SND-HC-R($\mathcal{D}_{\mathcal{T}}$):

$$z_R(\mathcal{D}_{\mathcal{T}}) = \min \sum_{((i,t),(j,\bar{t}))\in\mathcal{A}_{\mathcal{T}}} f_{ij} y_{ij}^{t\bar{t}} + \sum_{k\in\mathcal{K}} \sum_{((i,t),(j,\bar{t}))\in\mathcal{A}_{\mathcal{T}}} (c_{ij}^k q^k) x_{ij}^{kt\bar{t}} + \sum_{k\in\mathcal{K}} \sum_{i\in\mathcal{N}} (h_i^k q^k) w_i^k$$

SND($\mathcal{D}_{\mathcal{T}}^{\Delta}$) and

$$w_i^k \leq \begin{cases} \sum_{a=((i,t),(j,\bar{t}))\in\mathcal{A}_{\mathcal{T}}} \xi^k(a) x_{ij}^{kt\bar{t}} - e^k, & i = o^k, \\ l^k - \sum_{a=((j,\bar{t}),(i,t))\in\mathcal{A}_{\mathcal{T}}} \psi^k(a)\, x_{ji}^{k\bar{t}t}, & i = d^k, \\ \sum_{a=((i,t),(j,\bar{t}))\in\mathcal{A}_{\mathcal{T}}} \xi^k(a) x_{ij}^{kt\bar{t}} - \sum_{a=((j,\bar{t}),(i,t))\in\mathcal{A}_{\mathcal{T}}} \psi^k(a)\, x_{ji}^{k\bar{t}t}, & \text{otherwise}, \end{cases} \quad \forall\, i \in \mathcal{N}, \forall\, k \in \mathcal{K},$$

$$w_i^k \geq \begin{cases} \sum_{a=((i,t),(j,\bar{t}))\in\mathcal{A}_{\mathcal{T}}} \vartheta^k(a) x_{ij}^{kt\bar{t}} - e^k, & i = o^k, \\ l^k - \sum_{a=((j,\bar{t}),(i,t))\in\mathcal{A}_{\mathcal{T}}} \delta^k(a)\, x_{ji}^{k\bar{t}t}, & i = d^k, \\ \sum_{a=((i,t),(j,\bar{t}))\in\mathcal{A}_{\mathcal{T}}} \vartheta^k(a) x_{ij}^{kt\bar{t}} - \sum_{a=((j,\bar{t}),(i,t))\in\mathcal{A}_{\mathcal{T}}} \delta^k(a)\, x_{ji}^{k\bar{t}t}, & \text{otherwise}, \end{cases} \quad \forall\, i \in \mathcal{N}, \forall\, k \in \mathcal{K},$$

$$\sum_{i\in\mathcal{N}} w_i^k = l^k - e^k - \sum_{((i,t),(j,\bar{t}))\in\mathcal{A}_{\mathcal{T}}} \tau_{ij} x_{ij}^{kt\bar{t}}, \quad \forall\, k \in \mathcal{K}.$$

New Refinement Strategy for CTSNDP-HC

- Based on the same refinement operations of Boland et al. [2017] and Hewitt [2022], but using different refinement strategies

  - Add new time points to and modify arcs of the partial time-expanded network
    $\Rightarrow$ so that the existing relaxation optimal solution becomes infeasible
  - Lengthen short arcs (whose travel times are shorter than actual travel times, leading to infeasible consolidations)
  - Split the time intervals to improve accuracy of holding time to guarantee the convergence of the algorithm

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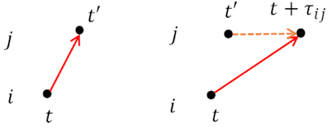

(a) Lengthen short arcs

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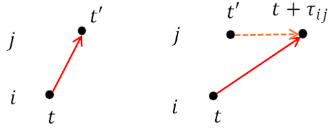
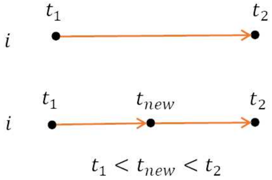


(a) Lengthen short arcs



(b) Split time interval

Computational Study

- We generated two classes of instances:
  - Class I: derived from Boland et al. [2017], evaluate the performance of the method in solving the CTSNDP and CTSNDP-HC
  - Class II: newly generated, analyse the factors that affect the complexity of the CTSNDP-HC
  - Holding costs defined for the less-than-truckload shipment case [Lai et al., 2022]
- Results grouped by "HC/LF", "HC/HF", "LC/LF" and "LC/HF"
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  - *Low Flexibility* (LF) (shipments' time requirements)
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- Gurobi (v.8.1.1) [Gurobi Optimization, 2021] MIP solver to solve relaxation SND-HC-R$(\mathcal{D}_{\mathcal{T}})$
- Seconds of an Intel(R) Core(TM) i7-8700 (3.20 GHz) Desktop PC, two hours of time limit

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Class I: Summary of the Results

- 558 instances [Boland et al., 2017] ($\leq |\mathcal{N}| = 30$, $|\mathcal{A}| = 683$, $|\mathcal{K}| = 400$)
- $LB0$: lower bound on the CTSNDP
- $UB0$: upper bound on the CTSNDP-HC based on the CTSNDP
- LB, UB: lower and upper bounds on the CTSNDP-HC

		%UB						%LB0	%UB0	
	%opt	min	max	avg	time	%tLB	iter	avg	avg	max
HC/LF	98.4	1.1	1.9	1.6	279.3	86.6	4.4	4.1	0.8	5.3
HC/HF	65.5	1.0	6.1	2.9	2902.7	94.4	6.2	10.6	3.9	17.9
LC/LF	100.0	-	-	-	0.7	62.3	1.8	0.7	0.0	1.0
LC/HF	100.0	-	-	-	0.2	57.0	2.3	0.8	1.1	8.0

$\%UB = 100\,(UB - LB)/LB$ - $\%UB0 = 100\,(UB0 - UB)/UB0$ - $\%LB0 = 100\,(LB - LB0)/LB0$

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Class II: Summary of the Results

- Based on new 1116 CTSNDP-HC derived from Crainic et al. [2001] (up to $|\mathcal{N}| = 30$, $|\mathcal{A}| = 683$ and $|\mathcal{K}| = 400$)
- Varying the connectivity level: networks $\mathcal{D}_1 > \mathcal{D}_2 > \mathcal{D}_3 > \mathcal{D}_4$
- Varying the flexibility level: distributions C > B > A

			%UB							%LB0	%UB0
		%opt	*min*	*max*	*avg*	*time*	*%tLB*	*iter*	*avg*	*avg*	*max*
A	$\mathcal{D}_1$	97.8	1.1	2.1	1.6	183.7	86.3	4.0	7.3	4.1	15.7
	$\mathcal{D}_2$	98.9	2.3	2.3	2.3	116.7	84.9	3.8	8.0	4.6	17.6
	$\mathcal{D}_3$	98.9	1.2	1.2	1.2	175.0	80.5	3.8	8.8	5.2	24.4
	$\mathcal{D}_4$	98.9	1.8	1.8	1.8	103.0	78.6	3.8	9.0	5.9	23.4
B	$\mathcal{D}_1$	84.9	1.1	6.8	3.0	1810.4	95.5	4.7	10.0	7.7	36.5
	$\mathcal{D}_2$	90.3	1.3	7.7	4.0	1246.9	94.6	4.8	11.8	9.9	34.8
	$\mathcal{D}_3$	89.2	1.0	6.3	2.7	906.9	89.8	4.9	11.3	10.1	37.1
	$\mathcal{D}_4$	95.7	1.3	2.8	1.9	348.2	85.1	5.1	10.7	10.4	37.1
C	$\mathcal{D}_1$	53.8	1.0	25.4	8.7	3586.5	98.5	4.0	6.9	6.5	44.7
	$\mathcal{D}_2$	59.1	1.0	19.7	4.6	3288.5	97.4	4.7	8.1	8.6	46.1
	$\mathcal{D}_3$	84.9	1.0	19.9	4.6	1650.8	94.0	5.1	11.0	11.8	43.1
	$\mathcal{D}_4$	94.6	3.0	3.0	3.0	452.3	87.5	5.6	10.6	14.1	45.8

Class II: Summary of the Results

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- Varying the flexibility level: distributions C > B > A

			%UB							%LB0	%UB0
		%opt	min	max	avg	time	%tLB	iter	avg	avg	max
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	$\mathcal{D}_2$	98.9	2.3	2.3	2.3	116.7	84.9	3.8	8.0	4.6	17.6
	$\mathcal{D}_3$	98.9	1.2	1.2	1.2	175.0	80.5	3.8	8.8	5.2	24.4
	$\mathcal{D}_4$	98.9	1.8	1.8	1.8	103.0	78.6	3.8	9.0	5.9	23.4
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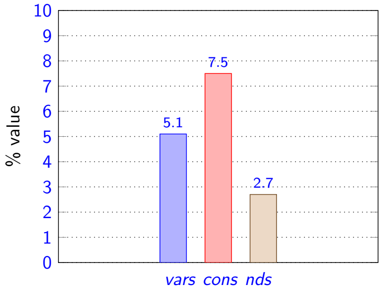
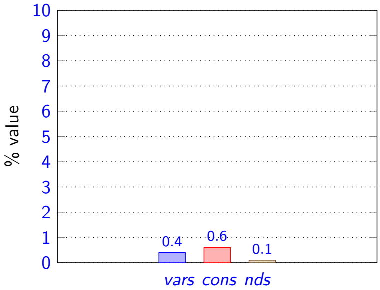
Class I: Comparison of Partially and Fully Time-expanded Networks

- $\%value = 100.0 \times x/y$
- $x \in \{variables, constraints, nodes\}$: partially time-expanded network
- $y \in \{variables, constraints, nodes\}$: fully time-expanded network



(a) Group HC/LF (98.4% solved to optimality)



(b) Group LC/HF (100.0% solved to optimality)

Conclusions

We have:

- Shown the importance of incorporating holding costs in CTSNDP
- Proved the existence of a complete time-indexed model for the CTSNDP-HC
- Designed a new effective DDD algorithm
- Shown that the benefits depends on the connectivity of the underlying physical network and on the flexibility of the shipments' time requirements

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Future work:

- Applications to other continuous time transportation planning problems, including problems under uncertainty
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Paper available at
http://www.optimization-online.org/DB_HTML/2021/10/8616.html

Thank you for your attention!

Class II: differences between $UB0$ and UB

- $\%dr$: percentage of the commodities that use different delivery routes
- $\%ds$: percentage of the commodities that use the same delivery routes but with different departure scheduling plans

Dist.	Network	$\%UB1$	$\%dp$	$\%da$	$\%dr$		$\%ds$		EXM-0			EXM		
					avg	max	avg	max	$\%hc$	$\%ht$	$\%cs$	$\%hc$	$\%ht$	$\%cs$
A	$\mathcal{D}_1$	1.6	22.4	14.1	1.0	7.5	21.4	42.5	3.2	6.7	40.9	1.2	3.9	32.9
	$\mathcal{D}_2$	2.0	25.7	17.0	1.0	7.0	24.8	38.0	3.7	7.4	43.7	1.2	4.1	35.2
	$\mathcal{D}_3$	2.4	29.8	19.9	0.4	2.6	29.4	42.5	4.5	8.3	49.1	1.4	4.4	38.0
	$\mathcal{D}_4$	2.7	34.0	22.8	0.0	0.0	34.0	56.0	4.9	9.2	52.1	1.5	4.8	40.6
B	$\mathcal{D}_1$	3.0	25.6	16.8	2.1	7.5	23.5	45.0	5.3	8.7	47.4	1.6	4.3	38.2
	$\mathcal{D}_2$	4.4	31.5	20.7	1.6	12.5	30.0	50.0	6.8	10.9	52.2	1.5	4.5	39.1
	$\mathcal{D}_3$	5.2	35.7	25.1	1.2	5.1	34.6	60.0	7.7	12.1	54.4	1.8	5.2	42.6
	$\mathcal{D}_4$	6.2	41.7	29.0	0.0	1.0	41.7	63.0	8.7	14.1	56.5	1.8	5.4	44.4
C	$\mathcal{D}_1$	5.1	31.2	20.9	3.3	11.0	27.9	44.0	7.7	10.3	50.6	1.7	4.1	37.2
	$\mathcal{D}_2$	5.5	34.9	24.3	3.0	12.5	31.9	50.0	8.1	11.4	54.9	1.7	4.4	40.0
	$\mathcal{D}_3$	7.3	39.7	26.2	1.6	10.0	38.1	57.5	9.9	15.1	58.0	1.6	5.1	41.5
	$\mathcal{D}_4$	8.7	47.6	34.0	0.0	0.0	47.6	72.5	11.1	17.9	61.5	1.7	5.8	45.6

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Dist.	Network	$\%UB1$	$\%dp$	$\%da$	$\%dr$		$\%ds$		EXM-0			EXM		
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	$\mathcal{D}_4$	8.7	47.6	34.0	0.0	0.0	47.6	72.5	11.1	17.9	61.5	1.7	5.8	45.6

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