Large-Scale optimization of the Unit Commitment Problem for medium-term Simulations of Energy Systems

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Outline

The Problem

Medium-Term Energy System simulation

Simulate an Energy System for *1 year* with *hourly resolution*

- 1 year = 8760 hours
- energy = electricity

Energy Systems at a glance:

- producers share demand via a **global energy exchange**
- **zones** connected by **capacitated network**
- **demand** and **supply** *must* be **accurately balanced**
  - small mismatch disruptive
  - electricity hard to store

Unit Commitment Problem (UCP)

Definition

Compute a **feasible** and **optimal** production schedule for (thermal) power plants.

Our objective: **minimize costs**

Assume

- competitive markets
- producers bid at production cost
- inelastic demand

then

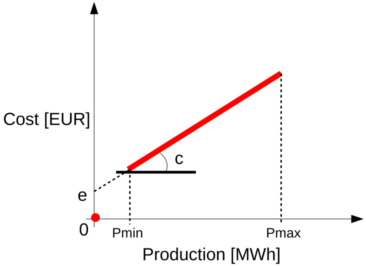
minimize costs = maximize global welfare *(Th. of Welfare)*

UCP simulates the **ideal** behaviour of the energy system

- prices close to clearance level
- identify technical and normative limits of the system

Thermal plants

- burn fuel
- significant production costs (fuel, maintenance, pollution)
- discrete activation patterns:
  - on/off state
  - minimum production level when active
- minimum up/down times
  - e.g. "once switched on stay active for the next 16h"
- described by MILP models
  - non-linearities for extra accuracy



Problem Description

Variables:

- thermal plants (TPP) $\rightarrow$ state: binary, production: continuous
- hydroelectric plants (HPP) $\rightarrow$ production: continuous
- network $\rightarrow$ flows: continuous

Constraints:

- hydroelectric energy balance: for each hydro plant and period water in $=$ production $+$ reservoir increase $+$ spillage
- zonal energy balance: for each period and zone

  $$\text{demand} = \text{local energy supply} + \text{supply from other zones}$$

- minimum up/down constraints for thermal plants
- plants production bounds

Objective:

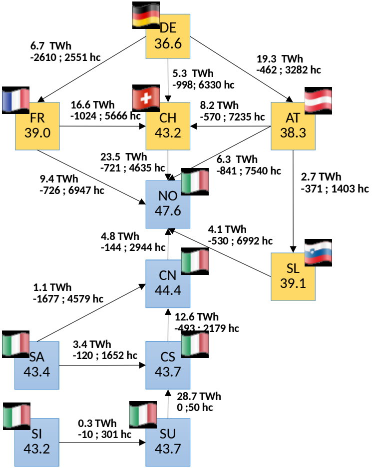
- minimize global production cost
  - TPP costs

Problem description

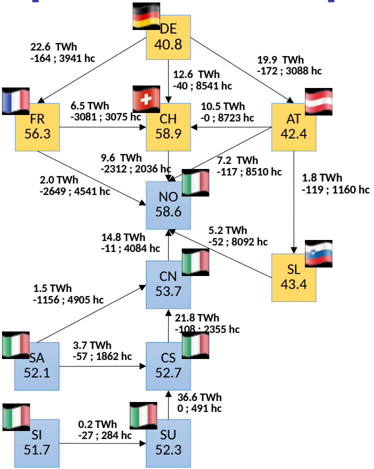
- *Linear* UCP
- over 1 year, with hourly resolution
  - 8760 h to simulate

⇒ a large-scale MILP

Example Europe 2020 - Results [Benini et al., 2014]



"Base"



"LessNuke", reduced nuclear power

Scenario	Italy		DE, FR, CH, AT, SL		CO_2 emissions w.r.t Base
	Coal **TWh**	**Gas** **TWh**	**Coal** **TWh**	**Gas** **TWh**	**Mt** CO_2
Base	67.3	29.1	234.1	2.1	0
LessNuke	74.9	35.8	328.0	12.8	+105.6

Outline

A heuristic method

Commit&Dispatch Algorithm (C&D)

0. Estimate: estimate production for each TPP via *ACR relaxation*
1. Commit: **for each group** $g \in G$ compute commitment $(y^i_{tzgm})_{M_g}$ to satisfy production levels x^i_{tzg}
2. Dispatch: fix commitment to (y^i_{tzgm}) and solve model. $\rightarrow$ obtain feasible solution
3. Repeat: If the new solution is different from the previous one let $x^{i+1}_{tzg} = x^i_{tzg}$, $i := i + 1$, and go to **Commit**. Otherwise STOP.

Key points:

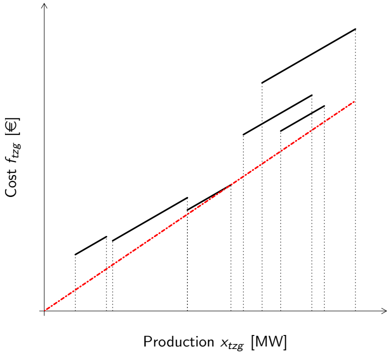
- at each iteration a new feasible solution not worse than the old one (non-strict monotonicity)
- finitely converges to local optimum

ACR

Aggregated Continuous Relaxation (ACR)

Cluster TPPs in groups and subgroups based on *location* and *technical characteristics*.

In each period t and for each group g in zone z with subgroups M_{zg} construct the best line (passing by the origin) which underestimates the original group cost function



$$f_{tzg}(x) = cx + \sum_{m \in M_g} e_m y_m$$

$$\rightarrow$$

$$\tilde{f}_{tzg}(x) = \tilde{c}x$$

ACR

Compared to the continuous relaxation (CR) of UCP the ACR is

- smaller and easier to optimize
  - equivalent to a Network Flow problem
- provably weaker

An exact approach

Rationale for accuracy

Why is high accuracy sought for simulations having strong simplifying assumptions?

High accuracy (gap $< 5\%$) is necessary to provide reliable results.

Then, quasi-optimal solutions (gap $< 2\%$) are useful to support simulations results:

- domain expert can address criticism of data and models.
- simulation errors are harder to discuss and defend

Column Generation (CG)

- For each subgroup $m \in M_{zg}$ consider every feasible commitment $u \in S_{zgm}$ (**pattern**)
- define α_u binary variable to select ($\alpha_u = 1$) pattern u.
- Master Problem: Substitute state variables (y_{tzgm}) with pattern variables $(\alpha_u)_{zgm}$.
  Constraints on patterns:

$$\text{(1a)} \qquad \sum_{\substack{m \in M_{zg}, \\ u \in S_{zgm}}} \hat{p}_{tzgmu} \alpha_u \leq x_{tzg} \leq \sum_{\substack{m \in M_{zg}, \\ u \in S_{zgm}}} \hat{P}_{tzgmu} \alpha_u \quad \forall t, z, g, \ldots$$

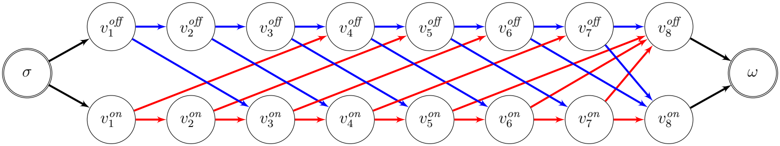
$$\text{(1b)} \qquad \sum_{u \in S_{zgm}} \alpha_u = 1 \qquad \forall z \in Z, g \in G_z, m \in M_{zg}$$

Pricing

An ILP for each subgroup in the *y* variables.
Model has the *integrality property*.

Solvable in linear-time with dynamic programming (see [Frangioni, 2006] for reference).

Possible commitments as a graph.
Optimal commitment as a shortest path on the graph.



Primal Heuristic

Rounding Heuristic (RH) to compute primal solutions at each CG iteration.

After solving the continuous relaxation of restricted master problem (RMP)

1. for each subgroup m select and fix the pattern u with the highest fractional price α_u
2. solve the RMP

Full-Dual optimization

Earlier experiments detected numerical instability while solving the RMP due to quasi-degeneracy.

To overcome the issue, at each CG iteration we try to

- generate *more columns*, to improve master's polytope description
- compute *more dual bounds*, possibly improving ones

Full-Dual optimization

At each Relax step we also solve the *Lagrangian relaxation* of the RMP.

- dualize constraints 1a which link patterns variables α_{tzgmu} with production levels x_{tzgm}.

$$\sum_{\substack{m \in M_{zg}, \\ u \in S_{zgm}}} \hat{p}_{tzgmu} \alpha_u \leq x_{tzg} \leq \sum_{\substack{m \in M_{zg}, \\ u \in S_{zgm}}} \hat{P}_{tzgmu} \alpha_u \quad \forall \ldots$$

RMP thus decomposes in:

- a LP model without pattern variables, equivalent to ACR
- a purely integer model identical to the Pricing problem

$\Rightarrow$ easier and more stable than the RMP.

Ad-hoc subgradient method (summary)

Due to degeneracy and flat objective, good subgradients are hard to find.

With standard subgradient methods, at each iteration the lagrangean lower bound is likely to

- worsen considerably, or
- change by a negligible amount

Ad-hoc subgradient method (summary)

1. classify subgradient step i comparing current lower bound LB_i and best known bound $\underline{LB}$. Let $\epsilon \to 0^+$
   - good: $LB_i - \underline{LB} > \epsilon$
   - poor: $|LB_i - \underline{LB}| \leq \epsilon$
   - bad: $LB_i - \underline{LB} < -\epsilon$
2. then if i is:
   - good: update duals with standard decreasing step length
   - poor: increase step length (escape the plateau)
   - bad: **rollback**
     - restore duals to the value of the latest good step
     - reduce step length

Results

Test

Instances: from Italy 2011.

Implementation: CPLEX 12 + SCIP 3.1.0 on dual core with 4G RAM.

Column Generation

- initialized with heuristic C&D solution
- 3h time limit
- rounding heuristic at 5^{th} iteration
- 3 strategies for solving the RMP
  - CP: Barrier + **C**rossover + **P**ricing
  - BP: **B**arrier + **P**ricing
  - BF: **B**arrier + **F**ull-Dual optimization
- Full-dual optimization: *ad-hoc* subgradient algorithm with max. 25 iterations

ACR+CD Heuristic

Id	Months	Dual [$M€$]	Primal [$M€$]	Gap %	ACR Time [s]	Tot. Time [s]
1.1	1	631.5	647.8	2.6 %	4.3	57.5
1.2		674.5	695.0	3.0 %	4.3	48.1
1.3		553.6	572.2	3.4 %	3.3	47.2
1.4		455.7	471.1	3.4 %	3.9	58.7
1.5		463.6	475.9	2.7 %	3.3	43.2
1.6		440.3	452.9	2.9 %	4.0	44.6
1.7		588.9	601.7	2.2 %	3.6	48.8
1.8		417.5	431.5	3.3 %	3.9	55.9
1.9		603.6	617.7	2.3 %	3.6	52.4
1.10		563.8	584.9	3.7 %	3.9	44.5
1.11		621.9	644.9	3.7 %	3.4	43.1
1.12		552.8	570.7	3.2 %	4.0	48.9
2.1	2	1305.7	1344.2	2.9 %	9.4	91.7
2.2		1009.0	1045.1	3.6 %	8.5	115.8
2.3		903.2	930.4	3.0 %	8.2	91.7
2.4		1004.7	1031.0	2.6 %	8.9	94.4
2.5		1167.3	1202.6	3.0 %	8.6	108.0
2.6		1174.5	1212.1	3.2 %	8.4	90.7
4.1	4	2314.2	2390.5	3.3 %	24.2	225.7
4.2		1907.8	1961.1	2.8 %	22.1	243.5
4.3		2341.5	2411.1	3.0 %	41.9	290.9
6.1	6	3216.9	3321.7	3.3 %	57.9	890.6
6.2		3345.5	3451.9	3.2 %	43.4	906.3

CG Dual Bounds - CP strategy

Id	Months	Final Dual [$M€$]	Iterations	Improvement %	Final RMP [$M€$]	Gap RMP/Dual %
1.1	1	630.6	10	-0.1%	635.5	0.8%
1.2		675.6	10	**0.2%**	678.5	0.4%
1.3		555.4	15	**0.3%**	557.1	0.3%
1.4		456.6	11	**0.2%**	459.3	0.6%
1.5		465.7	15	**0.4%**	466.8	0.2%
1.6		441.4	15	**0.2%**	442.5	0.2%
1.7		588.5	12	-0.1%	591.6	0.5%
1.8		417.6	9	**0.0%**	419.3	0.4%
1.9		603.8	10	**0.0%**	606.7	0.5%
1.10		564.2	12	**0.1%**	568.1	0.7%
1.11		623.3	11	**0.2%**	626.6	0.5%
1.12		552.1	11	-0.1%	556.9	0.9%
2.1	2	1300.0	5	-0.4%	1315.6	1.2%
2.2		1003.5	6	-0.5%	1017.3	1.4%
2.3		901.8	6	-0.2%	910.3	0.9%
2.4		989.4	6	-1.5%	1011.1	2.2%
2.5		1152.5	6	-1.3%	1176.4	2.1%
2.6		1155.7	4	-1.6%	1185.5	2.6%
4.1	4	2311.3	7	-0.1%	2331.7	0.9%
4.2		1874.1	5	-1.8%	1922.6	2.6%
4.3		2328.6	6	-0.6%	2360.9	1.4%
6.1	6	3169.9	4	-1.5%	3247.4	2.4%
6.2		3294.3	4	-1.5%	3376.6	2.5%

CG Dual Bounds - BP Strategy

* = numerical instabilities detected

Id	Months	Final Dual [$M€$]	Iterations	Improvement %	Final RMP [$M€$]	Gap RMP/Dual %
1.1	1	633.8	45	**0.4%**	634.9	0.2%
1.2		677.2	39	**0.4%**	678.1	0.1%
1.3		556.5	41	**0.5%**	557.0	0.1%
1.4		458.2	43	**0.6%**	458.9	0.1%
1.5		465.9	40	**0.5%**	466.6	0.2%
1.6		442.0	54	**0.4%**	442.3	0.1%
1.7		590.2	38	**0.2%**	591.4	0.2%
1.8		418.5	44	**0.2%**	419.0	0.1%
1.9		605.0	37	**0.2%**	606.3	0.2%
1.10		567.2	41	**0.6%**	567.9	0.1%
1.11		625.4	42	**0.6%**	626.3	0.1%
1.12		555.6	43	**0.5%**	556.5	0.2%
*2.1	2	1305.5	10	–	1314.1	–
*2.2		1011.9	13	–	1016.6	–
*2.3		901.0	7	–	910.2	–
*2.4		1004.4	15	–	1010.2	–
*2.5		1165.4	8	–	1175.6	–
*2.6		1167.2	4	–	1185.4	–
*4.1	4	2306.6	6	–	2332.9	–
*4.2		1900.4	8	–	1921.3	–
*4.3		2332.9	6	–	2361.1	–
6.1	6	3204.7	6	-0.4%	3244.0	1.2%
6.2		3331.9	6	-0.4%	3373.5	1.2%

CG Dual Bounds - BF Strategy

Id	Months	Final Dual [$M€$]	Iterations	Improvement %	Final RMP [$M€$]	Gap RMP/Dual %
1.1	1	631.8	18	**0.1%**	635.3	0.5%
1.2		675.9	18	**0.2%**	678.3	0.4%
1.3		555.3	19	**0.3%**	557.2	0.3%
1.4		458.1	28	**0.5%**	458.9	0.2%
1.5		466.0	31	**0.5%**	466.6	0.1%
1.6		441.9	36	**0.4%**	442.3	0.1%
1.7		588.0	18	-0.1%	591.7	0.6%
1.8		418.3	33	**0.2%**	419.0	0.2%
1.9		603.6	17	**0.0%**	606.5	0.5%
1.10		565.1	18	**0.2%**	568.2	0.6%
1.11		624.3	18	**0.4%**	626.5	0.4%
1.12		553.6	19	**0.1%**	556.9	0.6%
2.1	2	1302.4	8	-0.2%	1314.8	0.9%
2.2		1010.0	8	**0.1%**	1017.4	0.7%
2.3		904.2	15	**0.1%**	909.5	0.6%
2.4		1003.6	16	-0.1%	1010.0	0.6%
2.5		1164.9	8	-0.2%	1176.0	1.0%
2.6		1172.7	8	-0.2%	1184.5	1.0%
4.1	4	2315.3	5	0.0%	2335.3	0.9%
4.2		1902.3	7	-0.3%	1921.6	1.0%
4.3		2329.5	6	-0.5%	2361.6	1.4%
6.1	6	3192.9	5	-0.7%	3246.9	1.7%
6.2		3372.2	5	**0.8%**	3378.3	0.2%

Primal Bounds: Commit&Dispatch vs CG Rounding (BF strategy)

Id	Months	ACR+CD			RH			
		ACR+CD [$M€$]	Gap	Time [s]	RH [$M€$]	Gap	Time RH [s]	Time 5^{th} iteration [s]
1.1	1	647.8	2.5%	57.5	636.2	**0.1%**	86.8	407.0
1.2		694.9	2.8%	48.1	679.1	**0.1%**	101.0	445.0
1.3		572.0	3.0%	47.2	558.0	**0.1%**	88.1	406.0
1.4		471.1	2.9%	58.7	460.0	**0.2%**	85.6	466.0
1.5		475.9	2.1%	43.2	467.2	**0.1%**	54.7	406.0
1.6		453.0	2.5%	44.6	442.9	**0.1%**	37.0	282.0
1.7		601.6	2.2%	48.8	592.3	**0.2%**	106.9	488.0
1.8		431.0	3.0%	55.9	419.3	**0.1%**	59.1	247.0
1.9		617.7	2.3%	52.4	607.0	**0.2%**	147.5	503.0
1.10		584.9	3.5%	44.5	568.8	**0.2%**	117.0	447.0
1.11		644.7	3.3%	43.1	627.1	**0.2%**	123.1	478.0
1.12		571.1	3.2%	48.9	557.5	**0.1%**	83.5	436.0
2.1	2	1344.3	3.0%	91.7	1315.5	**0.3%**	476.9	1817.0
2.2		1044.9	3.5%	115.8	1017.9	**0.3%**	899.2	1790.0
2.3		930.6	2.9%	91.7	910.5	0.5%	159.8	1092.0
2.4		1031.4	2.7%	94.4	1011.3	0.5%	262.6	1171.0
2.5		1202.9	3.0%	108.0	1176.4	0.5%	1499.6	2252.0
2.6		1211.8	3.2%	90.7	1184.9	0.5%	526.7	1870.0
4.1	4	2390.3	3.2%	225.7	2380.2	0.8%	1810.5	9433.0
4.2		1961.2	2.8%	243.5	1922.2	0.7%	1209.8	4633.0
4.3		2411.1	3.0%	290.9	2363.8	0.7%	1531.3	7652.0
6.1	6	3322.0	3.3%	890.6	3246.9	1.7%	2561.3	8817.0
6.2		3451.6	2.4%	906.3	3430.0	2.4%	1164.5	9773.0

Results - Comments

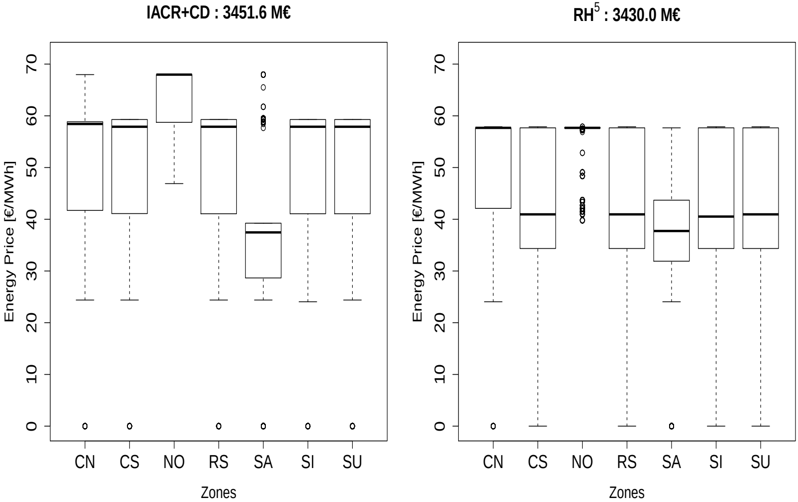
1. CG dual bounds do not improve a lot w.r.t. ACR
    - BF is better than CP and BP strategies
        - more iterations, more stability and better bounds

2. Primal bounds from RH are quasi-optimal
    - high-quality commitments with few iterations
    - close to the best known dual bound
    - which is almost the same as ACR

$\Rightarrow$ the ACR dual bound is almost optimal
   - that's why it is hard for CG to improve on it!

Accuracy comparison

Zonal Prices boxplots for primal solutions of instance 6.2 (0.7% bound difference)



Bottleneck

The continuous part of the model is needed to compute the value of the commitments and thus guide the optimization process.

Unfortunately, the LP part of the model has:

- due to its size: *flat objective function*
  - price of switching a few power plants for several hours has a small impact compared to global cost
- due to its structure: *degeneracy*

Better LP solvers?

Our model is degenerate and sparse.

Are there solvers that could handle it?

We considered:

- Interior Point: BlockIP [Castro, 2014]
  - C++ implementation available from the author
  - numerical problems when applied to original formulation
    - need presolving, non-trivial to implement
- Simplex: Space-Vector Decomposition [Desrosiers et al., 2015]
  - no available implementation

Answer: no(t yet?)

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