COLUMN GENERATION with DYNAMIC CONSTRAINT AGGREGATION in the MASTER PROBLEM

François Soumis
and team
GERAD

ColGen 2016
Overview

1. COLUMN GENERATION
   To deal with complex constraints and reduce number of variables

2. TASK AGREGATION
   To reduce number of constraints

3. RESULTS ON:
   Integrated pairing-rostering (50-80 tasks per columns)
   Large scale pairing (40 000 flights/month)
AIRLINE CREW SCHEDULING

COMPLEX COLECTIVE AGREEMENT CONSTRAINTS
- NON LINEAR
- NON CONVEX

COMPLEX NON LINEAR COST
- NON DECRESING FUNCTIONS

GLOBAL CONSTRAINTS LINKING CREW MEMBERS
- THOUSANDS OF FLIGHTS TO COVER
- BASE CONTRAINTS

INTEGRALITY
AIRLINE CREW SCHEDULING

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- NON LINEAR
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COMPLEX NON LINEAR COST
- NON DECREASING FUNCTIONS

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- THOUSANDS OF FLIGHTS TO COVER

INTEGRALITY

LARGE SCALE AND COMPLEX PROBLEMS
SET PARTITIONING FORMULATION

VARIABLES = FEASIBLE PAIRINGS

ADVANTAGES
- SIMPLER CONSTRAINTS
- LESS CONSTRAINTS
- COMPLEX COSTS CAN BE PRECALCULATED

DIFFICULTY
- MILLIONS OF MILLIONS OF VARIABLES
COLUMN GENERATION

- SUB-PROBLEM
  - MIN COST PATH WITH RESOURCES CONSTRAINTS
  - NON LINEAR, NON CONVEX BUT NON DECREASING FUNCTIONS
  - SOLVED AT INTEGRALITY BY DYNAMIC PROGRAMMING
ADVANTAGES OF COLUMN GENERATION

PROBLEM

\[ \text{MIN } CX \]
\[ AX \leq a \]
\[ BX \leq b \]
\[ X \text{ INTEGER} \]

ADVANTAGES

- SOLVE SUB-PROBLEM AT INTEGRALITY
- REDUCE INTEGRALITY GAP
- EASIER BRANCH AND BOUND

ColGen 2016
WEAKNESS of COLUMN GENERATION for LARGE SCALE PROBLEMS

• M.P. IS SLOW
  – SIMPLEX DEGENERATES WHEN SOLUTION IS CLOSE TO INTEGRALITY
    • PERTURBATIONS PRODUCE SMALL STEEPS
    • INTEGER POINTS METHODS PRODUCE MORE FRACTIONAL SOLUTIONS
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- S.P. IS SLOW
  - NUMBER OF ARCS GROW QUADRATICALY WITH NUMBER OF FLIGHTS
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WORST WHEN THE NUMBER OF TASKS PER COLUMN IS LARGE
TASK AGREGATION to OVERCOME WEAKNESS of COLUMN GENERATION for LARGE SCALE PROBLEMS

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    • PERTURBATIONS PRODUCE SMALL STEEPS
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    • REDUCE NUMBER OF CONTRAINTS AND DEGENERANCY
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  - REDUCE NUMBER OF FRACTIONAL VARIABLES
TASK AGREGATION

AGGREGATE TASKS IN CLUSTER
TASK AGREGATION

• AGGREGATE TASKS IN CLUSTER

• CLUSTERS CAN COME FROM ANY INITIAL SOLUTION
  - Crew follow aircrafts
  - Any heuristic (windowing, reduced problems, lazy B+B)
  - Solution to reoptimize
TASK AGREGATION

• AGGREGATE TASKS IN CLUSTERS

• OPTIMIZE
  – FAST OPT. ON CLUSTERS  Blue var. only
    • Smaller master problem (one constraint per cluster)
    • Smaller sub-problem network (less arcs)
TASK AGREGATION

• AGGREGATE LEGS IN CLUSTERS

• OPTIMIZE
  – FAST OPT. ON CLUSTERS  Blue var. only
  – MODIFY CLUSTERING TO REACH OPTIMALITY
    • Add some red var.
      – Arc with negative reduced cost indentified in the sub-problem
      – Solve the sub-problem with all arcs time to time
TASK AGREGATION

- AGGREGATE LEGS IN CLUSTERS

- OPTIMIZE
  - FAST OPT. ON CLUSTERS  Blue var. only
  - MODIFY CLUSTERING TO REACH OPTIMALITY
    - Add some red var.
      - Arc with negative reduced cost indentified in the sub-problem
      - Start with partial pricing in the sub-problem
    (arcs between clusters with large dual variables)
DUAL VARIABLES FOR PRICING IN THE SUB-PROBLEM
(m tasks, p clusters, n variables)

• p DUAL VARIABLES ARE GIVEN BY THE REDUCED PROBLEM
  – REDUCED COSTS OF p COLUMNS (without red arcs) ARE ZERO

• FIND m-p DUAL VARIABLES BY COMPLETING THE BASE
  – m-p SELECTED COLUMNS (with red arcs) WILL HAVE REDUCED COSTS = 0
  – $C^{m-p}_{n-p}$ WAYS TO SELECT m-p VARIABLES
  – REDUCED COSTS OF OTHERS VARIABLES (with red arcs) WILL VARY DEEPLY
DUAL VARIABLES FOR PRICING IN THE SUB-PROBLEM

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• **COMPLEMENTARY PROBLEM** (\( \pi_i \) are variables)
  - \( Z^{\text{MAX}} = \text{MAX } Z \)
  - REDUCED COSTS OF p COLUMNS (without red arcs) = 0
  - REDUCED COSTS OF COLUMNS (with red arcs generated up to date) \( \geq Z \)
QUALITY of the DUAL SOLUTION

- COMPLEMENTARY PROBLEM
  - \( Z^{\text{MAX}} = \text{MAX} \ Z \)
  - REDUCED COSTS OF \( p \) COLUMNS (without red arcs) = 0
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- PROPOSITION 1: THE COMPLEMENTARY PROBLEM
  PRODUCES CENTRAL REDUCED COSTS

\[ \bar{c}_j = c_j - \sum \pi_i a_{ij} \quad \text{decreasing linear relation} \; \pi_i <-----> \; \bar{c}_j \]
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\[ \sum \pi_i \cdot 1 = \sum c_j x_j = \text{constant} \quad \text{decreasing some dual variables} \]

increase some other dual variables
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\[
\bar{c}_j = c_j - \sum \pi_i a_{ij} \quad \text{decreasing linear relation } \pi_i \leftarrow \bar{c}_j
\]

\[
\sum \pi_i \cdot 1 = \sum c_j x_j = \text{constant} \quad \text{decreasing some dual variables}
\]

\[
\text{increasing some other dual variables}
\]

Maximizing the min reduced cost equalize the reduced costs

It stabilizes the column generation
QUALITY of the DUAL SOLUTION

- COMPLEMENTARY PROBLEM
  - $Z^{\text{MAX}} = \text{MAX } Z$
  - REDUCED COSTS OF p COLUMNS (without red arcs) = 0
  - REDUCED COSTS OF COLUMNS (with red arcs generated up to date) $\geq Z$

- PROPOSITION 2: AT LEAST m REDUCED COSTS = $Z$ in the COMPLEMENTARY PROBLEM SOLUTION

ANY DUAL SOLUTION

CP SOLUTION
INTERACTION BETWEEN: AGREG. PROB. COMP. PROB. and SUB-PROB.

- $Z^{\text{MAX}}$ SIGNIFICANTLY NEGATIVE
  - THE SOLUTION CAN BE SIGNIFICANTLY IMPROVED WITH EXISTING COLUMNS
INTERACTION BETWEEN: AGREG. PROB. COMP. PROB. and SUB-PROB.

- $Z^{\text{MAX}}$ SIGNIFICANTLY NEGATIVE
  - THE SOLUTION CAN BE SIGNIFICANTLY IMPROVED WITH EXISTING COLUMNS

- $Z^{\text{MAX}} = 0$
  - AGREGATED PROBLEM IS OPTIMAL FOR COLUMNS GENERATED UP TO DATE
INTERACTION BETWEEN: AGREG. PROB. COMP. PROB. and SUB-PROB.

- $Z_{\text{MAX}}$ SIGNIFICANTLY NEGATIVE
  - THE SOLUTION CAN BE SIGNIFICANTLY IMPROVED WITH EXISTING COLUMNS

- $Z_{\text{MAX}} = 0$
  - AGREGATED PROBLEM IS OPTIMAL FOR COLUMNS GENERATED UP TO DATE

- $Z_{\text{MAX}} = 0$ or SMALL NEGATIVE VALUE
  - SOLVE THE SUB-PROBLEM
  - $Z^{\text{SP}} << Z_{\text{MAX}}$ ADD THE GENERATED COLUMNS TO IMPROVE THE SOLUTION
  - $Z^{\text{SP}} \approx Z_{\text{MAX}} \approx 0$ STOP. THE SOLUTION IS NEAR OPTIMAL
EXPERIMENTATION

• INTEGRATED PAIRING-ROSTERING PROBLEMS
  – MONTHLY PROBLEMS
  – MEDIUM SIZE: 1000 – 8000 FLIGHTS/MONTH

• GLOBAL OPTIMIZATION for PAIRING PROBLEMS
  – MEDIUM SIZE: 1000 – 8000 FLIGHTS/MONTH
  – LARGE SCALE 40 000 FLIGHTS/MONTH
INTEGRATED CREW PLANNING

PAIRING

COVER FLIGHTS WITH PAIRINGS
(≈10-12 flights/column)

ROSTERING

COVER PAIRINGS WITH ROSTERS
(≈ 5-7 pairings/column)

INTEGRATED OPTIMISATION

COVER FLIGHTS WITH ROSTERS,
VERY DENSE COLUMNS
(50-80 flights/columns)
INTEGRATED PLANNING WITH CONSTRAINT AGGREGATION

- SOLVE PAIRING PROBLEM
- OPTIMIZE ROSTERS WITH FIXED PAIRINGS
- AGGREGATE FLIGHTS IN THE SAME PAIRING
- REOPTIMISE with CONSTRAINTS AGREGATION CHANGING THE PAIRINGS
- (REACH OPTIMAL SOLUTION BY SOLVING SMALL PROBLEMS)
# RESULTS WITH COL. GENERATION AND CONSTRAINT AGREGATION

<table>
<thead>
<tr>
<th>Problem</th>
<th>Sequential approach</th>
<th>Integrated approach</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>CPU (min)</td>
<td>Total cost</td>
</tr>
<tr>
<td>Instance</td>
<td>Flights</td>
<td></td>
</tr>
<tr>
<td>I-1</td>
<td>1011</td>
<td>4.0</td>
</tr>
<tr>
<td>I-2</td>
<td>1463</td>
<td>5.8</td>
</tr>
<tr>
<td>I-3</td>
<td>1793</td>
<td>11.4</td>
</tr>
<tr>
<td>I-4</td>
<td>5466</td>
<td>522.6</td>
</tr>
<tr>
<td>I-5</td>
<td>5639</td>
<td>231.9</td>
</tr>
<tr>
<td>I-6</td>
<td>5755</td>
<td>260.0</td>
</tr>
<tr>
<td>I-7</td>
<td>7527</td>
<td>507.6</td>
</tr>
<tr>
<td>Average</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

(*) NEAR OPTIMAL: L.P. TOLERANCE =10^-6, INTEGRALITY GAPS: PAIRING ~0.3%, BLOCS ~0.5%
GLOBAL OPT. for PAIRING PROB.

• SOLVED FIRST with a COMMERCIAL SOLVER
  ROLLING HORIZON: 3 DAYS WINDOWS, 1 DAY OVERLAP

• GLOBAL OPTIMIZATION for PAIRING PROBLEMS
  MEDIUM SIZE: 1000 – 8000 FLIGHTS/MONTH
  LARGE SCALE 10 000 FLIGHTS/WEEK

• ROLLING HORIZON for PAIRING PROBLEMS
  1 WEEK WINDOWS
  LARGE SCALE 40 000 FLIGHTS/MONTH
## MID-SIZE MONTHLY PROBLEMS

<table>
<thead>
<tr>
<th>Instance</th>
<th>Flights</th>
<th>Stations</th>
<th>CPU (min)</th>
<th>Gap (%)</th>
<th>No. Itrs</th>
<th>Degeneracy (%)</th>
<th>Fat reduction (%)</th>
<th>Deadheads reduction (%)</th>
<th>Reduction in cost (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>I-1</td>
<td>1011</td>
<td>26</td>
<td>17</td>
<td>0.15</td>
<td>6150</td>
<td>87.33</td>
<td>59.55</td>
<td>77.5</td>
<td>4.52</td>
</tr>
<tr>
<td>I-2</td>
<td>1463</td>
<td>35</td>
<td>25</td>
<td>0.29</td>
<td>4667</td>
<td>79.42</td>
<td>32.11</td>
<td>100</td>
<td>1.08</td>
</tr>
<tr>
<td>I-3</td>
<td>1793</td>
<td>41</td>
<td>28</td>
<td>0.01</td>
<td>2417</td>
<td>81.03</td>
<td>19.34</td>
<td>100</td>
<td>3.70</td>
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<tr>
<td>I-4</td>
<td>5466</td>
<td>49</td>
<td>278</td>
<td>0.36</td>
<td>1675</td>
<td>80.50</td>
<td>3.2</td>
<td>15.62</td>
<td>0.37</td>
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<tr>
<td>I-5</td>
<td>5639</td>
<td>34</td>
<td>56</td>
<td>0.00</td>
<td>1540</td>
<td>74.35</td>
<td>27.33</td>
<td>18.30</td>
<td>0.38</td>
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<tr>
<td>I-6</td>
<td>5755</td>
<td>52</td>
<td>237</td>
<td>0.13</td>
<td>19279</td>
<td>83.25</td>
<td>72.97</td>
<td>27.69</td>
<td>1.94</td>
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<tr>
<td>I-7</td>
<td>7527</td>
<td>54</td>
<td>141</td>
<td>0.37</td>
<td>1261</td>
<td>79.03</td>
<td>40.37</td>
<td>12.76</td>
<td>1.36</td>
</tr>
<tr>
<td><strong>Average</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>36.41</td>
<td>25.63</td>
<td>1.90</td>
<td></td>
</tr>
</tbody>
</table>

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WEEKLY PROBLEMS

• CYCLIC JUNE 2014 > 10 000 FLIGHTS
  • INITIAL SOLUTION 3552018, 170 DEAD HEADS

<table>
<thead>
<tr>
<th>Coût</th>
<th>DH</th>
<th>Temps</th>
<th>Borne N0</th>
<th>Var. Fract. N0</th>
<th>Noeuds</th>
<th>Version</th>
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</thead>
<tbody>
<tr>
<td>3515062.02</td>
<td>144</td>
<td>6h01m</td>
<td>3477624.6</td>
<td>1370</td>
<td>166</td>
<td>30/06/2015</td>
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<tr>
<td>3429615.70</td>
<td>133</td>
<td>4h38m</td>
<td>3473938.0</td>
<td>1347</td>
<td>39</td>
<td>04/02/2016</td>
</tr>
</tbody>
</table>

• CYCLIC JULY 2014 > 10 000 FLIGHTS
  • INITIAL SOLUTION 55156445, 88 DEAD HEADS
  • PENALTIES: BASE CONST., DISTRIBUTION OF DURATION OF PAIRINGS,…

<table>
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<tr>
<th>Coût</th>
<th>DH</th>
<th>Temps</th>
<th>Borne N0</th>
<th>Var. Fract. N0</th>
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<th>Version</th>
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<tbody>
<tr>
<td>4827421.57</td>
<td>101</td>
<td>4h09m</td>
<td>4966350.3</td>
<td>1093</td>
<td>65</td>
<td>30/06/2015</td>
</tr>
<tr>
<td>4572950.60</td>
<td>109</td>
<td>4h39m</td>
<td>4806035.0</td>
<td>2149</td>
<td>255</td>
<td>04/02/2016</td>
</tr>
</tbody>
</table>

• LARGE SAVING ON PENALTIES: 33%, 44%
MONTLY PROBLEM > 40 000 FLIGHTS

START WITH COPIES OF A WEEKLY SOLUTION
REOPTIMIZE WITH 5 WINDOWS OF 1 WEEK
WITHOUT GLOBAL CONSTRAINTS
WITH GLOBAL CONSTRAINTS

R= $\text{\$\$\$}$, S= SOFT COSTS, Contr. = PENALTY OF GLOBAL CONTR.

<table>
<thead>
<tr>
<th>Coût Total</th>
<th>Coût R+S</th>
<th>Coût Contr.</th>
<th>Temps</th>
<th>Cycl.</th>
<th>DCA</th>
<th>Version</th>
</tr>
</thead>
<tbody>
<tr>
<td>15517694.27 (0%)</td>
<td>15517658.27 (0%)</td>
<td>36.0 (0%)</td>
<td>60h12m</td>
<td>Non</td>
<td>Non</td>
<td>30/06/15</td>
</tr>
<tr>
<td>14877947.62 (4.12%)</td>
<td>14877947.62 (4.12%)</td>
<td>0.0 (100%)</td>
<td>46h35m</td>
<td>Non</td>
<td>Oui</td>
<td>30/06/15</td>
</tr>
<tr>
<td>20796331.80 (0%)</td>
<td>16718702.15 (0%)</td>
<td>4077629.65 (0%)</td>
<td>92h03m</td>
<td>Non</td>
<td>Non</td>
<td>30/06/15</td>
</tr>
<tr>
<td>18318902.66 (11.91%)</td>
<td>15805630.64 (5.46%)</td>
<td>2513272.02 (38.36%)</td>
<td>50h48m</td>
<td>Non</td>
<td>Oui</td>
<td>30/06/15</td>
</tr>
<tr>
<td>16756416.41 (19.43%)</td>
<td>15194680.80 (9.12%)</td>
<td>1561735.61 (61.70%)</td>
<td>87h28m</td>
<td>Oui</td>
<td>Non</td>
<td>30/06/15</td>
</tr>
<tr>
<td>16454739.40 (20.88%)</td>
<td>15171286.98 (9.26%)</td>
<td>1283452.42 (68.52%)</td>
<td>65h29m</td>
<td>Oui</td>
<td>Oui</td>
<td>30/06/15</td>
</tr>
<tr>
<td>17786735.91 (14.47%)</td>
<td>15542190.98 (7.04%)</td>
<td>2244544.93 (44.95%)</td>
<td>38h39m</td>
<td>Oui</td>
<td>Oui</td>
<td>24/01/16</td>
</tr>
<tr>
<td>16072276.35 (22.72%)</td>
<td>15163078.72 (9.30%)</td>
<td>909197.63 (77.70%)</td>
<td>62h44m</td>
<td>Oui</td>
<td>Oui</td>
<td>04/02/16</td>
</tr>
<tr>
<td>15837013.10 (23.85%)</td>
<td>15147603.21 (9.40%)</td>
<td>689409.89 (83.09%)</td>
<td>61h20m</td>
<td>Oui</td>
<td>Oui</td>
<td>04/02/16</td>
</tr>
</tbody>
</table>
CONCLUSIONS ON REDUCING THE NUMBER OF CONSTRAINTS

WE CAN SOLVE HUGE PROBLEMS

MILLIONS OF MILLIONS OF VARIABLES

10 000 CONSTRAINTS
CONCLUSIONS ON REDUCING THE NUMBER OF CONSTRAINTS

WE CAN SOLVE HUGE PROBLEMS

MILLIONS OF MILLIONS OF VARIABLES

10 000 CONSTRAINTS

KERNEL

• SOLVING ONLY A KERNEL PROBLEM MANY TIME
  • REDUCE NUMBER OF VARIABLES WITH COLUMN GENERATION
  • REDUCE NUMBER OF CONSTRAINTS WITH TASK AGGREGATION
• THE KERNEL PROBLEM IS ADJUSTED DYNAMICLY TO REACH OPTIMALITY

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REFERENCE LIST
REDUCING THE NUMBER OF CONSTRAINTS


