A novel column-generation matheuristic to Volumetric Modulated Arc Therapy (VMAT) treatment planning

Louis-Martin Rousseau

Canada Research Chair in Healthcare Analytics and Logistics
Department of Mathematics and Industrial Engineering
Polytechnique Montréal

Joint work with Mehdi Mahnam, Michel Gendreau, and Nadia Lahrichi

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Overview

Motivation

- 196,600 new cases of cancer in Canada in 2015.
- About 2 in 5 Canadians will develop cancer in their lifetimes and 1 in 4 will die of the disease (the same in the united states!).

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- About 2 in 5 Canadians will develop cancer in their lifetimes and 1 in 4 will die of the disease (the same in the united states!).
- **50%** of patients receive radiation therapy during their illness.
- This method works by damaging the genetic material within cancerous cells and destroying their ability to reproduce.

External radiation therapy

Definition

- Radiation is delivered by a linear accelerator mounted on a gantry to the patient, which is positioned on a couch.

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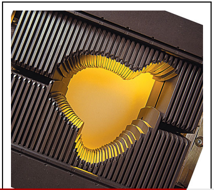
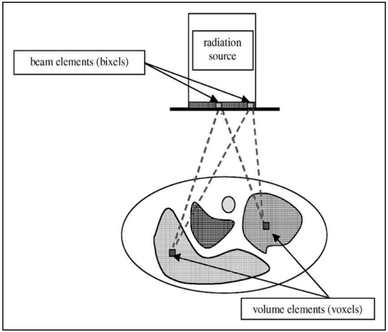
- Radiation is delivered by a linear accelerator mounted on a gantry to the patient, which is positioned on a couch.
- Intensity-Modulated Radiation Therapy (IMRT) is the most well-known method which adjust the **shape** of the beam using *Multi-Leaf Collimator (MLC)*.
- It consists of two opposite banks of metal leaves which can be shifted towards each other.



Dose Deposition

- For each gantry angle, each beam is decomposed into a rectangular grid of beamlets, I,, in order of $O(10^4)$.
- Each structure s is discretized into a finite number of cubes, J, called voxels, usually in order of $O(10^5)$.
- Dose Deposition Coefficient
  Dose received by voxel j from beamlet i in angle k is noted as D_{ij}^k.



Hamacher and Kufer (2002)

Volumetric Modulated Arc Therapy (VMAT)

Features

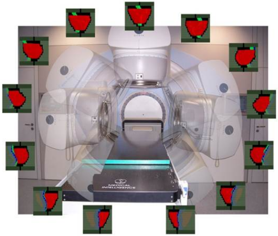
- A novel form of the arc therapy emerged in 2008 (Otto, 2008)
- Dynamic dose rate and gantry speed during rotation

Advantages of VMAT:

- better dose distribution,
- faster treatment, and
- deceasing the patient discomfort

Problem statement

- The 360° is divided into a finite number of equi-spaced **sectors**.
  At each control point h:
  - Aperture A^h
  - Dose rate ρ^h (in MU$/s$)
  - Gantry speed ν^h (in deg$/s$)
  - Fluence rate (in MU$/deg$): $\gamma^h = \rho^h/\nu^h$
- The machine parameters are constant between control points.



The VMAT treatment planning problem consists of following steps:

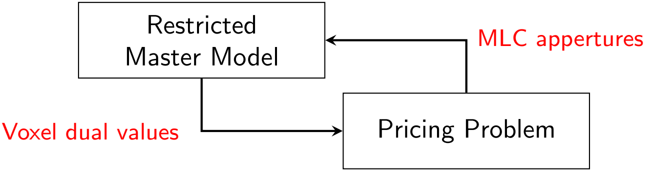
1. Selecting a sequence of collimator shapes, and
2. Determining the optimal dose rate, and
3. Determining the gantry rotation speed.

VMAT Literature Review

Article	Mode	Type	MO	Objectives	Model	Constraints	Method	Algorithm	Software	Treat. time	CPU time
Earl *et al.* (2003)	IMAT	DAO	Single	(1) Deviation of UB of OARs (least-square) (2) DVH-based penalties	Quadratic	(1) MLC const. (2) Arc start and stop angles (3) Speed of leaf motions (4) Speed of gantry rotation	Meta-heuristic	SA		10-20 min	
Otto (2008)	VMAT	DAO	Weighted Sum	Deviation of DVH bounds for OARs	Quadratic	(1) Non-negative beam intensity (2) Leaf overlapping (3) gantry motion (4) Dose variation	Heuristic	Progressive sampling and randomly changing the leaf positions	MATLAB		
Men *et al.* (2010b)	VMAT	DAO	Weighted Sum	(1) Voxel-based penalties; (2) Smoothing beam intensities	Quadratic	(1) Non-negative beam intensity (2) MLC deliverability (3) Max leaf speed	Heuristic	CG - RMP: gradient projection method, Pricing: KKT conditions	CUDA MATLAB		3-5 min
Peng *et al.* (2012)	VMAT	DAO	Single	Voxel-based penalties	Quadratic	(1) Max fluence rate (2) Max gantry speed (3) Interdigitation in MLC	Heuristic	CG-based greedy heuristic: RMP: gradient based algorithm, Pricing: DP	CUDA C		2-3 min
Craft *et al.* (2012)	VMAT	Two-Stage	Single	Mean Dose, SPG[1]	convex	(1) Voxel dose lower & upper bounds (2) Max gantry speed (3) Max leaf speed	Heuristic	1) 180 linear IMRT problems; 2) VMERGE heuristic and leaf sequencing	CERR	311s	35 min
Wala *et al.* (2012)	VMAT	Two-Stage	Single	SPG	convex	(1) Voxel lower & upper bounds (2) Ideal dose (3) planning goals	Heuristic	(1) 180 linear IMRT problems (2) VMERGE and PMERGE heauristics	Parallel	127s	40 min
Salari *et al.* (2012)	VMAT	Two-Stage	Pareto (Modified box Alg.)	(1) treatment time (2) Dose deviation of the ideal dose	Binary Quadratic	(1) Ideal dose (2) Network flow constraints	Exact, Heuristic	BQP	ILOG CPLEX		45 min
Papp et Unkelbach (2014)	VMAT	DLTO	Single	Voxel-based piecewise penalties	Convex	(1) Nonnegative beam intensity (2) Breakpoints in leaf trajectories (3) tailing and leading leaves (4) Max leaf speed	Heuristic	LP	MATLAB and CERR	4-5 min	

[1] Sum-of-Positive Gradients, a measure of the amount of variation

A column-generation-based heuristic



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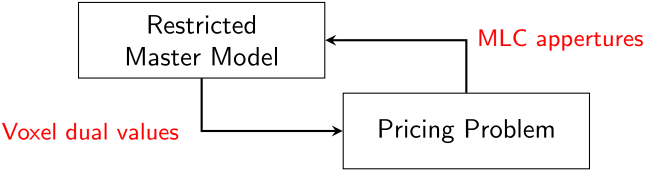

Proposed column-generation

- Pricing problem: Given a dual value for each voxel, find a new **arc** k, made up of the apertures of pair of leaves in a sequence of sectors, which can improve the solution.
- Master problem: Given a large set of arcs (K'), select a subset, determine their intensities γ_k^*, gantry speed t_h^* and dose distribution z^* to minimize deviation w.r.t. prescribed thresholds.

Pricing approach

- Graph approach to produce the aperture shape, inspired by Boland et al. (2004) and Romeijn et al. (2005).

Master Model Characteristics

- Each column k is from sector h_1 to h_2
- To be more precise, gantry speed, ν_g^h is assumed to be dependent to sectors rather than arc.
- To avoiding non-linearity, intensity, γ, is assumed to be arc dependent.
- After the CG algorithm and finding the final arcs, a post-optimization model determines the sector dependent intensities.

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- Variables:
  - z_j: dose received by voxe j,
  - y^k: priority of arc k,
  - γ^k: fluence rate of arc k,
  - t^h: time in sector h (reverse of gantry speed).

Master Model

$$\min \quad f(j) = \sum_{j \in V} \underline{w}([\underline{T}_j - z_j]_+)^2 + \overline{w}([z_j - \overline{T}_j]_+)^2 \qquad (1)$$

$$z_j = \sum_{k \in K'} u_j^k \gamma^k \qquad \forall j \in V \qquad (2)$$

Voxel dose calculation

$$\sum_{k \in K'} a^{kh} y^k = 1 \qquad \forall h \in H \qquad (3)$$

Each sector is covered by one sector

$$\sum_{k \in K'} \underline{\tau}^{kh} y^k \le t^h \qquad \forall h \in H \qquad (4)$$

Feasibility of Gantry Speed

$$\sum_{h \in H} t^h \le T_{max} \qquad (5)$$

Total treatment time

$$\sum_{k \in K'} a^{kh} \gamma^k \le t^h R \qquad \forall h \in H \qquad (6)$$

$$\gamma^k \le y^k R\overline{T} \qquad \forall k \in K' \qquad (7)$$

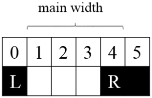
Restricting the max arc fluence rate by sector time and max dose rate

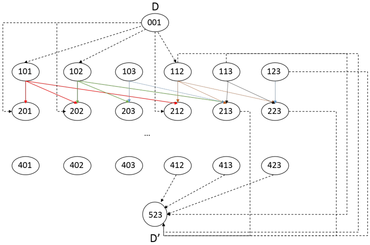
$$y^k, \gamma^k \ge 0 \qquad \forall k \in K' \qquad (8)$$

Pricing approach

- Graph approach to produce the aperture shape, inspired by Boland et al. (2004) and Romeijn et al. (2005).

The situation of each row in each sector is indicated as a node (h, l, r); e.g. node $(90, 0, 4)_5$ is the position of leaves of row 5 in sector 90:





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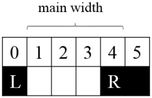
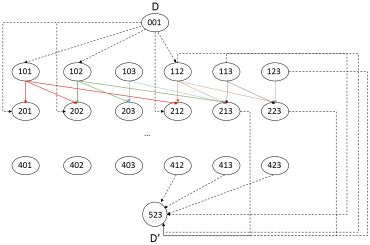




Notes

- maximum leaf motion constraint is considered.
- The weight in node $(h, l, r)_m$ is $\sum_{j \in Vox} \sum_{i=t+1}^{l-1} \widehat{\pi}_j D^h_{jmi}$
- A polynomial shortest path algorithm easily obtain the best solution.

Parallel programming

- To increase the speed of pricing algorithm, the arcs are generated in parallel
- Independent arcs around the patient allows us to implement pricing in parallel using openMP

# threads	Avg. Pricing Time (sec.)
1	38.19
2	12.36
3	7.64
4	5.11

- However, The columns are not *disjoint* because the voxel evaluation is in the master model!

Heuristic Algorithms

Algorithm 1 Greedy Heuristic

Initial random heuristic columns (arcs)
while stopping criteria not satisfied **do**
    Master Model (Arc based)
    Fix the best arc to 1
    Pricing
        arcs with equal length l
Post-Optimization model (Sector-based)
    Re-optimize gantry speed & intensities
Stop

Algorithm 2 Diving Heuristic

Initial random heuristic columns (arcs)
while All sectors be covered **do**
    while CG converges **do**
        Master Model (Arc based)
        Pricing
            arcs with equal length l
    Fix the best arc to 1
Post-Optimization model (Sector-based)
    Re-optimize gantry speed & intensities
Stop

Aggregation

- To decrease the number of constraints, an aggregation method implemented in the **beginning** of the algorithm
- Similar voxels could be distinguished using **K-MEANS** algorithm.
- However, original K-MEANS is so time-consuming to compare all about 100,000 voxels to each other based on vectors of 150 beamlets in about 180 sectors.
- Then, since we expect similarity among neighbors more in our problem, we calculate the distance of a voxel to only its **neighbor clusters**.
- We considered the criterion to calculate the distance as *Dose Depoition Matrix* (D) when all beamlets are open (full radiation).

Modified K-Means example

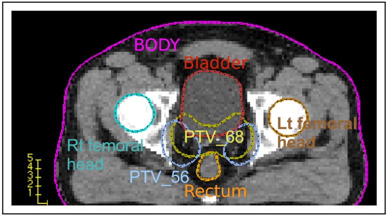
- For instance, considering 60% of tumor voxels and 40% of normal voxels, we have the following clustering results:

Iter	# Transfer Iter	Time (Sec.)	Avg.Dist
0			54.27054
1	19265	0.952947	11.57258
2	2160	0.74742	10.51782
3	301	0.733188	10.44499
4	40	0.729702	10.44053
5	13	0.738105	10.43969

- The first iteration looks enough!

Prostate case

- CORT dataset (Craft et al, 2014)
- 180 equispaced sectors
- All algorithm is implemented in C++/CPLEX and evaluated in CERR
- Computational time is restricted to 30 min



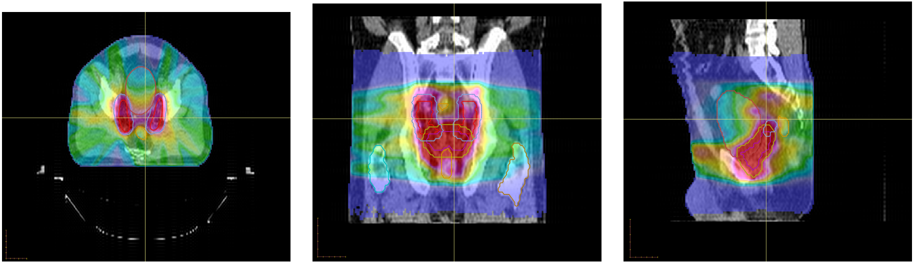
Case Characteristics

Total # beamlets	25,404
Beamlet size (mm)	1 1
Voxel resolution (mm)	3, 3, 3
# Target voxels	9491
# Body voxels	690,373

Algorithm Parameters

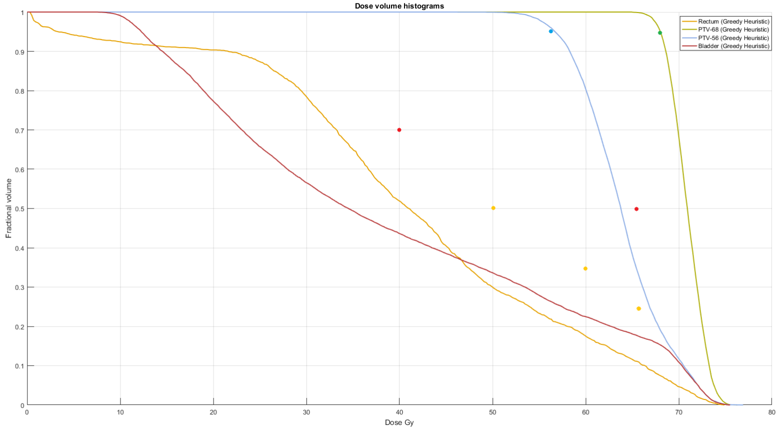
max treat. time	300 sec
max dose rate	600 MU/min
max leaf speed	6 cm/sec
gantry speed	$[1 \quad 6]^\circ$/sec

Greedy heuristic results



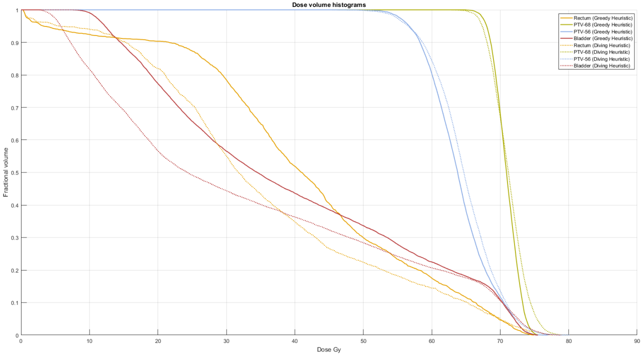
Algorithm	Time (min)
Pricing	3.95
Maser Model	10.50
Post Optimization	2.07
Total	16.69

Greedy heuristic dosimetry measures



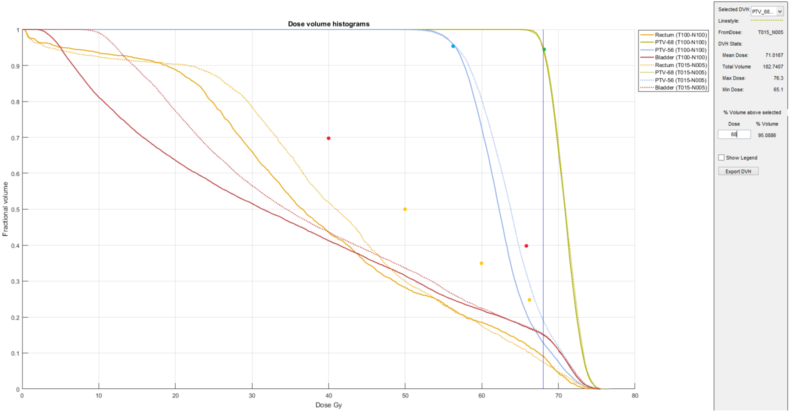
Organ	Measure	Organ	Measure
PTV 68	$V68 = 95.08\ \% > 95\%$	Rectum	$V65 = 11.59\% < 25\%$
PTV 56	$V56 = 96.46\ \% > 95\%$		$V60 = 17.52\% < 35\%$
Bladder	$V65 = 18.09\% < 50\%$		$V50 = 29.96\% < 50\%$
	$V40 = 43.64\% < 70\%$		

Diving vs. Greedy



Algorithm	time (min)
Diving	50.89
Greedy	16.69

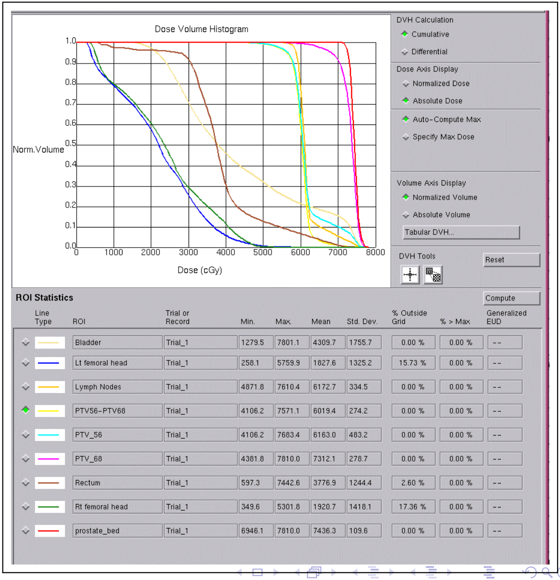
Aggregation effect



Tumor tissue	Normal tissue	time (min)
100 %	100 %	165.97
15 %	5 %	16.69

Comparison with a commercial software

- An expert in Pinnacle in CICL clinic.
- To take less time, voxel resolution (4,4,4) is considered
- The obtained plan was not better than our problem with the same assumptions!



Novel contributions

Master Model

- Proposing a new mathematical model considering the dose rate, gantry speed, and apertures, **simultaneously**.
- Proposing a new **aggregation** algorithm to decrease the number of constraints inspired by K-MEANS algorithm in clustering and data mining.
- Proposing a new column-generation based heuristic considering many arc alternatives.

Pricing Sub-problem

- Proposing the pricing model based on **graph theory** in VMAT
- the **MLC leaf movement** constraints in the pricing algorithm, rather than the master model.
- Regarding considering **voxels in the pricing** (not approximations), we have better evaluation of the impact of new arcs.

Main challenges

- Although the aggregation reduced a large number of voxels, still the master model is the main bottleneck of the algorithm which takes 60% of the computational time.
- Columns generated at each iteration from different sectors are not disjoint and have similar effects on voxels.

References

- Craft, D., McQuaid, D., Wala, J., Chen, W., Salari, E. and Bortfeld, T., 2012. Multicriteria VMAT optimization. *Medical physics*, 39(2), pp.686-696.
- Peng, F., Jia, X., Gu, X., Epelman, M.A., Romeijn, H.E. and Jiang, S.B., 2012. A new column-generation-based algorithm for VMAT treatment plan optimization. *Physics in medicine and biology*, 57(14), p.4569.
- Unkelbach, J., Bortfeld, T., Craft, D., Alber, M., Bangert, M., Bokrantz, R., Chen, D., Li, R., Xing, L., Men, C. and Nill, S., 2015. Optimization approaches to volumetric modulated arc therapy planning. *Medical physics*, 42(3), pp.1367-1377.
- Craft, D., Bangert, M., Long, T., Papp, D. and Unkelbach, J., 2014. Shared data for intensity modulated radiation therapy (IMRT) optimization research: the CORT dataset. *GigaScience*, 3(1), pp.1-12.

Acknowledgement

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Thanks for Your Attention