The Cross-dock Door Assignment Problem

A comparison of Lagrangean relaxation and Column Generation

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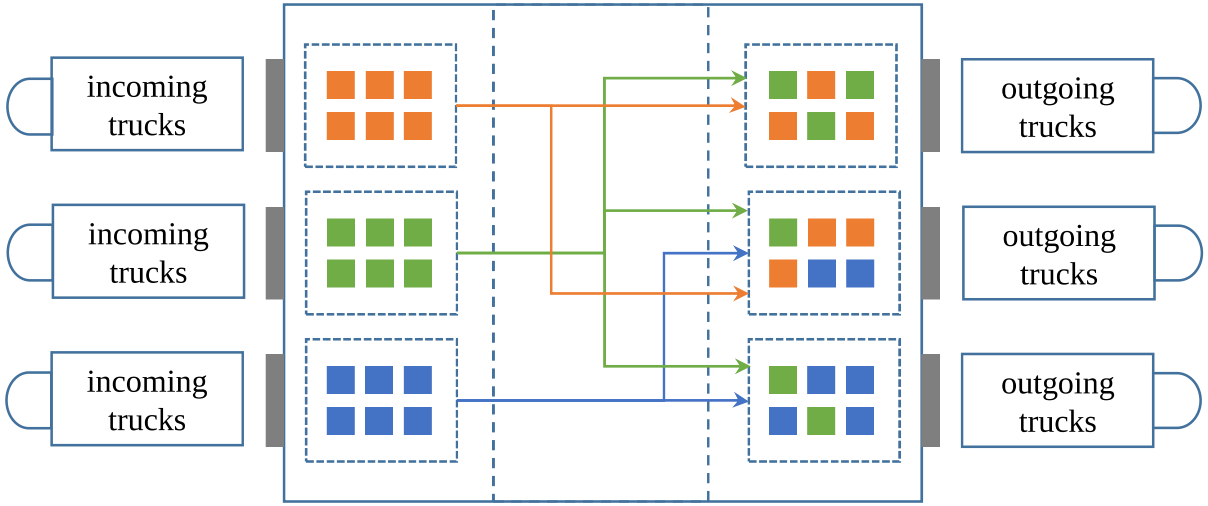
Agenda

- Introduction and Literature
- Problem statement
- MIP formulations
- Algorithms
- Results
- References and Conclusion

Introduction

Cross-docking: it is a logistics strategy, where goods get

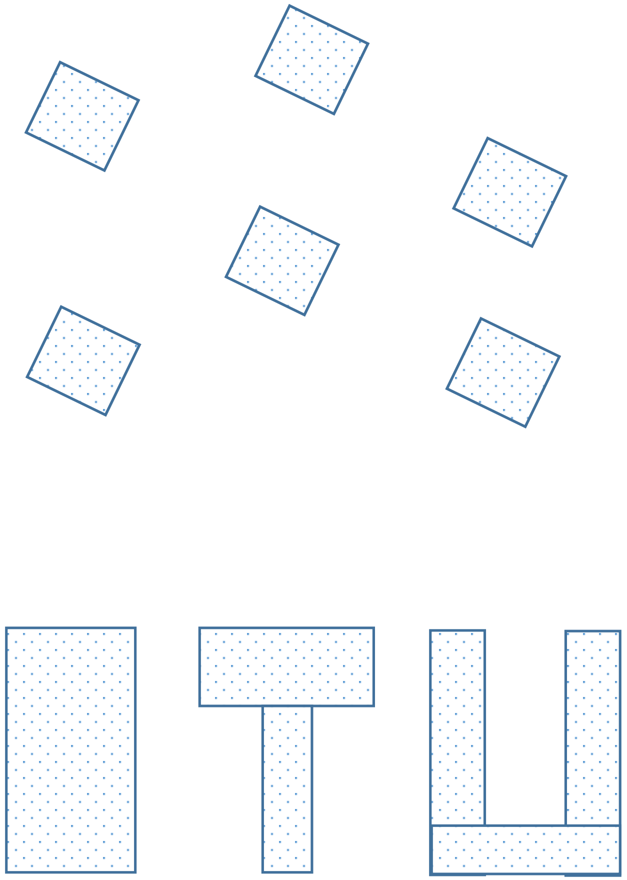
1. **Unloaded** from incoming trucks
2. **Sorted**, and
3. **Loaded** into outgoing trucks, within 24 hours.



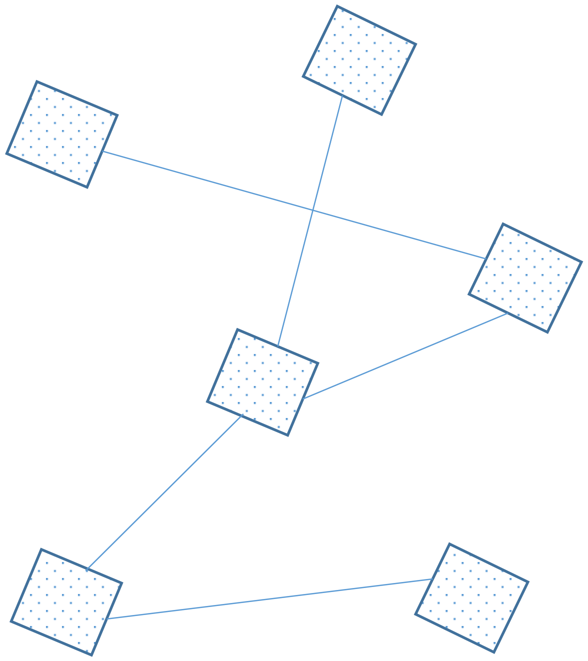
- ✓ In Hong Kong, almost all 3PLs use Crossdocking.
- ✓ More than 80% of publications in the last decade.
- ✓ Cross-docking is implemented successfully in:
  - ✓ Retailing, Manufacturing, Automotive, Courier, and Photography.

Cross-docking Problems

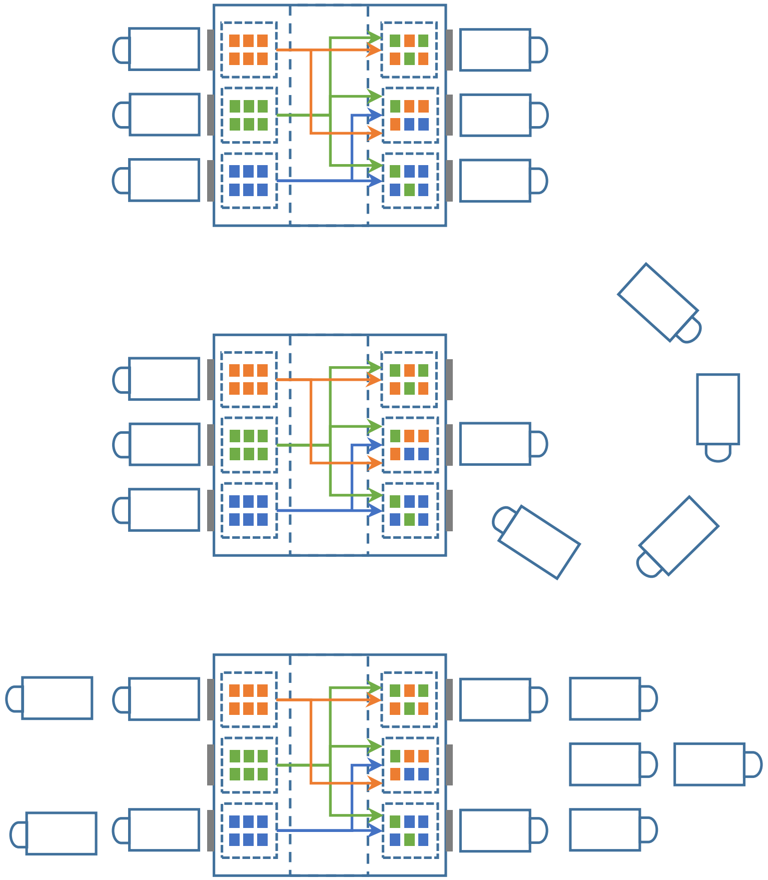
Strategic



Tactical



Operational



Literature

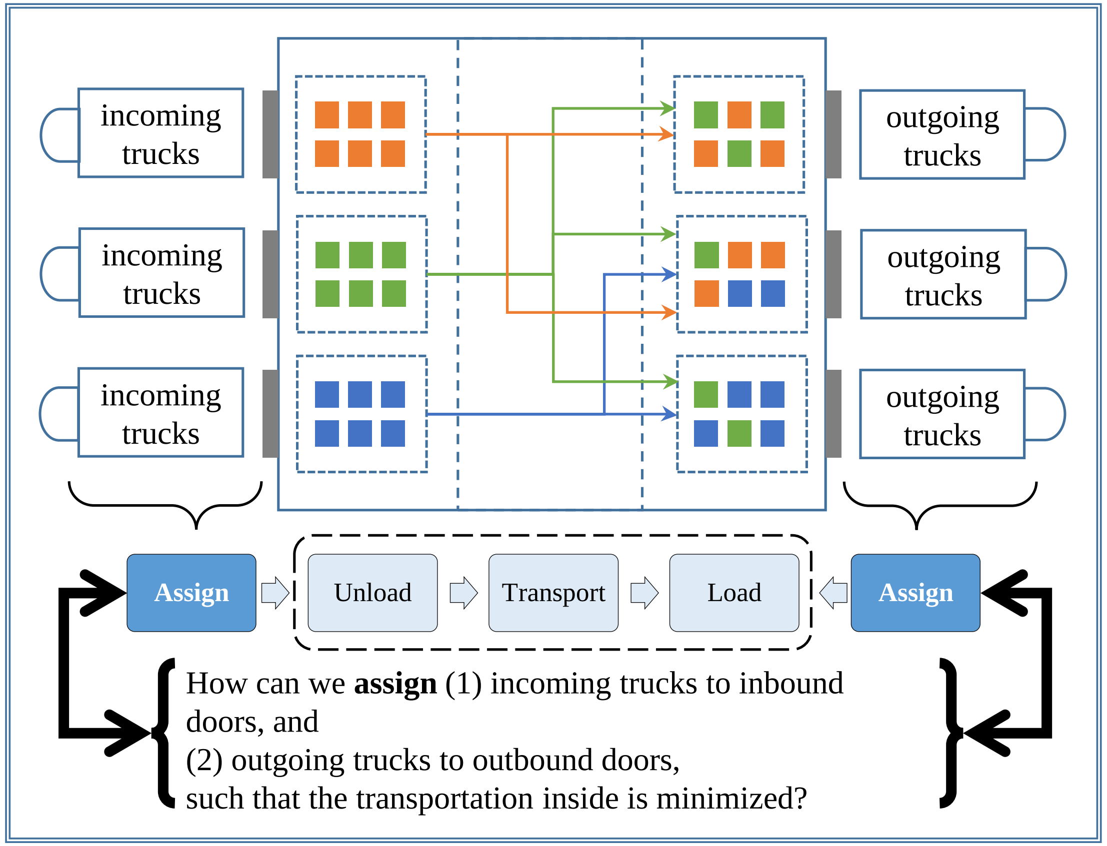
1983	Peck [1]			simulation
			MIP	greedy heuristic
1990	Tsui and Chang [2]	1-1 assignments	MIP	
1992			Linearization	B&B
2006	Oh, Hwang, Cha and Lee [3]		MIP	two heuristics
2008	Bozer and Carlo [4]		MIP	simulated annealing
2009	Cohen and Keren [5]		MIP	heuristic algorithm
2009	Zhu, Hahn, Liu, Guignard [6]	Introduced CDAP	MIP	
2012	Guignard, Hahn, Pessoa and Cardoso da Silva [7]		RLT - MIP	3 heuristics and B&B
2015	Nassief, Contreras and As'ad [8]		MIP	LR + Local search
2016	Nassief, Contreras and Jaumard [9]	Comparisons	MIPs	CG + Tabu search

Problem Statement

Sets:
- Set of origins (incoming trucks) $m \in M$
- Set of destinations (outgoing trucks) $n \in N$
- Set of inbound (strip) doors $i \in I$
- Set of outbound (stack) doors $j \in J$

Parameters:
- Commodities between origins and destinations w_{mn}
- Distances between doors d_{ij}
- Capacities on inbound (strip) doors S_i
- Capacities on outbound (stack) doors R_j

Binary decision variables:
- Assignment of origins to strip doors x_{mi}
- Assignment of destinations to stack doors y_{nj}
- Paths of commodities. z_{mijn}



Classical formulation: (Zhu et al 2009) [6]

$$\min \quad \sum_{m\in M}\sum_{i\in I}\sum_{n\in N}\sum_{j\in J} w_k d_{ij} x_{mj} y_{nj}$$

$$s.t. \quad \sum_{i\in I} x_{mi} = 1 \qquad \forall\, m \in M$$

$$\sum_{j\in J} y_{nj} = 1 \qquad \forall\, n \in N$$

$$\sum_{m\in M} s_m x_{mi} \le S_i \qquad \forall\, i \in I$$

$$\sum_{n\in N} r_n y_{nj} \le R_j \qquad \forall\, j \in J$$

$$x_{mi} \in \{0,1\} \qquad \forall\, m \in M,\ i \in I$$

$$y_{nj} \in \{0,1\} \qquad \forall\, n \in N,\ j \in J$$

MIP Formulation: M1 (Nassief et al 2015) [8]

$$\min \quad \sum_{k\in K}\sum_{i\in I}\sum_{j\in J} w_k d_{ij} z_{kij}$$

$$s.t. \quad \sum_{i\in I}\sum_{j\in J} z_{kij} = 1 \qquad \forall\, k \in K \qquad (1)$$

$$\sum_{j\in J} z_{kij} = x_{o(k)i} \qquad \forall\, k \in K,\ i \in I \qquad (2)$$

$$\sum_{i\in I} z_{kij} = y_{d(k)j} \qquad \forall\, k \in K,\ j \in J \qquad (3)$$

$$\sum_{m\in M} s_m x_{mi} \le S_i \qquad \forall\, i \in I \qquad (4)$$

$$\sum_{n\in N} r_n y_{nj} \le R_j \qquad \forall\, j \in J \qquad (5)$$

$$\sum_{k\in K: d(k)=n} s_{o(k)} z_{kij} \le S_i y_{nj} \qquad \forall\, n \in N,\ j \in J,\ i \in I \qquad (6)$$

$$\sum_{k\in K: o(k)=m} r_{d(k)} z_{kij} \le R_j x_{mi} \qquad \forall\, m \in M,\ i \in I,\ j \in J \qquad (7)$$

$$x_{mi} \in \{0,1\} \qquad \forall\, m \in M,\ i \in I \qquad (8)$$

$$y_{nj} \in \{0,1\} \qquad \forall\, n \in N,\ j \in J \qquad (9)$$

$$z_{kij} \ge 0 \qquad \forall\, k \in K,\ i \in I,\ j \in J \qquad (10)$$

MIP Formulation: M2

$$\min \quad \sum_{k\in K}\sum_{i\in I}\sum_{j\in J} w_k d_{ij} z_{kij}$$

s.t.

$$\sum_{i\in I}\sum_{j\in J} z_{kij} = 1 \qquad \forall\, k \in K \qquad (1)$$

$$\sum_{j\in J} z_{kij} = \sum_{c\in C_i : o(k)\in C} x_i^c \qquad \forall\, k \in K,\ i \in I \qquad (11)$$

$$\sum_{i\in I} z_{kij} = \sum_{l\in L_j : d(k)\in L} y_j^l \qquad \forall\, k \in K,\ j \in J \qquad (12)$$

$$\sum_{c\in C_i} x_i^c = 1 \qquad \forall\, i \in I \qquad (13)$$

$$\sum_{l\in L_j} y_j^l = 1 \qquad \forall\, j \in J \qquad (14)$$

$$\sum_{k\in K : d(k)=n} s_{o(k)} z_{kij} \le S_i \sum_{l\in L_j : n\in L} y_j^l \qquad \forall\, n \in N,\ j \in J,\ i \in I \qquad (15)$$

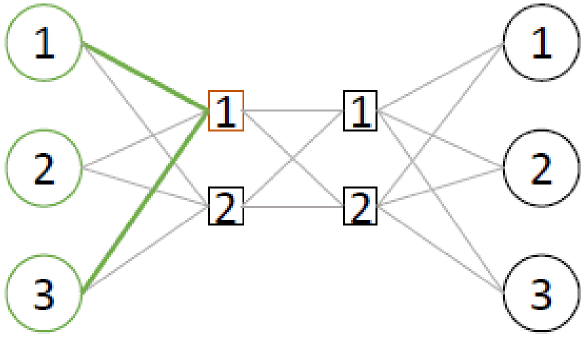
$$\sum_{k\in K : o(k)=m} r_{d(k)} z_{kij} \le R_j \sum_{c\in C_i : m\in C} x_i^c \qquad \forall\, m \in M,\ i \in I,\ j \in J \qquad (16)$$

$$x_i^c \in \{0,1\} \qquad \forall\, c \in C_i,\ i \in I \qquad (17)$$

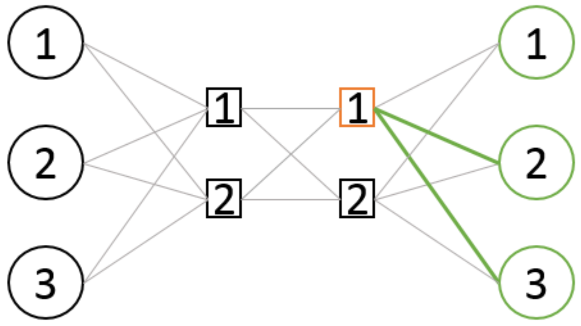
$$y_j^l \in \{0,1\} \qquad \forall\, l \in L_j,\ j \in J \qquad (18)$$

$$z_{kij} \ge 0 \qquad \forall\, k \in K,\ i \in I,\ j \in J \qquad (10)$$

$$C_i = \left\{ M' \subseteq M : \sum_{m\in M'} s_m x_{mi} \le S_i \right\}$$



$$L_j = \left\{ N' \subseteq N : \sum_{n\in N'} r_n y_{nj} \le R_j \right\}$$



Lagrangean Relaxation on M1

$$\min \quad \sum_{k \in K} \sum_{i \in I} \sum_{j \in J} w_k d_{ij} z_{kij}$$

$s.t.$

$\sum_{i \in I} \sum_{j \in J} z_{kij} = 1$	$\forall\, k \in K$	(1)
$\sum_{j \in J} z_{kij} = x_{o(k)i}$	$\forall\, k \in K,\ i \in I$	(2)
$\sum_{i \in I} z_{kij} = y_{d(k)j}$	$\forall\, k \in K,\ j \in J$	(3)
$\sum_{m \in M} s_m x_{mi} \leq S_i$	$\forall\, i \in I$	(4)
$\sum_{n \in N} r_n y_{nj} \leq R_j$	$\forall\, j \in J$	(5)
$\sum_{k \in K: d(k)=n} s_{o(k)} z_{kij} \leq S_i y_{nj}$	$\forall\, n \in N,\ j \in J,\ i \in I$	(6)
$\sum_{k \in K: o(k)=m} r_{d(k)} z_{kij} \leq R_j x_{mi}$	$\forall\, m \in M,\ i \in I,\ j \in J$	(7)
$x_{mi} \in \{0,1\}$	$\forall\, m \in M,\ i \in I$	(8)
$y_{nj} \in \{0,1\}$	$\forall\, n \in N,\ j \in J$	(9)
$z_{kij} \geq 0$	$\forall\, k \in K,\ i \in I,\ j \in J$	(10)

=> LD function:

- Three independent sub-problems.
- No integrality property.

A column-and-row generation algorithm for M2

$(RMP)^t \quad min \quad \sum_{k\in K}\sum_{(i,j)\in D_k^t} w_k d_{ij}\, z_{kij}^t$

$s.t.$

$\sum_{(i,j)\in D_k^t} z_{kij}^t = 1$	$\forall\, k \in K$	(19)
$\sum_{(i,j)\in D_k^t} z_{kij}^t = \sum_{c\in C_i^t} a_c^t x_c^t$	$\forall\, k \in K,\ i \in I$	(20)
$\sum_{(i,j)\in D_k^t} z_{kij}^t = \sum_{l\in L_j^t} b_c^t y_c^t$	$\forall\, k \in K,\ j \in J$	(21)
$\sum_{k\in K}\sum_{(i,j)\in D_k^t} s_{o(k)} z_{kij} \le S_i \sum_{l\in L_j^t : n\in l} y_l$	$\forall\, n, j, i \in CP_1^t$	(22)
$\sum_{k\in K}\sum_{(i,j)\in D_k^t} r_{d(k)} z_{kij} \le R_j \sum_{c\in C_i^t : m\in c} x_c$	$\forall\, m, i, j \in CP_2^t$	(23)
$\sum_{c\in C_i} x_i^c = 1$	$\forall\, i \in I$	(24)
$\sum_{l\in L_j} y_j^l = 1$	$\forall\, j \in J$	(25)
$x_c^t \in \{0,1\}$	$\forall\, c \in C_i^t$	(26)
$y_l^t \in \{0,1\}$	$\forall\, l \in L_j^t$	(27)
$z_{kij}^t \geq 0$	$\forall\, k \in K, (i,j) \in D_k^t$	(28)

1 Heuristic => initial columns

2

3

PP_i^X 0-1 Knapsack (strip door)

PP_j^y 0-1 Knapsack (stack door)

PP_k^z Combinatorial problem

4 Constructive phase

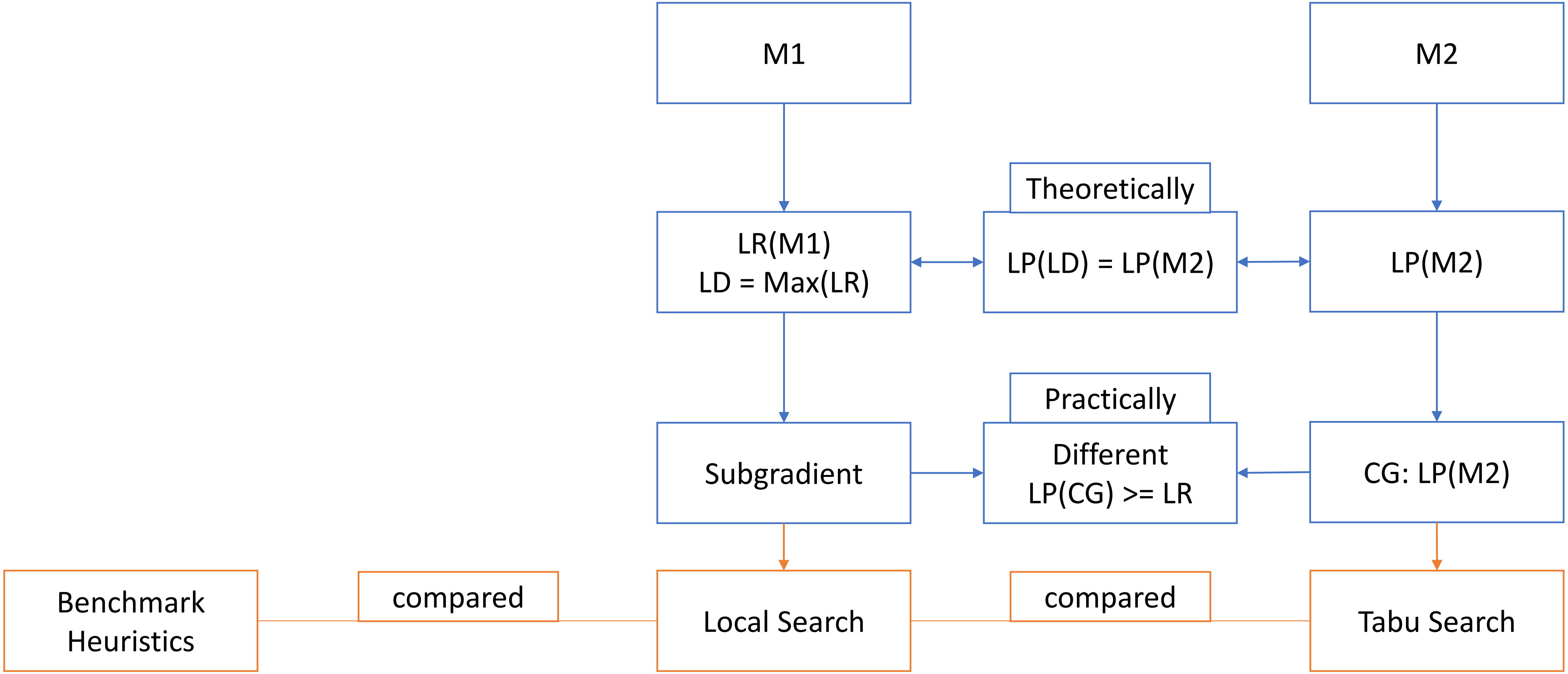
5

6 Add (22) and (23) if violated

7

8 Tabu Search

Comparisons



Results:

No	Instance	LR(M1)	CG(M2)	LR(M1)	CG(M2)	H1	H2	H3	LR(M1)	CG(M2)
		%LB		%UB					Time (sec.)	
1	8x4	3.23	2.47	0.00	0.00	0.00	0.00	0.00	5.56	0.38
2	9x4	3.93	3.08	0.00	0.00	0.00	0.00	0.00	10.53	0.49
3	10x4	4.80	3.80	0.00	0.06	0.00	0.00	0.00	11.65	0.57
4	10x5	5.40	4.25	0.00	0.00	0.15	0.15	0.12	11.79	0.84
5	11x5	6.13	4.96	0.00	0.12	0.11	0.10	0.12	16.52	1.12
6	12x5	5.83	4.60	0.00	0.00	0.05	0.03	0.04	15.87	1.34
7	12x6	7.19	6.27	0.00	0.06	0.14	0.25	0.19	21.08	1.84
8	15x6	6.93	5.81	0.00	0.00	0.20	0.17	0.27	32.06	3.21
9	15x7	8.39	6.98	0.11	0.05	0.19	0.09	0.12	29.63	5.51
10	20x10	11.73	10.66	0.31	0.02	0.44	0.16	0.56	78.63	39.42
	Average	6.36	5.29	0.04	0.03	0.13	0.09	0.14	23.33	5.47

Results:

No	Instance	LR(M1)	CG(M2)	LR(M1)	CG(M2)	LR(M1)	CG(M2)
		%LB		%UB		Time (sec.)	
11	10x4	4.01	3.29	0.00	0.00	6.03	0.62
12	15x6	6.20	4.96	0.00	0.12	16.73	3.33
13	20x10	10.15	8.88	0.11	0.24	53.60	38.96
14	25x10	11.29	10.01	0.22	0.00	96.05	76.90
15	50x10	15.90	14.89	0.04	0.01	873.54	5810.87
16	50x20	25.57	23.96	0.17	0.38	1623.42	36811.33
	Average	12.19	11.00	0.09	0.12	444.89	7123.67

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