The tail assignment problem with look-ahead maintenance constraints

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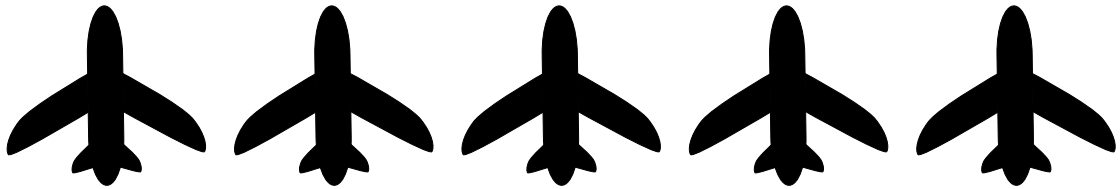
24th May 2016

Tail assignment problem



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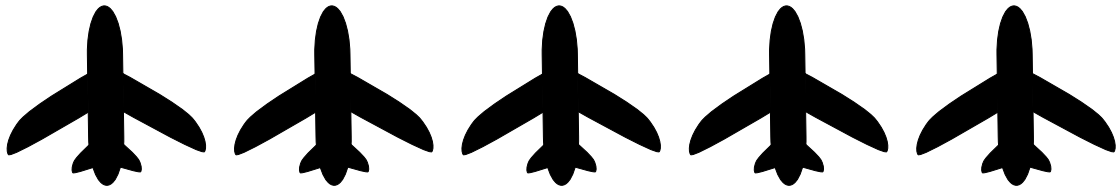

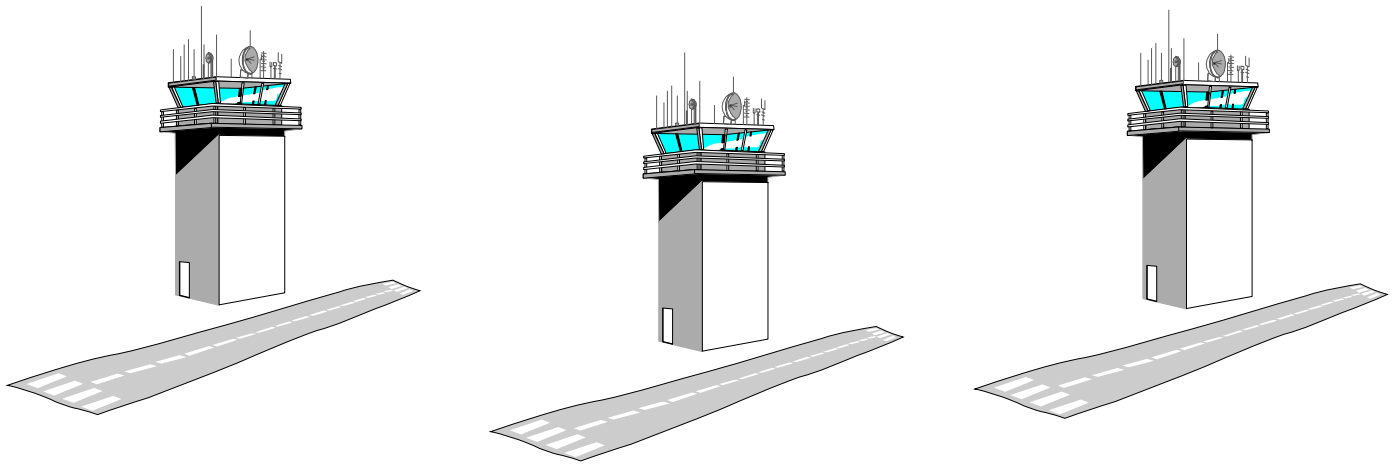

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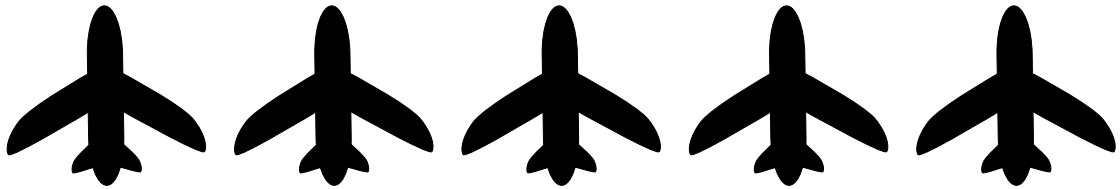

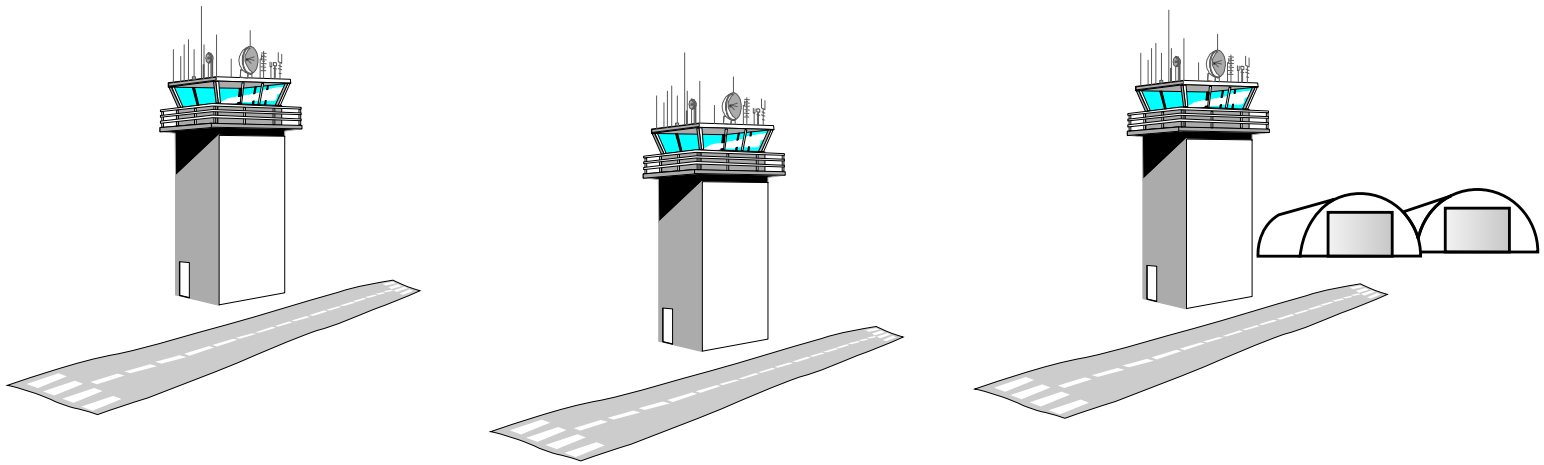

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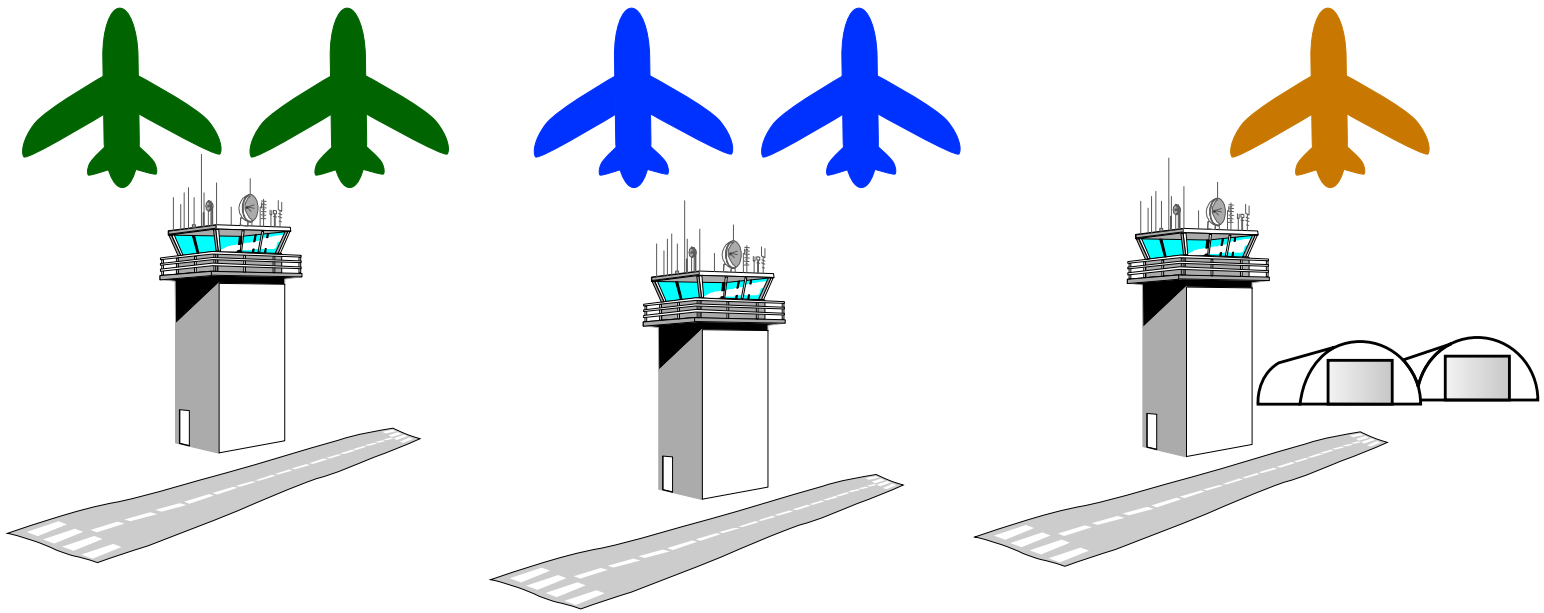

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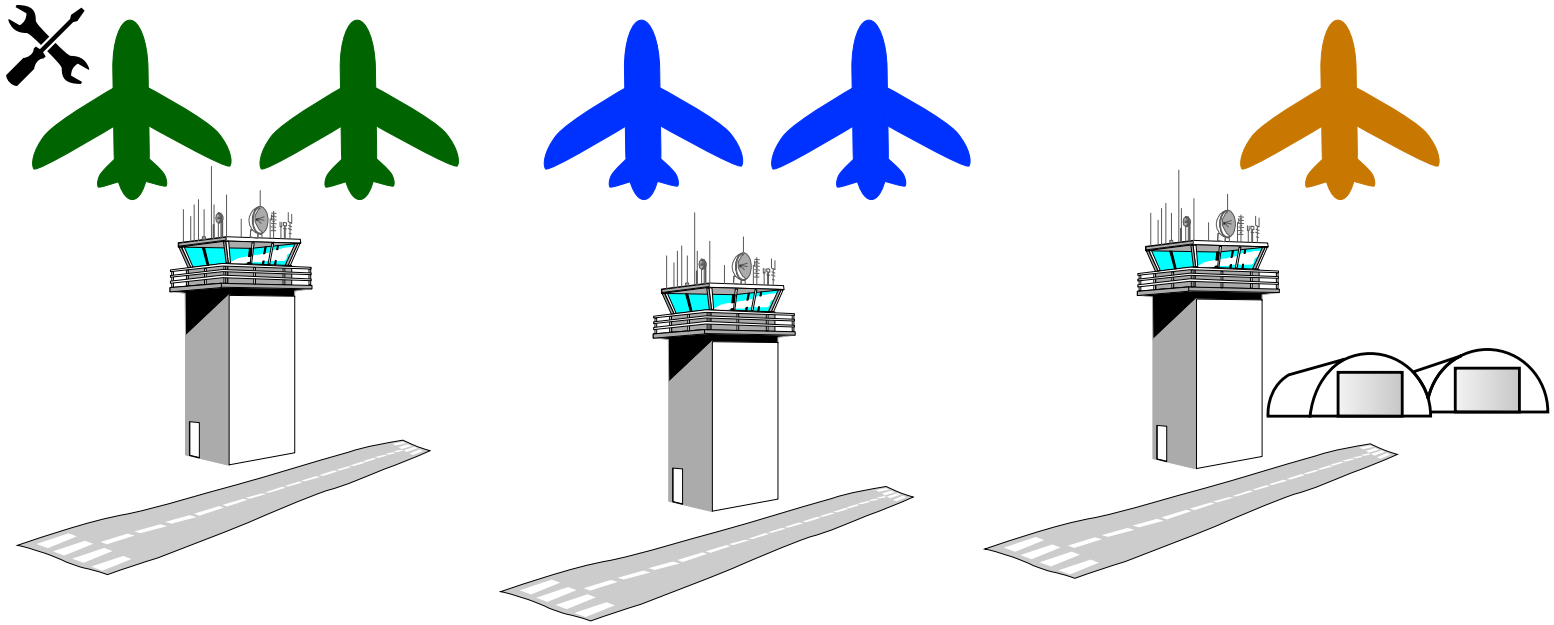
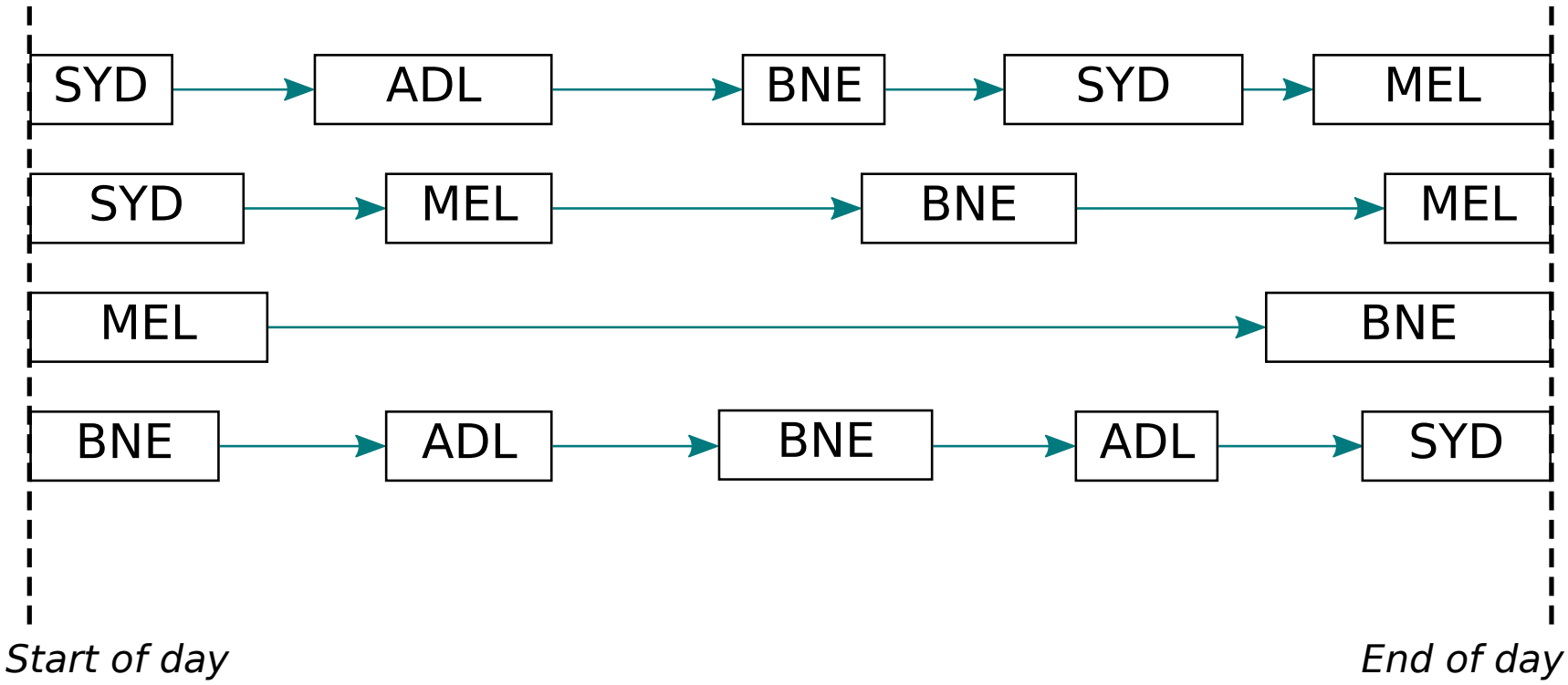


Line-of-flight



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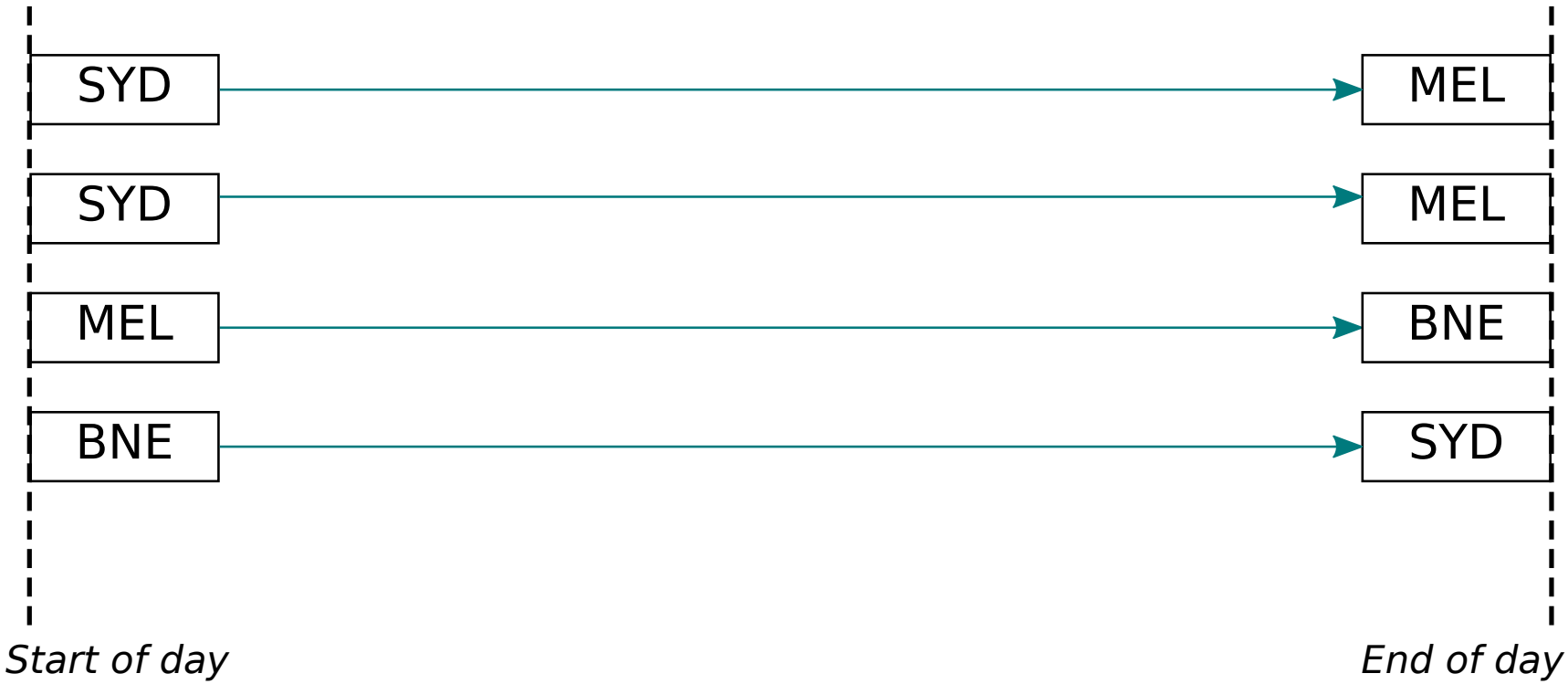


Fixed line-of-flight input

Benefits

- Separate aircraft routing and tail assignment problems.
- Results in a simple problem for the tail assignment.

Difficulties

- Unable to plan for all realisations of the maintenance schedule and aircraft locations.
- No recourse when disruptions occur.
- May result in aircraft not satisfying maintenance requirements.

The tail assignment problem

Problem definition

The assignment of LOFs to aircraft, while:

- minimising the number of maintenance misalignments of days one, two and three.

Day-one

- Introduce a constraint with slack variable to assign maintenance critical aircraft an MLOF.

$$\sum_{p \in P^r} o_p y_p^r + s_1^r \geq \theta_1^r \quad \forall b \in B, \forall r \in R^b.$$

Day-two

- Introduce look-ahead constraint.
- Count the number of terminating maintenance critical aircraft.
- Ensure this does not exceed the available maintenance routes on day two.

$$\sum_{b_1 \in B} \sum_{r \in R^{b_1}} \sum_{\substack{p \in P^{b_1} \mid \\ term(p) = b_2}} \theta_2^r y_p^r - s_2^{b_2} \leq M_{b_2}^2 \quad \forall b_2 \in B,$$

$$s_2^{b_2} \geq 0 \quad \forall b_2 \in B.$$

Day-three

- Introduce look-ahead constraint.
- Count the number of terminating maintenance critical aircraft.
- Ensure this does not exceed the available paths to maintenance.
- Ensure that the assigned paths through each base does not exceed the available maintenance routes on day three.

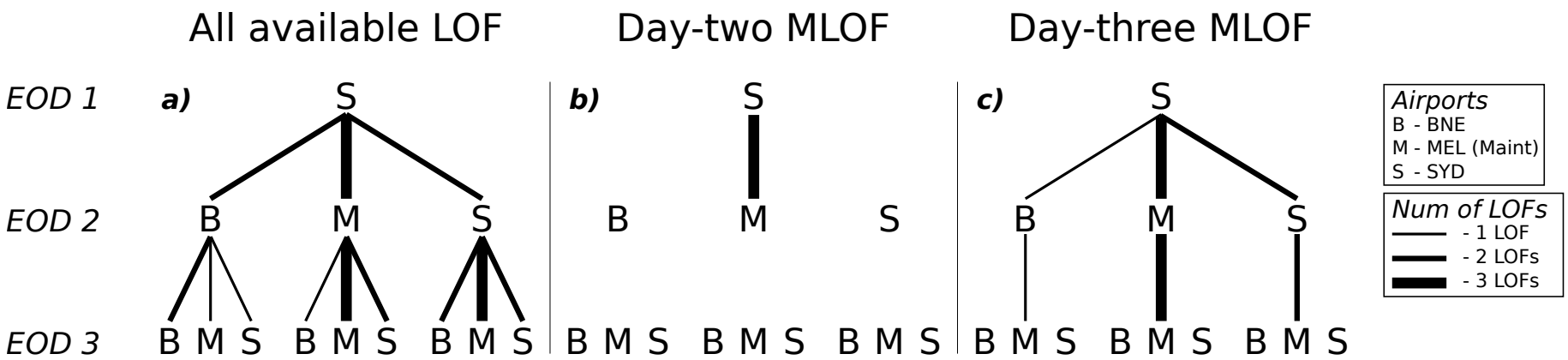
$$\sum_{b_1 \in B} \sum_{r \in R^{b_1}} \sum_{\substack{p \in P^{b_1} \mid \\ term(p)=b_2}} \theta_3^r y_p^r - s_3^{b_2} = \sum_{b_3 \in B} \rho_{b_2 b_3} \quad \forall b_2 \in B,$$

$$\sum_{b_3 \in \hat{B}} \rho_{b_2 b_3} + \sum_{b_1 \in B} \sum_{r \in R^{b_1}} \sum_{\substack{p \in P^{b_1} \mid \\ term(p)=b_2}} \theta_2^r y_p^r - s_2^{b_2} \leq M_{b_2}^2 \quad \forall b_2 \in B,$$

$$\sum_{b_2 \in B} \rho_{b_2 b_3} \leq M_{b_3}^3 \quad \forall b_3 \in B,$$

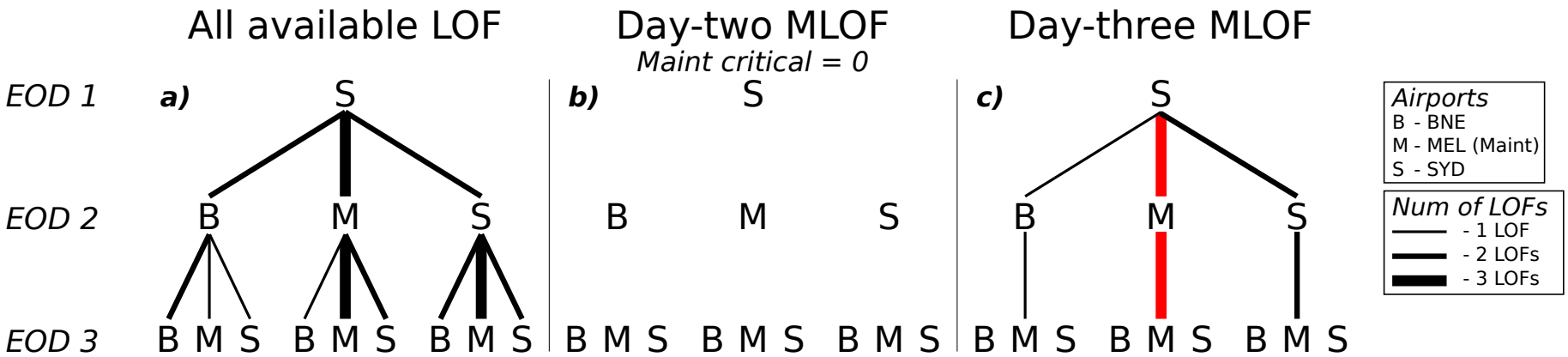
$$\rho_{b_2 b_3} \in [0, N^{b_2 b_3}] \; \forall b_2, b_3 \in B, \quad s_3^{b_3} \geq 0 \; \forall b_3 \in B.$$

Day-two and -three maintenance misalignments



- Define number of maintenance LOF: $M^2_{b_2}$ and $M^3_{b_3}$.
- Define number of day-three maintenance routes: $N^{b_2 b_3}$.

Day-two and -three maintenance misalignments



- Constrain number of aircraft assigned to maintenance routes:

$$\sum_{b_3 \in \hat{B}} \rho_{b_2 b_3} + \sum_{b_1 \in B} \sum_{r \in R^{b_1}} \sum_{\substack{p \in P^{b_1} \mid \\ term(p) = b_2}} \theta_2^r y_p^r - s_2^{b_2} \leq M_{b_2}^2 \quad \forall b_2 \in B.$$

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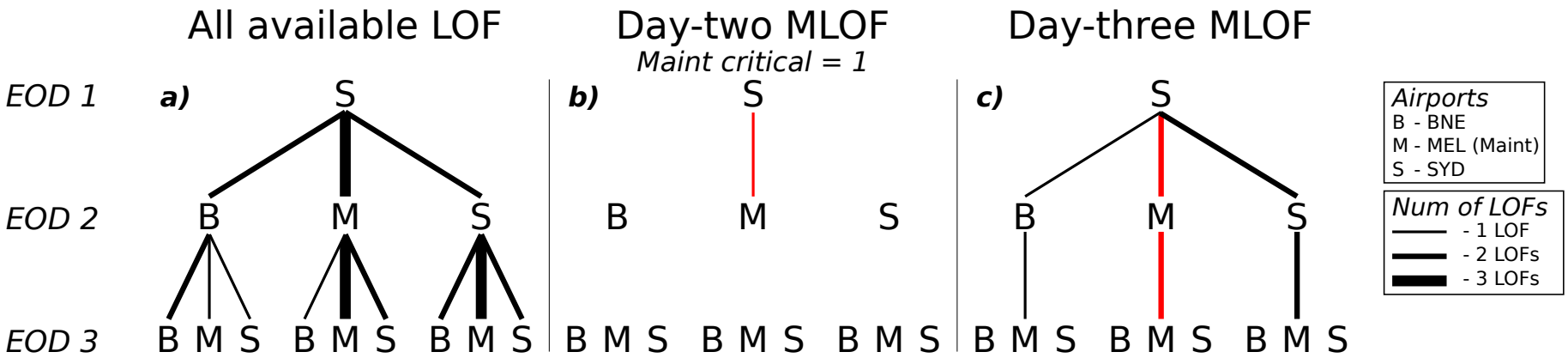

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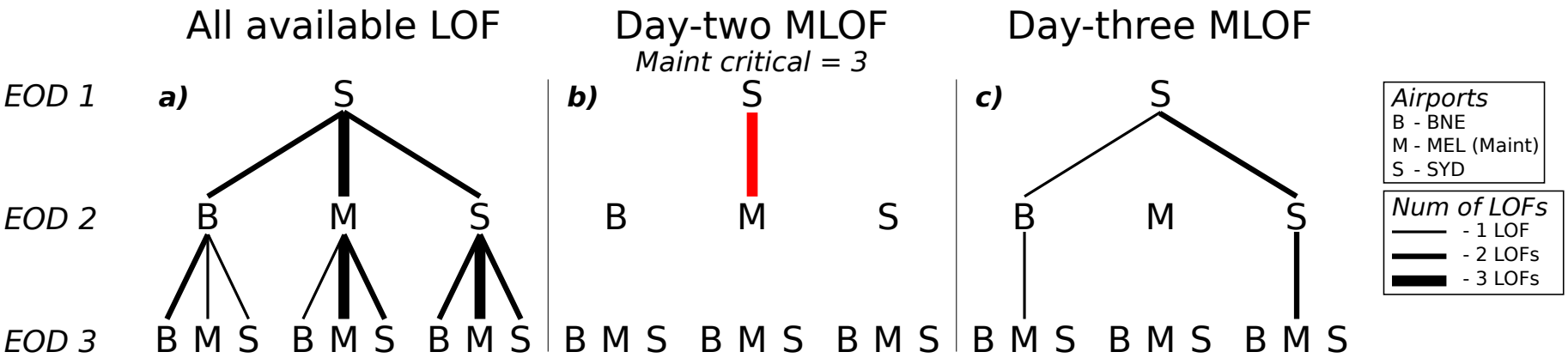

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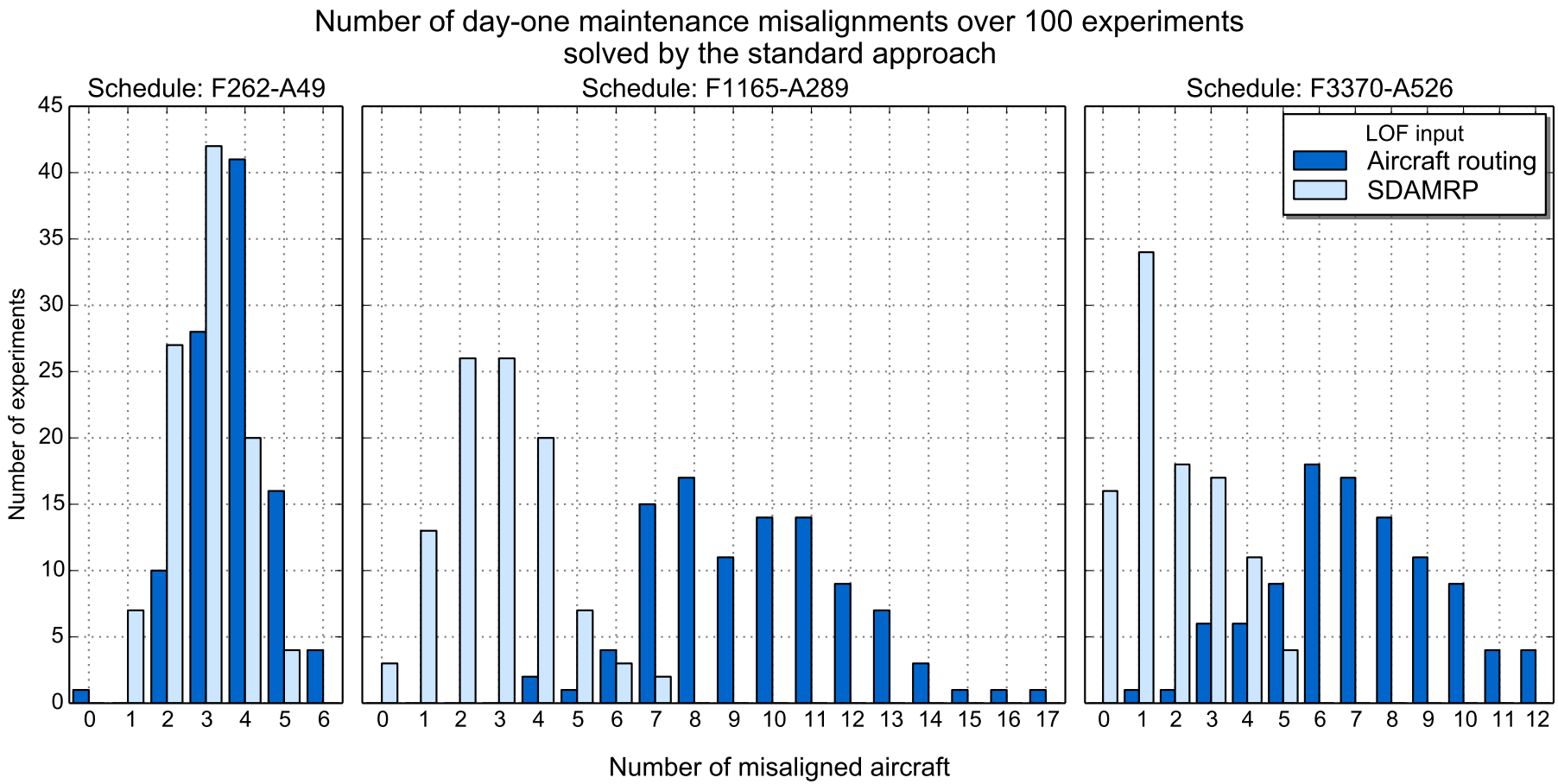
Day-one maintenance misalignments



Realised maintenance misalignment

SDAMRP LOFs

	F267-A49	F1165-A289	F3370-A526
Without look-ahead	3.2527	3.3001	2.2902
With look-ahead	1.6667	0.9666	0.6159
Improvement	48.75%	70.71%	73.11%

Table : The average number of day-one maintenance misalignments per day after successively solving the TAP for three days with and without look-ahead constraints.

Route adjustment process

- Need to modify input LOFs to reduce the number of maintenance misalignments.
- Traditionally achieved by performing aircraft swaps.

Alternative approaches

- Solve the TAP using column generation to construct LOFs minimising maintenance misalignments.
- Apply column generation heuristically by generating LOFs for a subset of aircraft and flights.

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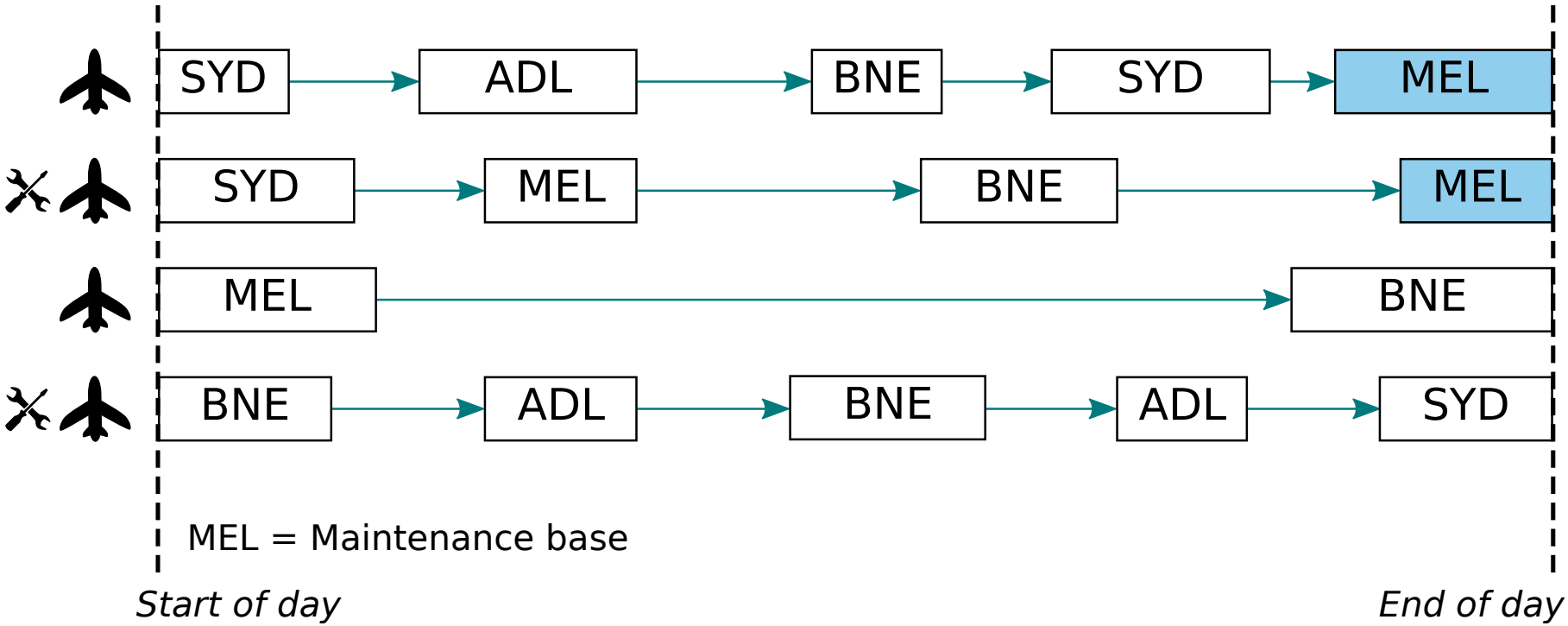
Iterative Algorithm

1. Solve the tail assignment problem.
2. Check
   - First run: If misalignments exists, select aircraft for route adjustment,
   - Other runs: If *improvement* occurs, update selected aircraft.
3. Solve route adjustment problem.
4. Update lines-of-flight for the tail assignment problem.

Selecting aircraft

After first TAP solve



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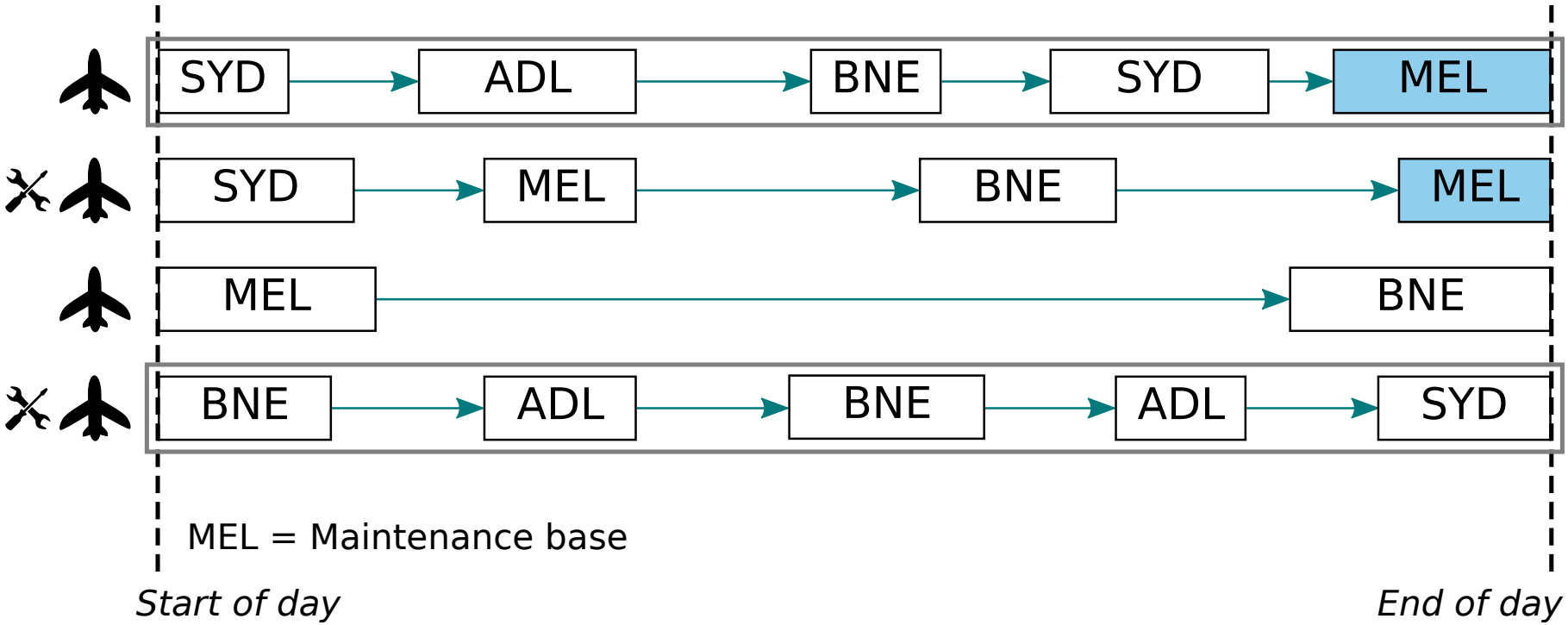
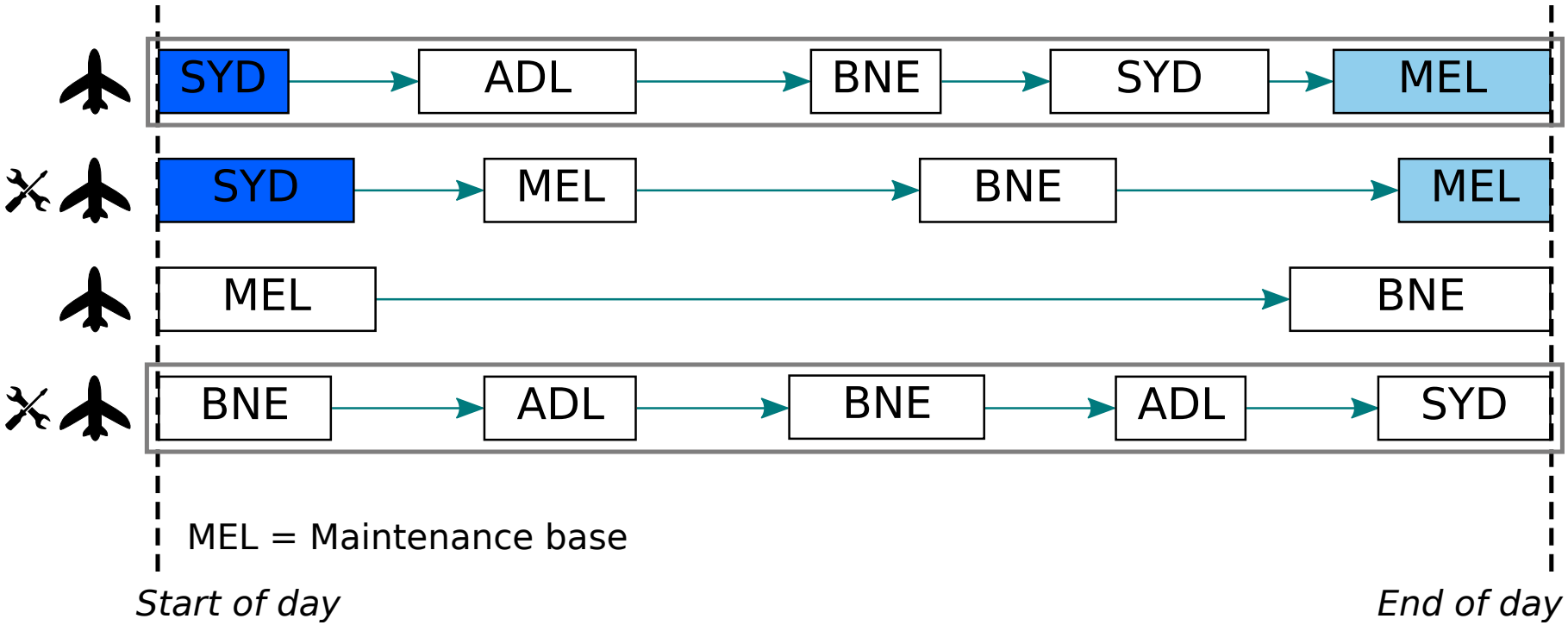


Updating selected aircraft

After subsequent TAP solves



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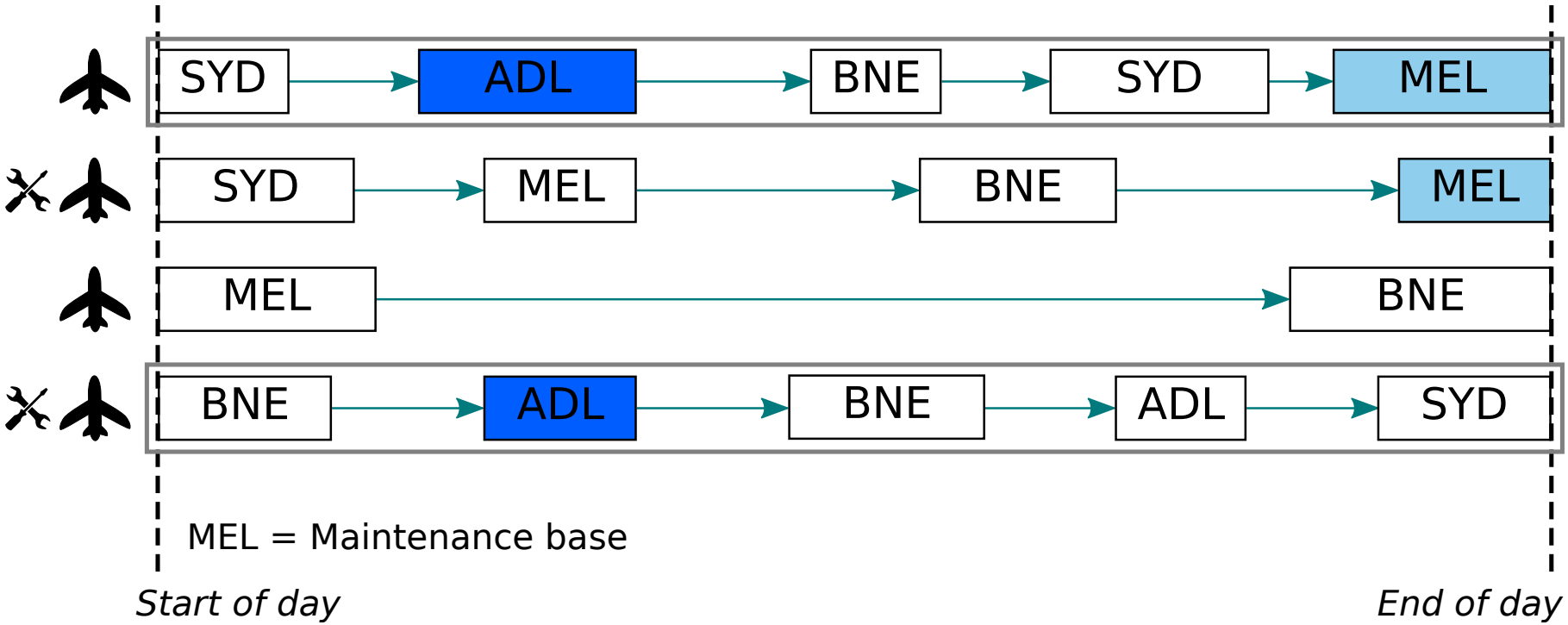
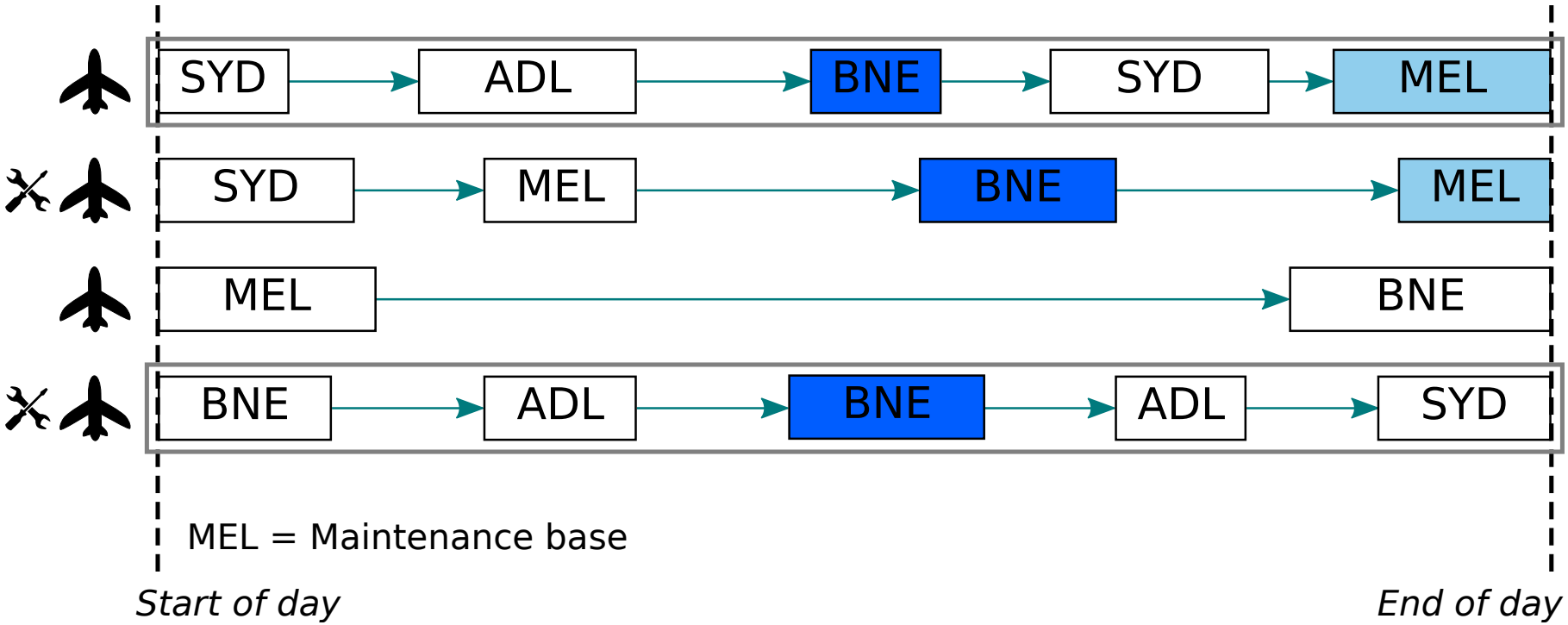


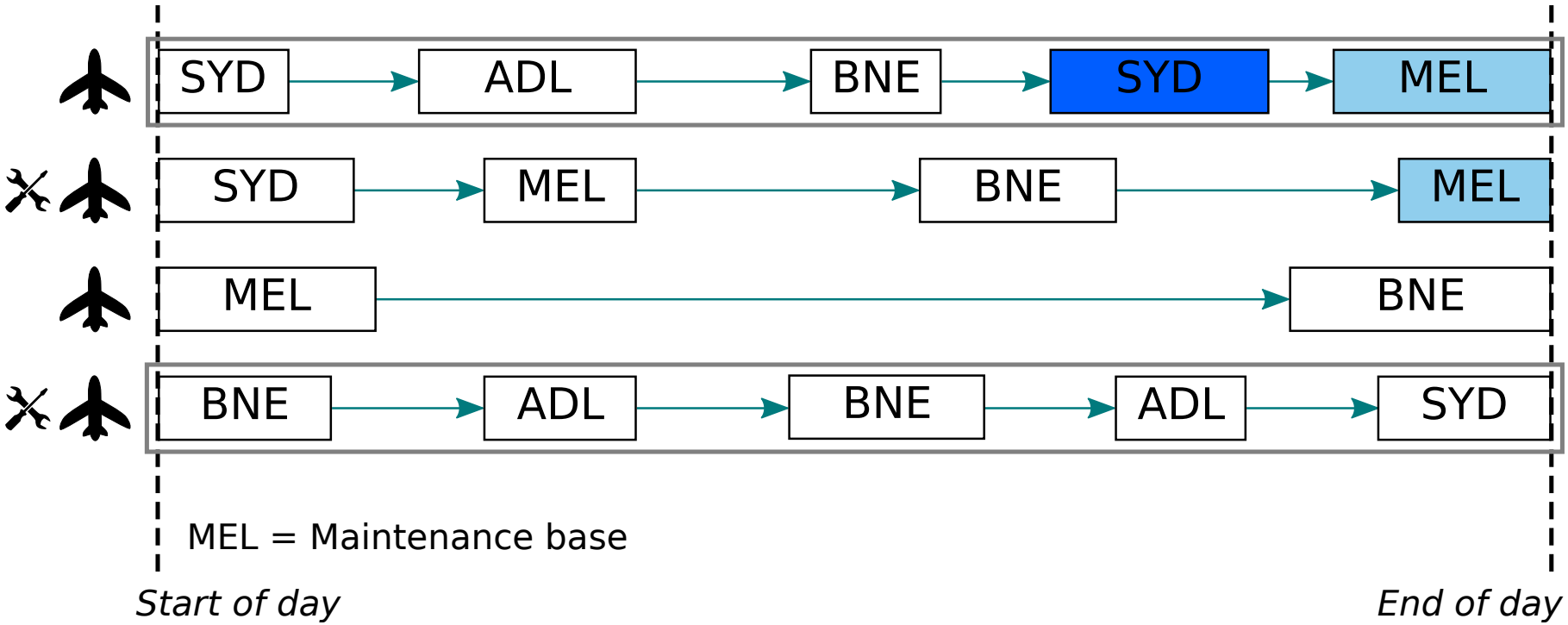
Updating selected aircraft

After subsequent TAP solves



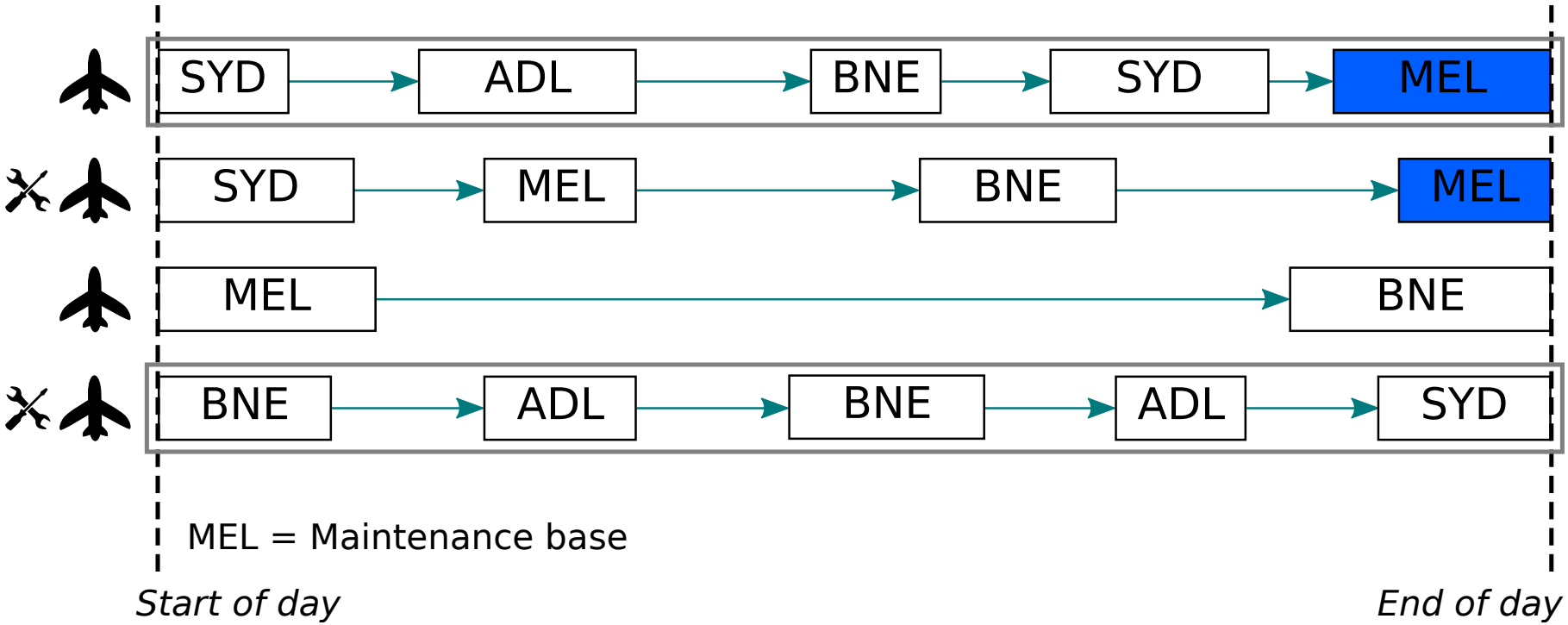
Updating selected aircraft

After subsequent TAP solves



Updating selected aircraft

After subsequent TAP solves



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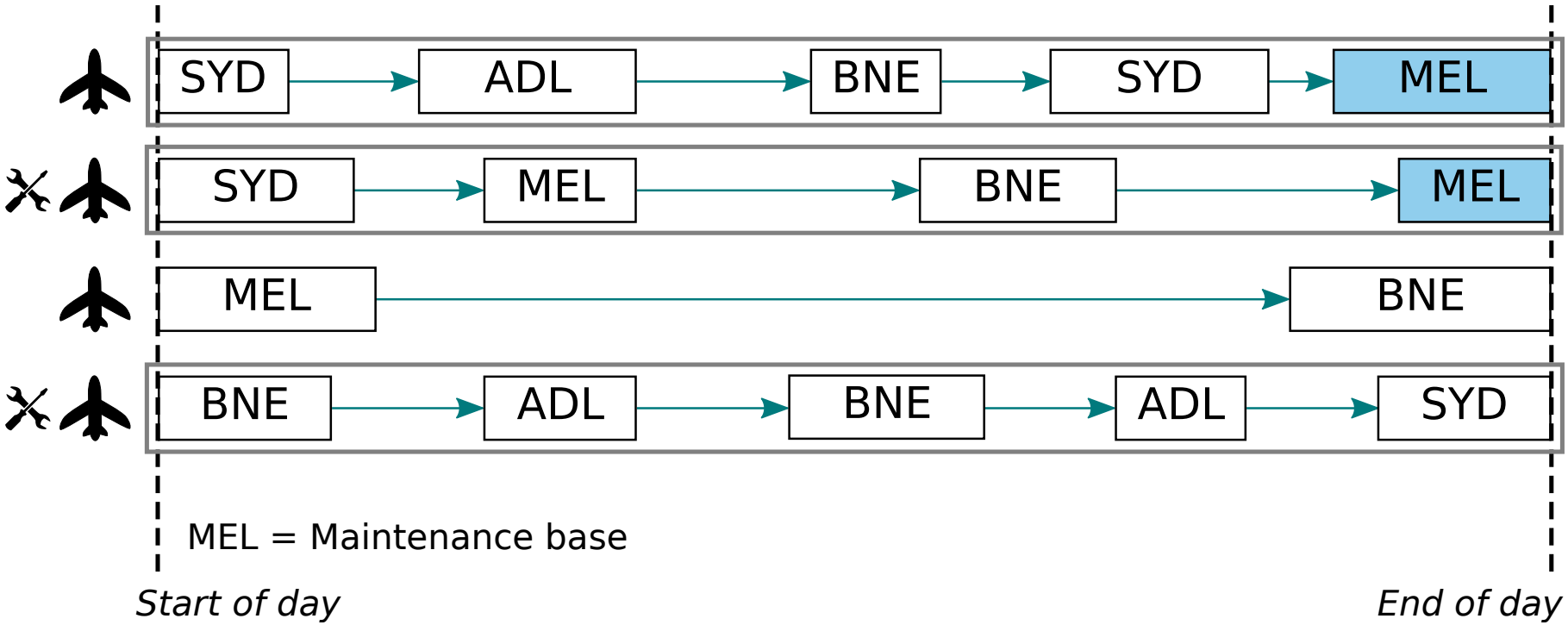
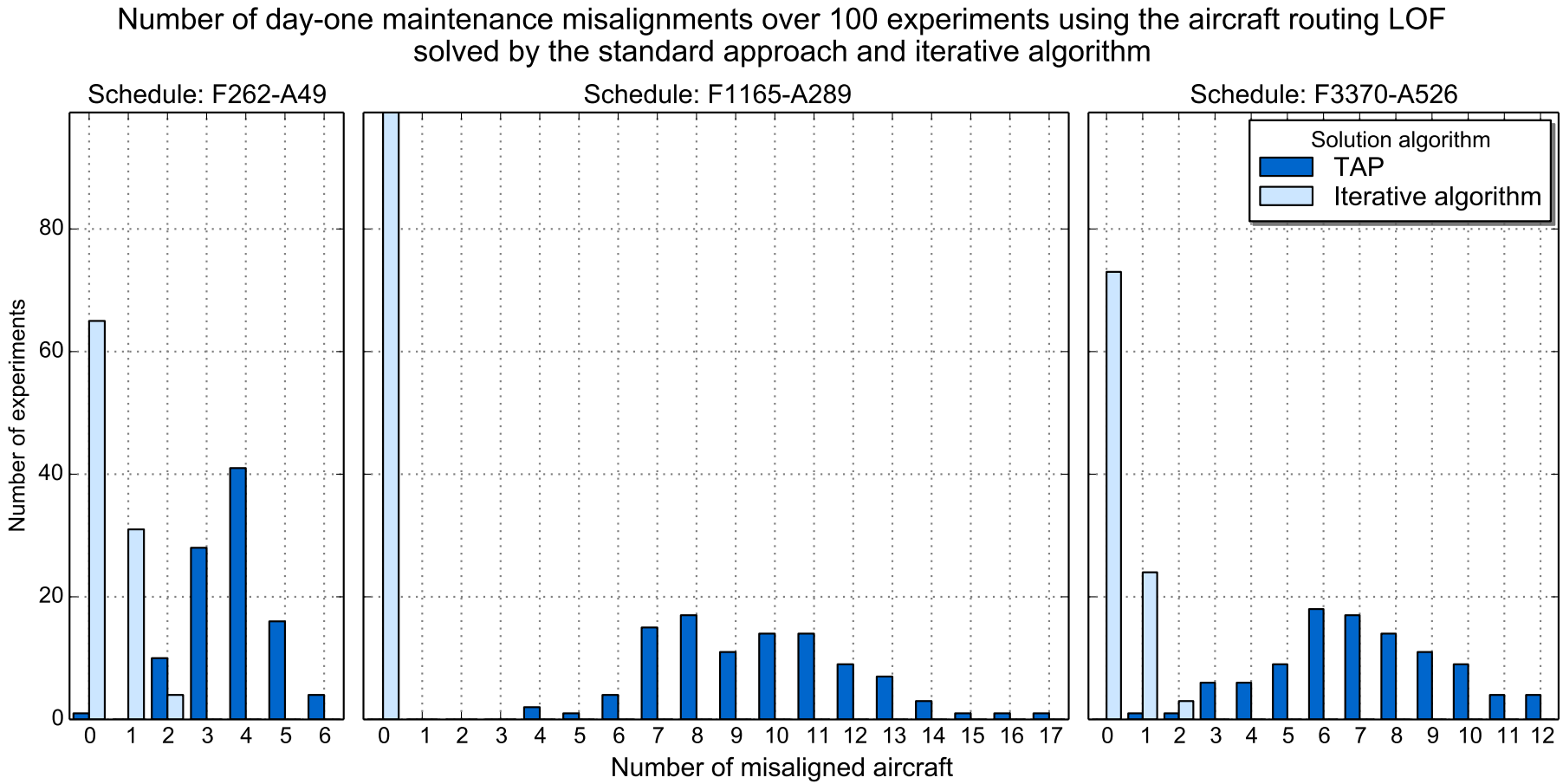


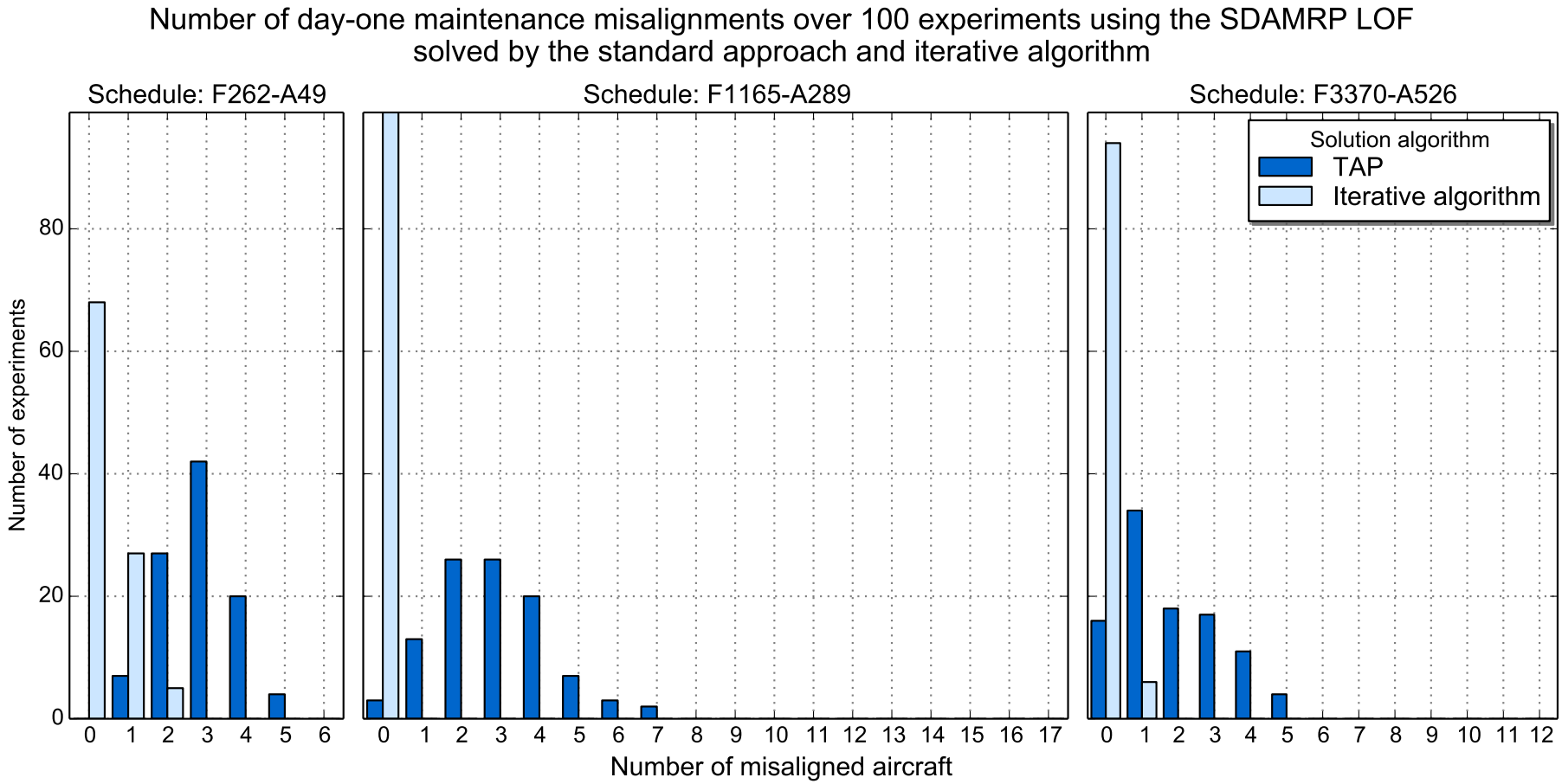
Compare TAP and Iterative Algorithm

Aircraft routing LOF



Compare TAP and Iterative Algorithm

SDAMRP LOF



Aircraft routing LOF

Misalignments	F267-A49		F1165-A289		F3370-A526	
	Col. Gen.	Iter.	Col. Gen.	Iter.	Col. Gen.	Iter.
0	88	65	100	100	76	73
1	12	31	0	0	21	24
2	0	4	0	0	3	3
≥ 3	0	0	0	0	0	0

Table : The number of experiments exhibiting different levels of maintenance misalignment at the end of day-one over a set of 100 experiments.

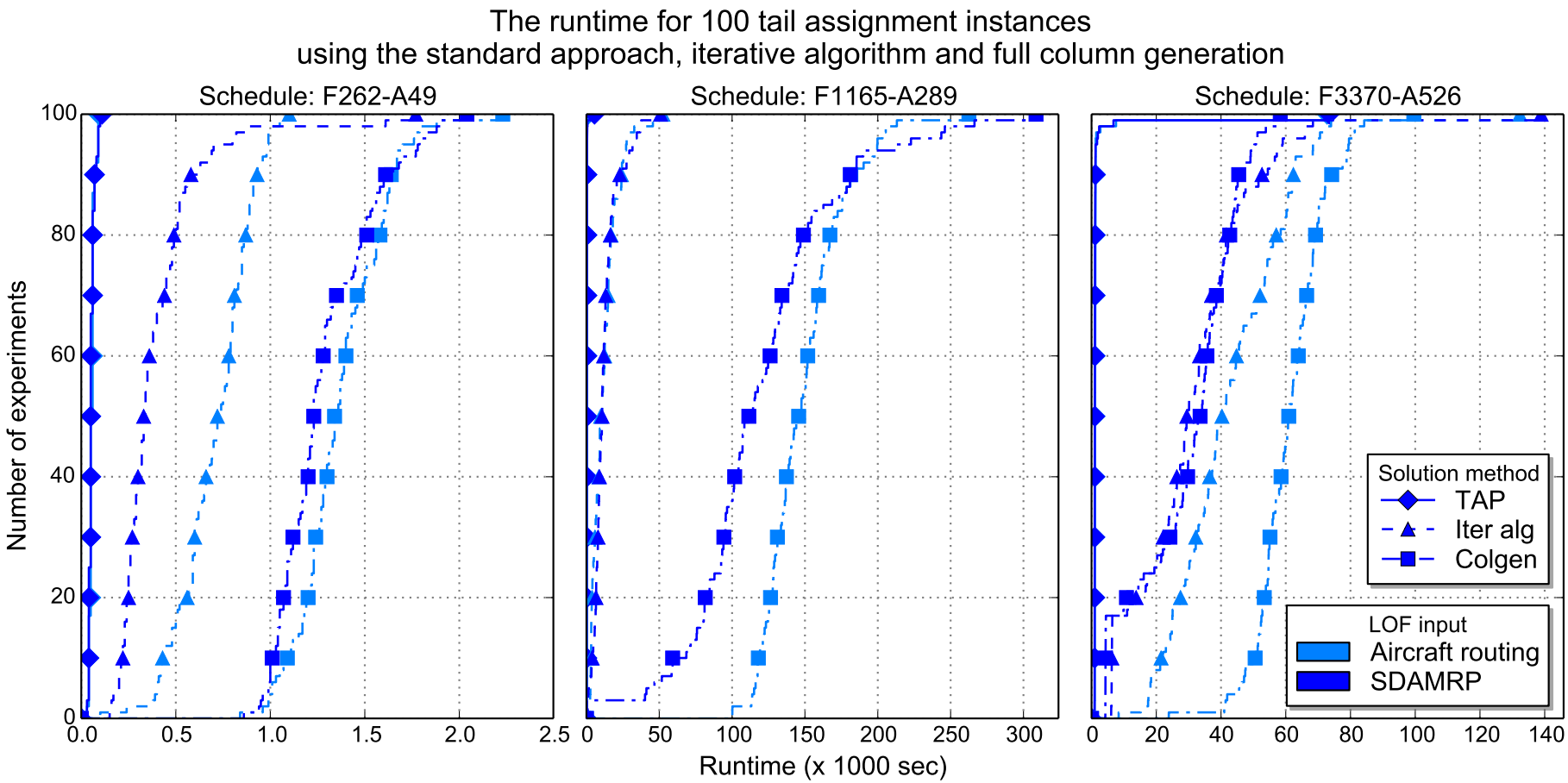
Column Generation and Iterative Algorithm

SDAMRP LOF

Misalignments	F267-A49		F1165-A289		F3370-A526	
	Col. Gen.	Iter.	Col. Gen.	Iter.	Col. Gen.	Iter.
0	100	68	100	100	95	94
1	0	27	0	0	5	6
2	0	5	0	0	0	0
≥ 3	0	0	0	0	0	0

Table : The number of experiments exhibiting different levels of maintenance misalignment at the end of day-one over a set of 100 experiments.

Runtime


The runtime for 100 tail assignment instances
using the standard approach, iterative algorithm and full column generation
Schedule: F262-A49
Schedule: F1165-A289
Schedule: F3370-A526
Number of experiments
Runtime (x 1000 sec)
Solution method
TAP
Iter alg
Colgen
LOF input
Aircraft routing
SDAMRP

Conclusions

- Presented a formulation for the tail assignment problem with look-ahead maintenance constraints using fixed one-day routes LOFs.
- Showed the ability of the look-ahead constraints to reduce maintenance misalignments.
- Developed an iterative algorithm to reduce the maintenance misalignments arising from using fixed LOF inputs.
- Demonstrated the potential of the column generation and iterative algorithm to satisfy maintenance requirements.