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A Branch and Price Approach for Deployment of Cloud Services

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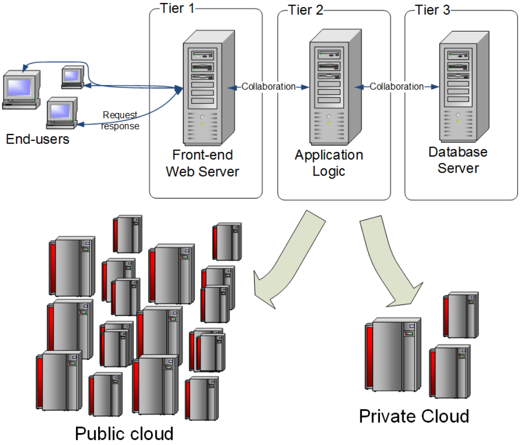
Problem Overview

Focus on the service deployment problem of a provider of cloud software services



Placement in private cloud

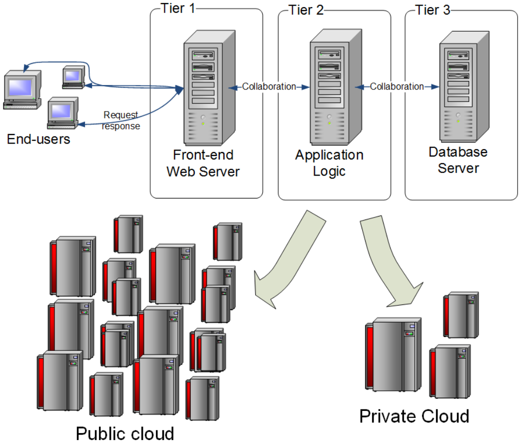
- mapping between the virtual machines (VMs) and the nodes

Placement in public cloud

- pool of infinite resources
- use only if private cloud is fully utilized

Problem Overview

Focus on the service deployment problem of a provider of cloud software services



Goal: obtain a cost-efficient service deployment while ensuring a satisfactory quality of service (QoS).

- provide a tolerable performance under the presence of failures

Static and dynamic problems

Demand patterns:

- recurring time periods with a stationary demand
- continuously fluctuating

Static problem

- solutions for a stationary demand period

Dynamic problem

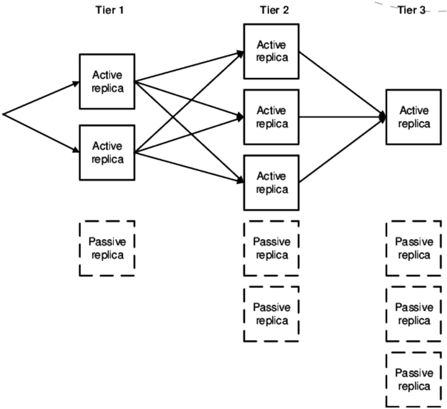
- strategies to adapt to a continuously changing demand

Fault tolerance and replication

Failure-prone infrastructure

- introduction of replicated software components
- standby redundancy with passive replicas allocated shared backup resources

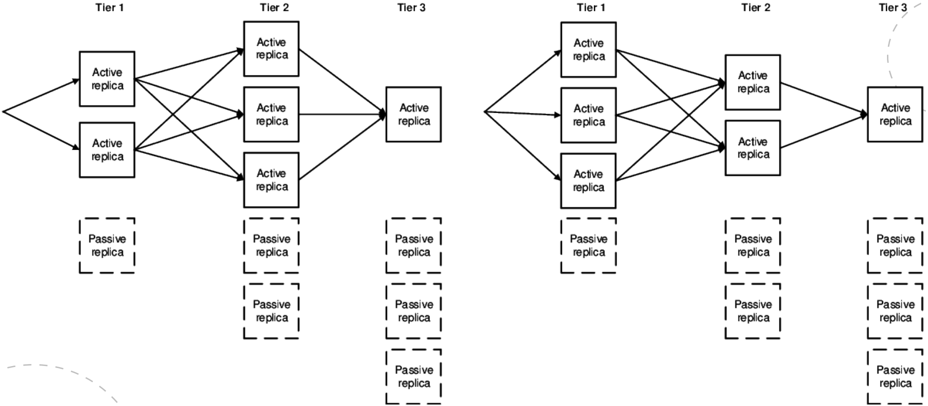
- Active replicas: balance the load between active replicas of a tier
- Passive replicas: backup replicas (idle in a failure-free situation)



Modeling of the replication

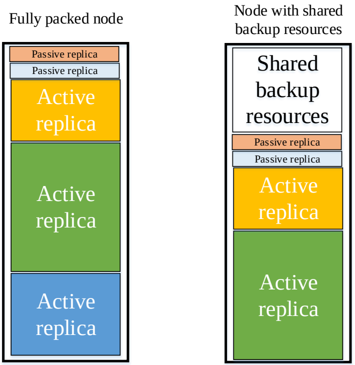
Replication patterns

- define the numbers of active and passive replicas of each tier
- input to the optimization model (Gullhav et al., 2013)
- select one for each service



Private Cloud Placement

- Active replicas are assigned more resources than passives
- Need to ensure that passive replicas can be activated and serve demand



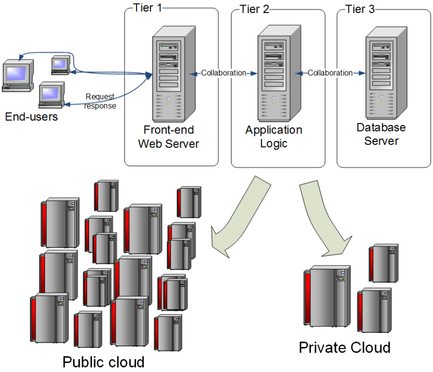
Node pattern: a packing of a node

- set of active replicas
- set of passive replicas
- shared backup resources

Problem statement

Simultaneously:

- decide the appropriate replication level for all services of the SP, and
- deploy the resulting replicas (VMs);
- to minimize cost of placement in the public cloud



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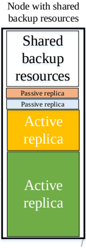
Math Formulation with Node Patterns



$\mathcal{B}$: Set of *node patterns* (representing packed nodes)
$\mathcal{S}$: Set of services
$\mathcal{Q}_i$: Set of components (tiers) of service $i \in \mathcal{S}$
$\mathcal{R}_i$: Set of replication patterns of service i
x_b= the number of times node pattern $b \in \mathcal{B}$ is selected
y_{ir}= 1, if replication pattern r is selected for service i; = 0, otherwise
t_{iq}= the number of active replicas of component (i, q) placed in the public cloud

$$\min z = \sum_{i \in \mathcal{S}} \sum_{q \in \mathcal{Q}_i} C_{iq} t_{iq} \tag{1}$$

$$\sum_{r \in \mathcal{R}_i} y_{ir} = 1, \quad \forall i \in \mathcal{S} \tag{2}$$

$$\sum_{b \in \mathcal{B}} W_{biq} x_b + t_{iq} - \sum_{r \in \mathcal{R}_i} R^A_{iqr} y_{ir} = 0, \quad \forall i \in \mathcal{S}, \forall q \in \mathcal{Q}_i \tag{3}$$

$$\sum_{b \in \mathcal{B}} V_{biq} x_b - \sum_{r \in \mathcal{R}_i} R^P_{iqr} y_{ir} = 0, \quad \forall i \in \mathcal{S}, \forall q \in \mathcal{Q}_i \tag{4}$$

$$\sum_{b \in \mathcal{B}} x_b \leq N \tag{5}$$

$$x_b \in \mathbb{Z}_+ \ \forall b \in \mathcal{B}; \ y_{ir} \in \{0,1\} \ \forall i \in \mathcal{S}, \forall r \in \mathcal{R}_i; \ t_{iq} \in \mathbb{Z}_+ \ \forall i \in \mathcal{S}, \forall q \in \mathcal{Q}_i \tag{6}$$

Generation of Node Patterns

(1) Apriori generation (pre-generation) (Gullhav and Nygreen, 2015)

- the number of feasible node patterns is very large, even for small problems

(2) Solve a subproblem by

- a MIP solver
- a label setting algorithm solving a shortest path problem with resource constraints (SPPRC)
  - exact label setting algorithm
  - heuristic label setting algorithm

Restricted Master Problem

$$\min z = \sum_{i \in \mathcal{S}} \sum_{q \in \mathcal{Q}_i} C_{iq} t_{iq} \tag{7}$$

$$\sum_{r \in \mathcal{R}_i} y_{ir} = 1, \quad \forall i \in \mathcal{S} \tag{8}$$

$$\sum_{b \in \mathcal{B}} W_{biq} x_b + t_{iq} - \sum_{r \in \mathcal{R}_i} R^A_{iqr} y_{ir} = 0, \quad \forall i \in \mathcal{S}, \forall q \in \mathcal{Q}_i \quad (\alpha_{iq}) \tag{9}$$

$$\sum_{b \in \mathcal{B}} V_{biq} x_b - \sum_{r \in \mathcal{R}_i} R^P_{iqr} y_{ir} = 0, \quad \forall i \in \mathcal{S}, \forall q \in \mathcal{Q}_i \quad (\beta_{iq}) \tag{10}$$

$$\sum_{b \in \mathcal{B}} x_b \leq N \quad (\eta) \tag{11}$$

$$x_b \geq 0 \quad \forall b \in \mathcal{B} \tag{12}$$

$$0 \leq y_{ir} \leq 1 \quad \forall i \in \mathcal{S}, \forall r \in \mathcal{R}_i \tag{13}$$

$$t_{iq} \geq 0, \quad \forall i \in \mathcal{S}, \forall q \in \mathcal{Q}_i \tag{14}$$

(α_{iq}, β_{iq} and η are dual variables)

Subproblem MIP Formulation

Minimize reduced cost:

$$\min z^S = -\eta - \sum_{i \in \mathcal{S}} \sum_{q \in \mathcal{Q}_i} \alpha_{iq} w_{iq} - \sum_{i \in \mathcal{S}} \sum_{q \in \mathcal{Q}_i} \beta_{iq} v_{iq} \tag{15}$$

Subject to:

- Node-disjoint placement of active and passive replicas
- Upper bound on the number of passive replicas on a node
- Upper bound on the number of different services on a node
- Node resource constraint
  - — including shared backup resources for passive replicas

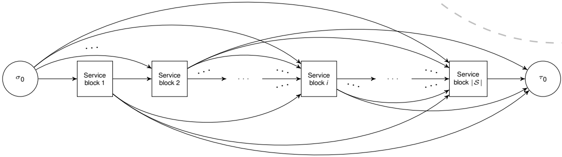
$$w_{iq} = \begin{cases} 1 & \text{if an active replica of component } q \text{ of service } i \text{ is included} \\ 0 & \text{otherwise} \end{cases}$$

$$v_{iq} = \begin{cases} 1 & \text{if a passive replica of component } q \text{ of service } i \text{ is included} \\ 0 & \text{otherwise} \end{cases}$$

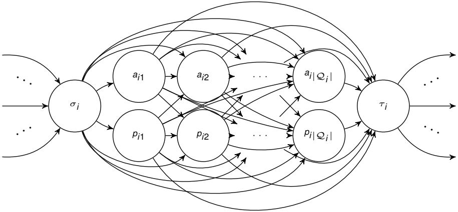
Shortest Path Problem Formulation

The subproblem can be formulated as a *shortest path problem with resource constraints* (SPPRC) and solved by a *label setting algorithm*.

Upper layer network

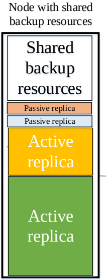


The subnet of a service block



Dominance criteria:

- label ℓ dominates label k if:
  - — $\text{cost}(\ell) \leq \text{cost}(k)$
  - — $\text{totalResources}(\ell) \leq \text{totalResources}(k)$
  - — $\text{sharedBackup}(\ell) \geq \text{sharedBackup}(k)$
  - — ...
- **Issue**: the total resource allocation vs. the allocation of shared backup resources



Heuristic dominance and network reduction:

- Search for new node patterns on two reduced networks:
  1. a network with only active replicas; and
  2. a network where the amount of shared backup resources is fixed to a positive value
- This heuristic label setting algorithm is used together with the (exact) MIP formulation

Branching

- ordinary branching for the replication pattern variables (y_{ir})
- the branching rule for the node pattern variables (x_b) is based on ideas used for the cutting stock problem (Vanderbeck and Wolsey, 1996; Vance, 1998)
  - Consider pairs of components: eg. (i, q) and (i', q')
  - Select the pairs such that

$$\sum_{\substack{b \in \mathcal{B}: \\ W_{biq}=1 \\ W_{bi'q'}=1}} x_b = F \text{ is fractional}$$

  - Branch on

$$\sum_{\substack{b \in \mathcal{B}: \\ W_{biq}=1 \\ W_{bi'q'}=1}} x_b \leq \lfloor F \rfloor \text{ and } \sum_{\substack{b \in \mathcal{B}: \\ W_{biq}=1 \\ W_{bi'q'}=1}} x_b \geq \lceil F \rceil$$

Implications in the Subproblem

In a given branch and bound node we will have a set of branching constraints in the master problem

- Account for the dual variables
- MIP: introduce one binary variable and one constraint
- SPPRC: the costs are compensated by the potential penalties and bonuses that can be obtained by the label extension
- More branches, more difficult subproblem

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Experimental Setup

- Artificial data
- The cases differ from 20 services (80 component types) to 50 services (200 component types)
- we have only considered one node resource type (CPU)
- each node can at most run 4 passive replicas and 3 different services
- integer solutions are found by regularly solving the RMP as an IP

Table : Public cloud VM types: cost and capacity

Provider 1		Provider 2	
Cost	Capacity	Cost	Capacity
10	10%	15	15%
20	20%	30	30%
40	40%	60	60%

Results (I)

The branch and price approach compared to the pre-generation algorithm

75 % private cloud coverage

Table : Average relative gap (in %) between best found solution and best bound at different time steps (seconds)

	Pre-generation			B&P		
	3600	7200	10800	3600	7200	10800
`H20`	10.89	10.17	9.410	2.007	1.769	1.709
`H30`	14.34	13.05	12.51	4.126	3.933	3.576
`H40`	16.74	14.84	14.73	7.208	5.847	5.489
`H50`	37.53	16.13	15.99	N/A	7.641	6.738

Results (II)

90 % private cloud coverage

Table : Average relative gap (in %) between best found solution and best bound at different time steps (seconds)

	Pre-generation			**B&P**		
	3600	7200	10800	3600	7200	10800
`H20`	28.99	27.20	25.63	8.044	6.775	5.146
`H30`	41.43	39.13	37.28	17.64	14.85	12.08
`H40`	107.9	45.14	44.30	23.49	18.39	17.64
`H50`	62.38	48.44	46.00	25.74	22.39	22.33

Results (III)

Full private cloud coverage

- only private cloud
- minimize the number of node patterns used

Table : Average relative gap (in %) between best found solution and best bound at different time steps (seconds)

	Pre-generation			B&P		
	3600	7200	10800	3600	7200	10800
`P20`	3.895	3.895	3.592	1.553	0.924	0.620
`P30`	8.196	5.423	4.663	2.522	1.729	1.343
`P40`	7.903	6.566	5.552	3.958	3.081	2.938
`P50`	9.119	9.119	7.224	5.144	4.494	4.137

Outline

Concluding Remarks

- The branch and price approach gives better solutions, more quickly, than the pre-generation algorithm
- Speed-up by using a heuristic label setting algorithm
- Large private cloud coverage ⇒ larger gaps
- Only private cloud ⇒ cutting stock-like model and easier to solve

More details are found in Gullhav A.N., Nygreen B., A branch and price approach for deployment of multi-tier software services in clouds, *Computers & Operations Research, http://dx.doi.org/10.1016/j.cor.2016.05.007*.

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