Branch-and-cut (and-price) for the chance constrained vehicle routing problem

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joint work with Thai Dinh and James Luedtke
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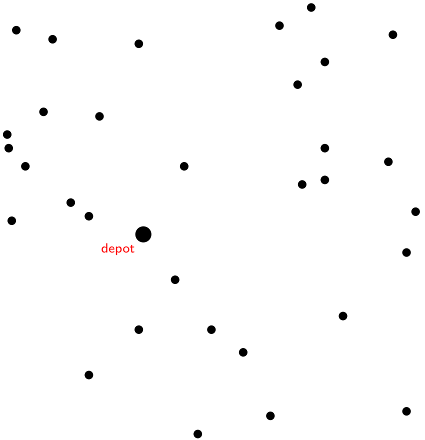
and Optimization

The deterministic vehicle routing problem



- $G = (V, E)$
- $V = \{0\} \cup V_+$
- Edge lengths $\ell_e,\ e \in E$
- K vehicles, capacity b
- Find a set of K routes with minimum total length
- Client demands $d_i, \forall i \in V_+$

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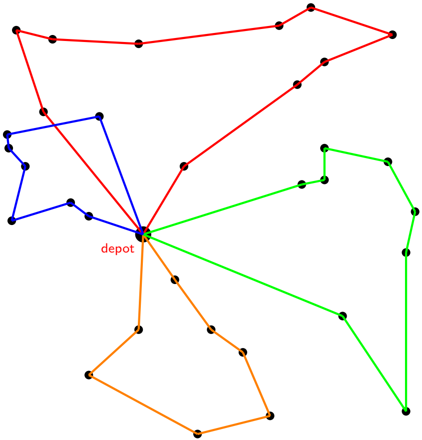

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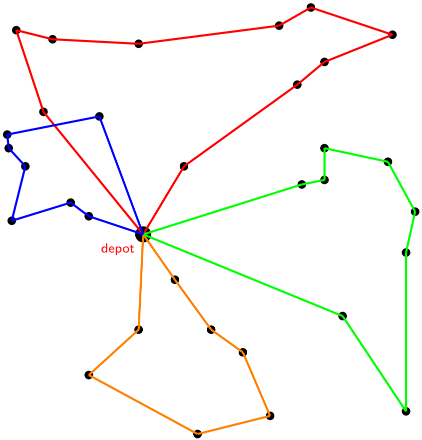
- Let S_j be the set of clients served by route j.
  Then $d(S_j) \leq b$

The stochastic vehicle routing problem



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- ~~Client demands $d_i, \forall i \in V_+$~~
- Demands $D_i, \forall i \in V_+$: random variables that only get realized after routes have been decided
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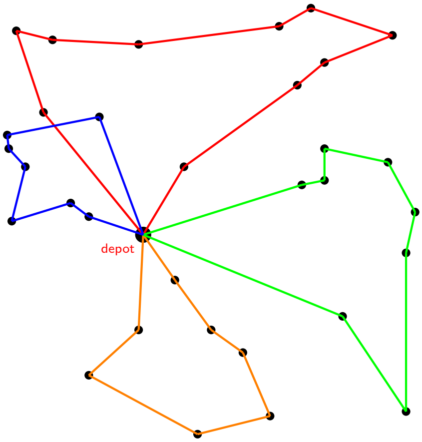
The chance-constrained vehicle routing problem



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  Then $\mathbb{P}\{D(S_j) \leq b\} \geq 1 - \epsilon$

Literature review

Deterministic VRP

- State-of-the-art methods use branch-and-cut-and-price
- Citation:

Literature review

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Stochastic VRP (2-stage)

- Heuristics: Stewart & Golden (1983), Dror & Trudeau (1986), Savelsbergh & Goetschalckx (1995), Novoa et al. (2006), Secomandi and Margot (2009), . . .
- Integer L-Shaped: Gendreau et al. (1994), Laporte et al. (2002), . . .
- Branch-and-cut: Laporte et al. (1989), . . .
- Branch-and-price: Christiansen et al. (2007)
- Branch-and-cut-and-price: Gauvin et al. (2014)

Stochastic VRP (chance-constrained)

- Reduction to deterministic case: Stewart & Golden (1983)
- Branch-and-cut: Laporte et al. (1989)
- Branch-and-cut: Beraldi et al. (2015)
- Branch-and-cut for Robust VRP: Gounaris, Wiesemann, Floudas (2013)

State of CCVRP

Distribution		BC	BP	BCP
Deterministic		✓	✓	✓
Independent	$*$	✓	✓	✓
	Normal	✓		
Correlated	Normal	✓		

- $*$ - Stewart and Golden (83): Reduction to deterministic, only applies to some distributions e.g. Poisson, Binomial.

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Goal

Develop exact methods for chance-constrained SVRP with very few assumptions on the demand uncertainty.

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Assumption: Quantile

$$Q_p(S) := \inf \left\{ b' : \mathbb{P}\{\sum_{i \in S} D_i \leq b'\} \geq p \right\}$$

can be computed for any $S \subseteq V_+$ and any $p \in [0, 1]$.

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Edge formulation for deterministic VRP

d_i: deterministic demand at customer $i \in V_+$
$r(S)$: number of trucks required to serve $S \subseteq V_+$
x_e: number of times a vehicle traverses edge $e \in E$

$$\begin{aligned}
\min_x \quad & \sum_{e \in E} \ell_e x_e \\
\text{s.t.} \quad & \sum_{e \in \delta(\{i\})} x_e = 2, && \forall i \in V_+ \\
& \sum_{e \in \delta(\{0\})} x_e = 2K \\
& \sum_{e \in \delta(S)} x_e \geq 2r(S), && \forall S \subseteq V_+ \\
& x_e \leq 1, && \forall e \in E \setminus \delta(\{0\}) \\
& x_e \in \mathbb{Z}_+, && \forall e \in E.
\end{aligned}$$

Edge formulation for chance-constrained VRP

Modified capacity inequalities

$$\sum_{e \in \delta(S)} x_e \geq 2r_\epsilon(S), \quad \forall S \subseteq V_+$$

- $r_\epsilon(S)$: Minimum number of trucks required to serve customer set S, where probability of capacity violation is at most ϵ for each truck
- Requires solving stochastic bin-packing

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Challenge

How to obtain valid lower bounds on $r_\epsilon(S)$?

- Laporte et al. (1989): If demands are independent normal, can use

$$\left\lceil \frac{Q_{1-\epsilon}(S)}{b} \right\rceil$$

where $Q_p(S)$ be pth quantile of the random variable $\sum_{i \in S} D_i$, i.e. $Q_p(S) := \inf\{b' : \mathbb{P}\{\sum_{i \in S} D_i \leq b'\} \geq p\}$.

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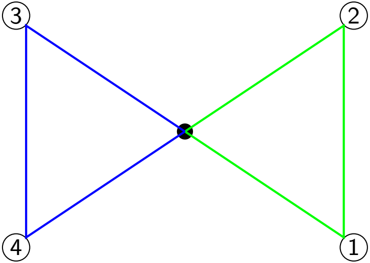
- Not valid in general.

Bad example for Laporte et al. bound



		Scenarios		
		1	2	3
Clients	1	1	2	1
	2	1	1	1
	3	1	1	2
	4	1	1	1
Probability		0.8	0.1	0.1

Table: Demands in each scenario

- $b = 2$
- $\epsilon = 0.1$

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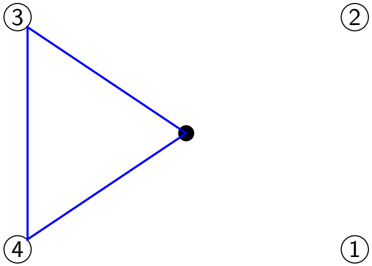

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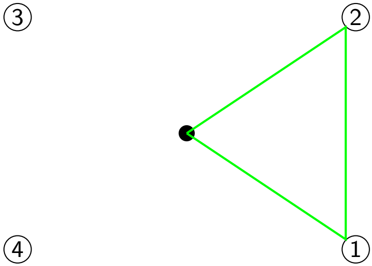

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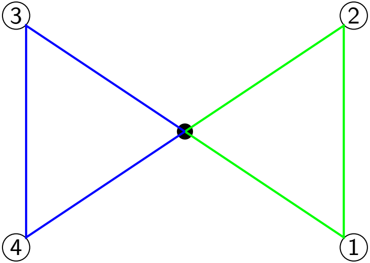

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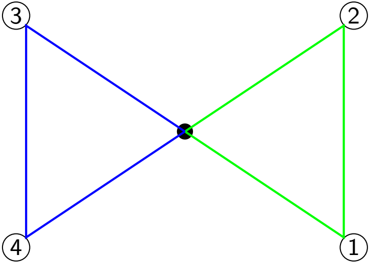

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- Solution depicted is feasible
- However, for $S = \{1, 2, 3, 4\}$, $Q_{0.9}(S) = 5$

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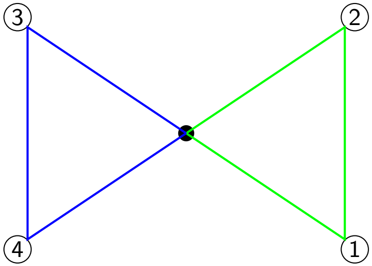

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- $b = 2$
- $\epsilon = 0.1$
- Solution depicted is feasible
- However, for $S = \{1, 2, 3, 4\}$, $Q_{0.9}(S) = 5$
- Thus using $\left\lceil \frac{Q_{1-\epsilon}(S)}{b} \right\rceil$ requires 3 vehicles to enter $S = \{1, 2, 3, 4\}$

Bounds on required trucks more generally

Simple general bound

$$k_\epsilon(S) = \begin{cases} 1 & \mathbb{P}\Big\{\sum_{i\in S} D_i \le b\Big\} \ge 1-\epsilon \\ 2 & otherwise \end{cases}$$

$$\sum_{e\in\delta(S)} x_e \ge 2k_\epsilon(S), \quad \forall S \subseteq V_+$$

- $k_\epsilon(S) \le r_\epsilon(S)$ but sufficient to define a valid formulation
- Cheap to compute for a given set S (thus easy for $x \in \mathbb{Z}_+^E$)
- Cuts may be weak

Improved general bound

Lemma

$$r_\epsilon(S) \geq \left\lceil \frac{Q_{1-\epsilon r_\epsilon(S)}(S)}{b} \right\rceil$$

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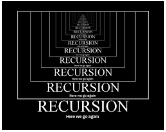

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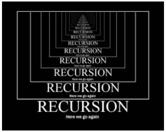
- But we don't know $r_\epsilon(S)$! 

Lemma

For any $k \geq 2$,

$$r_\epsilon(S) \geq \min\left\{k, \left\lceil \frac{Q_{1-\epsilon(k-1)}(S)}{b} \right\rceil \right\}$$

Proof: Either $r_\epsilon(S) \geq k$ or $r_\epsilon(S) \leq k-1$.

Improved general bound (2)

Use best k:

$$k_\epsilon^*(S) = \max\left\{\min\left\{k, \left\lceil \frac{Q_{1-\epsilon(k-1)}(S)}{b} \right\rceil\right\} : k = 2, \ldots, K\right\}.$$

- Always at least as good as first simple bound

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Improvements are possible for special cases:

- Independent and Correlated normal: Can use a stronger closed form formula (derived from robust CVRP).

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Deterministic		✓	✓	✓
Independent	*	✓	✓	✓
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Correlated	Normal	✓		
	Computable $Q_p(S)$	✓		

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Set partitioning formulation for DETERMINISTIC

Sets :
Ω: set of elementary routes satisfying capacity

Parameters :
a_{ir}: number of times vertex i appears in route r

Variables :
λ_r: (binary) whether to choose route r

$$\begin{aligned}
\min_{\lambda} \quad & \sum_{r \in \Omega} c_r \lambda_r \\
\text{s.t.} \quad & \sum_{r \in \Omega} a_{ir} \lambda_r = 1, \ \forall i \in V_+ \\
& \sum_{r \in \Omega} \lambda_r = K \\
& \lambda_r \in \{0, 1\}, \ \forall r \in \Omega
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\end{aligned}$$

- Pseudo-polynomial pricing.

Set partitioning formulation for STOCHASTIC

Sets :
Ω_s: set of elementary routes satisfying chance-constraint

Parameters :
a_{ir}: number of times vertex i appears in route r

Variables :
λ_r: (binary) whether to choose route r

$$
\begin{aligned}
\min_{\lambda} \quad & \sum_{r \in \Omega_s} c_r \lambda_r \\
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$$\text{s.t.} \quad \sum_{r \in \Omega'_s} a_{ir} \lambda_r = 1, \ \forall i \in V_+$$

$$\sum_{r \in \Omega'_s} \lambda_r = K$$

$$\lambda_r \in \{0, 1\}, \ \forall r \in \Omega$$

Theorem

Finding the least cost non-elementary route in a graph that respects the capacity chance constraint under the finite distribution model is strongly NP-hard.

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Finding the least cost non-elementary route in a graph that respects the capacity chance constraint under the independent normal distribution model is strongly NP-hard.

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Proof Idea:
Use chance-constraint to enforce elementarity.

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	Computable $Q_p(S)$	✓	Hard	

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BCP idea

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Chance-constrained

- Elementary (strongly NP-hard) $\rightarrow$ Non-elementary (strongly NP-hard)
  $\rightarrow$ Relax chance-constraint

Relaxed pricing scheme

Exact capacity chance constraint

- y_i: binary indicator of whether or not node i is visited

$$F^\epsilon = \left\{ y \in \{0,1\}^{V_+} : \mathbb{P}\left\{ D^T y \leq b \right\} \geq 1 - \epsilon \right\}$$

Idea

Find $w \in \mathbb{Z}_+^{V_+}$ and $\tau \in \mathbb{Z}_+$ such that:

$$F^\epsilon \subseteq R(w,\tau) := \left\{ y \in \mathbb{Z}^{V_+} : w^T y \leq \tau \right\}$$

Use $R(w,\tau)$ instead of F^ϵ:

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Use $R(w, \tau)$ instead of F^{ϵ}:

- Capacity cuts ensure only solutions to F^{ϵ} will be picked

Generic relaxed pricing scheme (cont'd)

How to choose coefficients?

- Natural choice: $w_i = \mathbb{E}[D_i]$

Given w, optimize τ in preprocessing phase:

$$\tau = \max\left\{w^T y \ : \ \mathbb{P}\left\{D^T y \leq b\right\} \geq 1 - \epsilon, y \in \{0,1\}^{V_+}\right\}$$

- Stochastic binary knapsack problem
- Joint normal random demands ⇒ Binary second-order cone program
- Scenario model of random demands ⇒ Structured binary integer program (Song et al., 2014)
- Any easily computable upper bound on the above maximum can be used.

Relaxed pricing with joint normal demands

- With joint normal random demands, binary second-order cone program can be replaced with a semidefinite program
- With mean vector μ and covariance matrix Σ:

$$P\left\{D^T y \le b\right\} \ge 1 - \epsilon \iff \mu^T y + \Phi^{-1}(1-\epsilon)\sqrt{y^T \Sigma y} \le b$$

Idea

- Get a lower bound on $y^T \Sigma y$ in terms of $\mu^T y$

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- η found by solving an SDP

$$\eta^* = \max_{\eta, p, Q} \eta \tag{3a}$$

$$\text{s.t.} \quad \mu_i \eta \leq p_i \qquad i \in V_+ \tag{3b}$$

$$\Sigma = \text{diag}(p_1, ..., p_n) + Q \tag{3c}$$

$$Q \succeq 0, \tag{3d}$$

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$$\text{s.t.} \quad \mu_i \eta \le p_i \qquad i \in V_+ \tag{3b}$$

$$\Sigma = \text{diag}(p_1, ..., p_n) + Q \tag{3c}$$

$$Q \succeq 0, \tag{3d}$$

- Solve RCSP using constraint (2) on resource $\mu^T y$.

Pricing with independent normal demands

Pricing for independent normal with mean vector μ and variance vector σ^2

$$\mathbb{P}\left\{D^T y \leq b\right\} \geq 1 - \epsilon \iff \mu^T y + \Phi^{-1}(1-\epsilon)\sqrt{\sum_{i \in V_+} y_i^2 \sigma_i^2} \leq b$$

Relax to:

$$\mu^T y + \Phi^{-1}(1-\epsilon)\sqrt{y^T \sigma^2} \leq b$$

- Resources: $\mu^T y$ and $y^T \sigma^2$

Computational tests overview

Test instances

- Based on deterministic VRP instances
- 32 to 55 customers
- Two variance settings: "low" ($\approx$ 10% of mean) and "high" ($\approx$ 20% of mean)
- Three distribution assumptions: independent normal, joint normal, scenario

Implementation details

- Cplex 12.4.0
- Implemented in BCP code based from F. et al. (2006)
- 7200 second time limit

BC	BC^*	BC^J	BCP^r	BCP^i
$k_\epsilon(S)$	$k_\epsilon^*(S)$	$k_\epsilon^J(S)$	Rel. pricing	Rel. pricing (for ind. normal)

Table: Strategies used

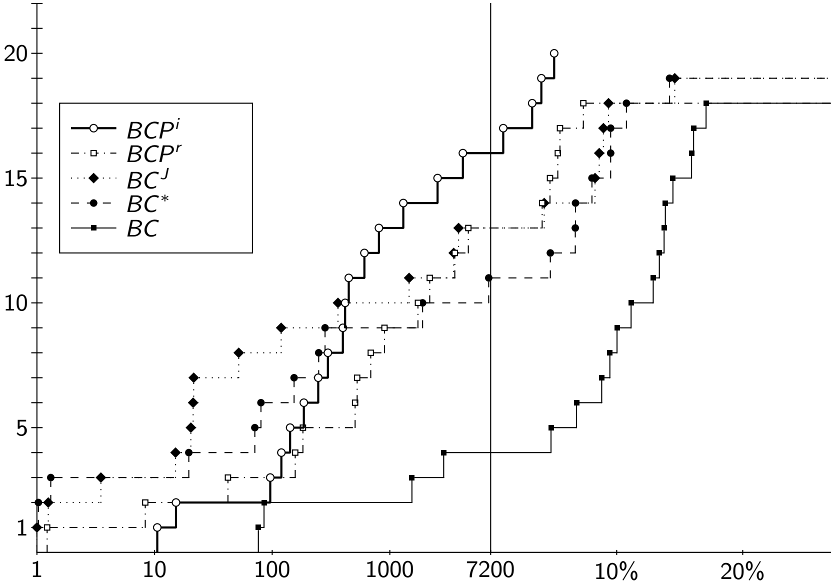

Figure: Summary of results for instances with independent normal distribution.

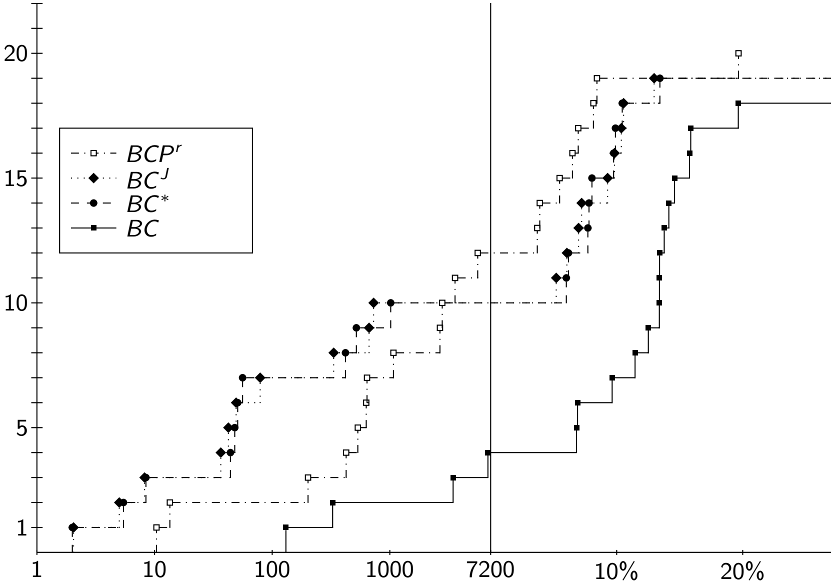

Figure: Summary of results for instances with joint normal distribution.

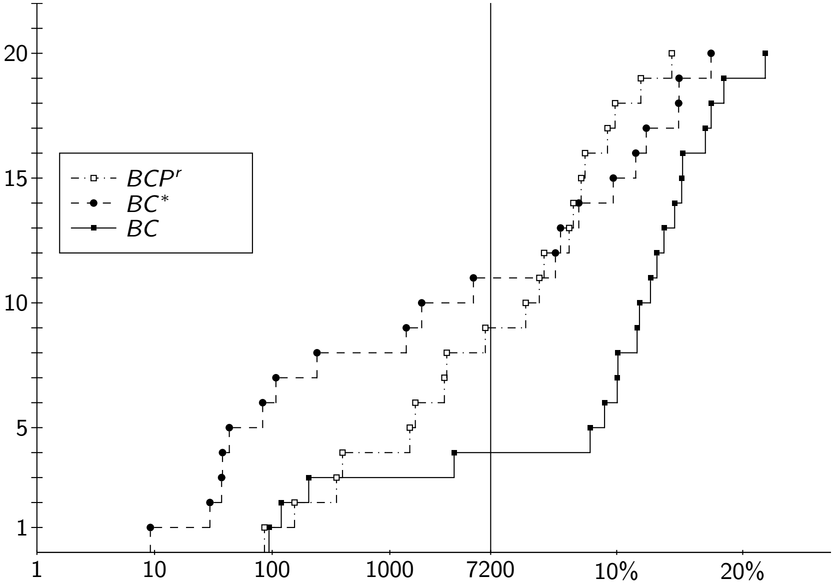

Figure: Summary of results for instances with scenario distribution.

Concluding remarks

Summary

- Chance-constrained formulation avoids difficulties in modeling recourse actions
- Proposed method can solve chance-constrained VRP with correlations
- Builds on successful approaches for solving deterministic VRP
- Can be extended to other variants

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<table>
<tr><th colspan="2">Distribution</th><th>BC</th><th>BP</th><th>BCP</th></tr>
<tr><td>Deterministic</td><td></td><td>✓</td><td>✓</td><td>✓</td></tr>
<tr><td rowspan="2">Independent</td><td>*</td><td>✓</td><td>✓</td><td>✓</td></tr>
<tr><td>Normal</td><td>✓</td><td>Hard</td><td>✓</td></tr>
<tr><td rowspan="2">Correlated</td><td>Normal</td><td>✓</td><td>Hard</td><td>✓</td></tr>
<tr><td>Computable $Q_p(S)$</td><td>✓</td><td>Hard</td><td>✓</td></tr>
</table>

Future work

Future work

- Incorporate more "advanced features" of deterministic VRP into solution approach
- Seek improved "pricing friendly" relaxation of chance-constrained capacity constraint
- Other models of handling uncertainty
- How "well" can deterministic constraints "approximate" chance-constraints?

THANK YOU!

Comparing solutions

Experiment:

- For an instance, obtain chance-constrained and recourse model solutions
- Evaluate each solution in both model metrics

Four instances, size up to 22 nodes, all independent normal

	Max Violation Prob. %		% Increase
Var	CC Sol	Rec Sol	Expected Cost
Low	1.7	50.0	2.3%
	5.0	7.8	0.9%
	2.4	2.4	0
	3.1	6.4	0.6%
High	4.0	8.3	3.4%
	3.6	23.7	2.9%
	1.0	1.0	0
	0.7	16.9	0.3%

Comparing solutions: Correlated demands

- Recourse solution: Ignore correlation
- Evaluate each solution in both model metrics using true distribution

	Max Violation Prob.		% Increase in
Var	CC Sol	Rec Sol	Expected Cost
Low	4.0	50.7	1.1%
	2.4	13.3	2.2%
	0.2	6.3	0.2%
	0.6	16.5	0.1%
High	4.6	12.1	3.6%
	5.0	28.9	3.1%
	1.2	8.6	-0.3%
	2.5	21.5	-0.1%