Branch-and-Price for Creating an Annual Delivery Program of Multi-Product Liquefied Natural Gas

Jørgen Glomvik Rakke[a,b], Guy Desaulniers[c], Henrik Andersson[a], and Marielle Christiansen[a]

[a] Department of Industrial Economics and Technology Management, NTNU, Trondheim, Norway

[b] Norwegian Marine Technology Research Institute (MARINTEK), Trondheim, Norway

[c] Département de mathématiques et génie industriel, École Polytechnique

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Outline

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Cuts

Results

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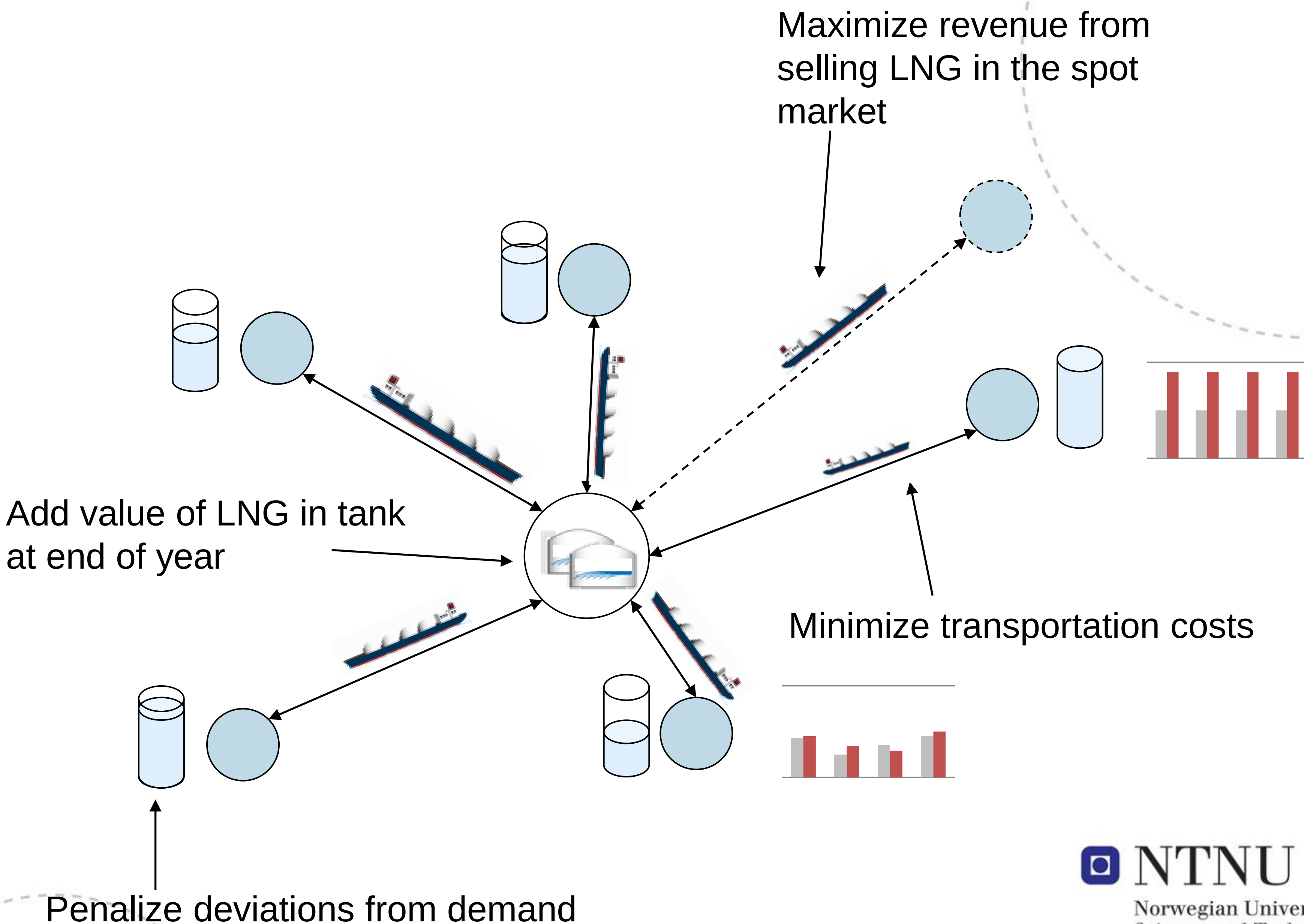

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Time partitions

Full planning horizon			Partition 1
Interval 1	Interval 2	Interval 3	Partition 2

C_{c1}^{D+} C_{c1}^{D-}			Partition 1
C_{c2}^{D+} C_{c2}^{D-}	C_{c3}^{D+} C_{c3}^{D-}	C_{c4}^{D+} C_{c4}^{D-}	Partition 2

Basic Voyage Formulation (BVF)

- MIP model with scheduled voyages as binary variables
  - A scheduled voyage consists of:
    - Ship
    - Destination (Contract + terminal)
    - Departure day
- Based on pre-generation of all scheduled voyage
- The main decision is which combination of ships should deliver LNG to which contracts, and when the deliveries are to be made.

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Mathematical Model

$$\min \sum_{c\in\mathcal{C}} \sum_{v\in\mathcal{V}_c} \sum_{t\in\mathcal{T}_v} C^T_{cv} x_{cvt} + \sum_{c\in\mathcal{C}^{LT}} \sum_{i\in\mathcal{I}_c} C^{D+}_{ci} y^+_{ci} + \sum_{c\in\mathcal{C}^{LT}} \sum_{i\in\mathcal{I}_c} C^{D-}_{ci} y^-_{ci} - \sum_{c\in\mathcal{C}^S} \sum_{v\in\mathcal{V}_c} \sum_{t\in\mathcal{T}_v} R^S_c L_v x_{cvt} - \sum_{g\in\mathcal{G}} R^I_g q_{gT}, \quad (1)$$

Objective

subject to:

$$\sum_{c\in\mathcal{C}_g\setminus\mathcal{C}^M} \sum_{v\in\mathcal{V}_c} x_{cvt} + \sum_{c\in\mathcal{C}_g\cap\mathcal{C}^M} \sum_{\substack{v\in\mathcal{V}^M \\ : t-T_{cv}-1\in\mathcal{T}_v}} x_{cv(t-T_{cv}+1)} \leq B_g, \quad \forall g\in\mathcal{G}, t\in\mathcal{T}, \quad (2)$$

Berth constraints

$$q_{g,t-1} + P_{gt} - q_{gt} - \sum_{c\in\mathcal{C}_g\setminus\mathcal{C}^M} \sum_{v\in\mathcal{V}_c} L_v x_{cvt} = 0, \quad \forall g\in\mathcal{G}, t\in\mathcal{T}, \quad (3)$$

$$\underline{Q}_{gt} \leq q_{gt} \leq \overline{Q}_{gt}, \quad \forall g\in\mathcal{G}, t\in\mathcal{T}, \quad (4)$$

Inventory constraints

$$\sum_{c\in\mathcal{C}} \sum_{\substack{\tau\in\mathcal{T}_v \\ : t-T_{cv}<\tau\leq t}} x_{cv\tau} \leq 1, \quad \forall v\in\mathcal{V}, t\in\mathcal{T}_v, \quad (5)$$

Routing constraints

$$\sum_{c\in\mathcal{C}^M} \sum_{t\in\mathcal{T}^M_v} x_{cvt} = 1, \quad \forall v\in\mathcal{V}^M, \quad (6)$$

Maintenance constraints

$$\sum_{v\in\mathcal{V}_c} \sum_{t\in\mathcal{T}^I_i\cap\mathcal{T}_v} L_v x_{cvt} + y^-_{ci} - y^+_{ci} = D_{ci}, \quad \forall c\in\mathcal{C}^{LT}, i\in\mathcal{I}_c, \quad (7)$$

Contractual constraints

$$x_{cvt} \in \{0,1\}, \quad \forall c\in\mathcal{C}, v\in\mathcal{V}_c, t\in\mathcal{T}_v, \quad (8)$$

$$y^+_{ci}, y^-_{ci} \geq 0, \quad \forall c\in\mathcal{C}^{LT}, i\in\mathcal{I}_c. \quad (9)$$

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Objective function

$$\min \sum_{c\in\mathcal{C}} \sum_{v\in\mathcal{V}_c} \sum_{t\in\mathcal{T}_v} C^T_{cv} x_{cvt}$$

Transportation cost

$$+ \sum_{c\in\mathcal{C}^{LT}} \sum_{i\in\mathcal{I}_c} C^{D+}_{ci} y^+_{ci} + \sum_{c\in\mathcal{C}^{LT}} \sum_{i\in\mathcal{I}_c} C^{D-}_{ci} y^-_{ci}$$

Penalty costs

$$- \sum_{c\in\mathcal{C}^S} \sum_{v\in\mathcal{V}_c} \sum_{t\in\mathcal{T}_v} R^S_c L_v x_{cvt} - \sum_{g\in\mathcal{G}} R^I_g q_{g\overline{T}},$$

Spot revenue

Problems

- Weak LP-relaxation
  - Mainly introduced by the penalties for over and under delivery

$$\min \sum_{c \in \mathcal{C}} \sum_{v \in \mathcal{V}_c} \sum_{t \in \mathcal{T}_v} C^T_{cv} x_{cvt}$$

$$+ \sum_{c \in \mathcal{C}^{LT}} \sum_{i \in \mathcal{I}_c} C^{D+}_{ci} y^+_{ci} + \sum_{c \in \mathcal{C}^{LT}} \sum_{i \in \mathcal{I}_c} C^{D-}_{ci} y^-_{ci}$$

$$- \sum_{c \in \mathcal{C}^S} \sum_{v \in \mathcal{V}_c} \sum_{t \in \mathcal{T}_v} R^S_c L_v x_{cvt} - \sum_{g \in \mathcal{G}} R^I_g q_{g\overline{T}},$$

Example

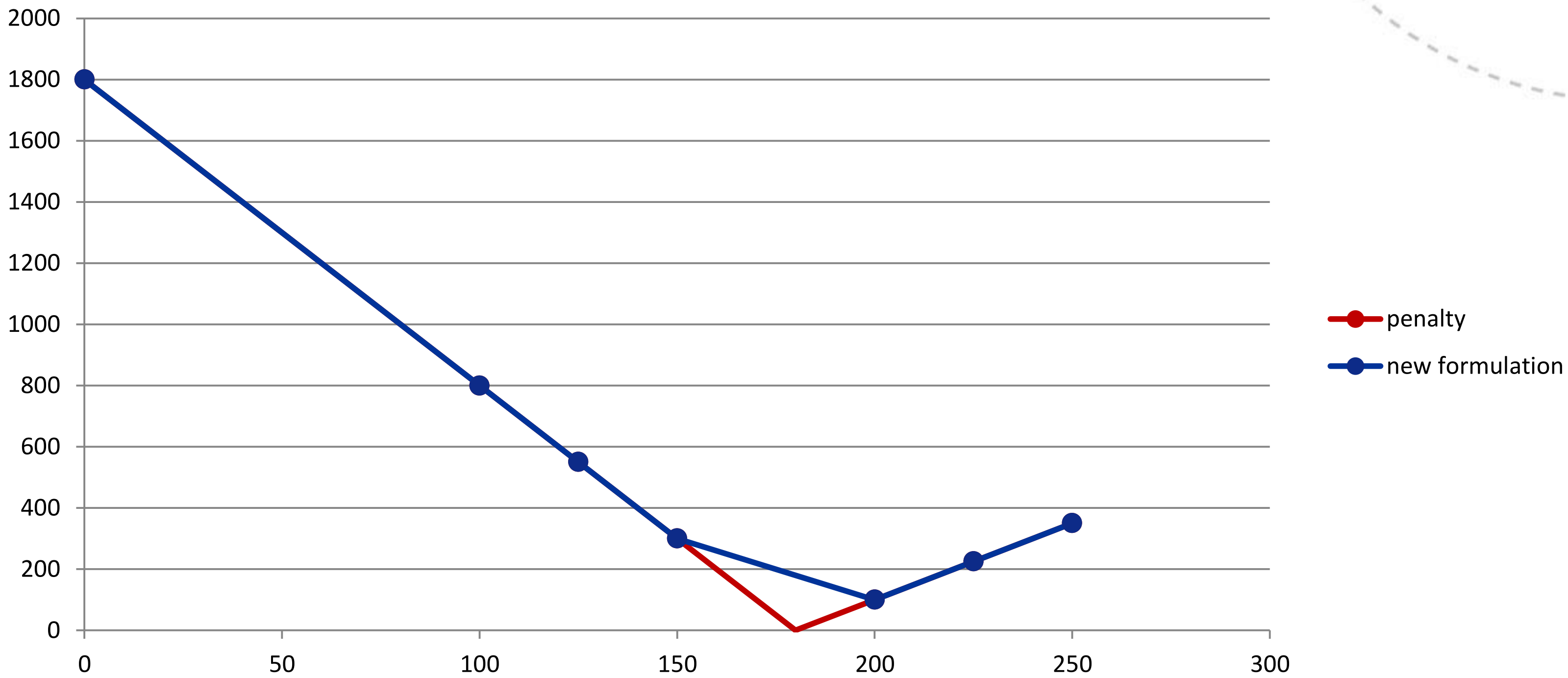
- Heterogeneous fleet, ship capacities = 100, 125, 150
- Demand = 180
- Under-delivery penalty = 10
- Over-delivery penalty = 5
- Minimum under-delivery = 180 - 150 = 30
- Minimum over-delivery = 100 + 100 - 180 = 20

Penalty function

How to deal with this problem

- Create delivery patterns
  - A delivery pattern specifies:
    - Contract
    - Time period
    - Number of deliveries for all ships

Penalty function



Pattern Based Formulation (PBF)

- Set of delivery patterns for each time period and contract $\mathcal{P}_{ci}$
- Pattern variable z_{cip}

$$\begin{aligned}\min \sum_{c\in\mathcal{C}}\sum_{v\in\mathcal{V}_c}\sum_{t\in\mathcal{T}_v} C^T_{cv} x_{cvt} + \sum_{c\in\mathcal{C}^{LT}}\sum_{i\in\mathcal{I}_c}\sum_{p\in\mathcal{P}_{ci}} C^P_{cip} z_{cip} \\ - \sum_{c\in\mathcal{C}^S}\sum_{v\in\mathcal{V}_c}\sum_{t\in\mathcal{T}_v} R^S_c L_v x_{cvt} - \sum_{g\in\mathcal{G}} R^I_g q_{g\overline{T}},\end{aligned} \qquad (10)$$

subject to:

$$\text{constraints (2)–(6)},$$

$$\sum_{t\in\mathcal{T}_i\cap\mathcal{T}^I_v} x_{cvt} - \sum_{i\in\mathcal{P}_{ci}} N_{cipv} z_{cip} = 0, \quad \forall c\in\mathcal{C}^{LT}, i\in\mathcal{I}_c, v\in\mathcal{V}_c, \qquad (11)$$

Linking constraints

$$\sum_{i\in\mathcal{P}_{ci}} z_{cip} = 1, \quad \forall c\in\mathcal{C}^{LT}, i\in\mathcal{I}_c, \qquad (12)$$

Convexity constraints

$$x_{cvt}\in\{0,1\}, \quad \forall c\in\mathcal{C}, v\in\mathcal{V}_c, t\in\mathcal{T}_v, \qquad (13)$$

$$z_{cip}\geq 0, \quad \forall c\in\mathcal{C}^{LT}, i\in\mathcal{I}_c, p\in\mathcal{P}_{ci}. \qquad (14)$$

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How to generate delivery patterns

- Pre generate all possible delivery patterns
  - Only feasible for really small test instances
- Column generation
  - Split the problem into a master and sub problems to generate delivery patterns with negative reduced cost

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Subproblem

$$\overline{C}_{cip} = C^P_{cip} - \alpha_{ci} + \sum_{v \in \mathcal{V}_c} N_{cipv} \beta_{civ}$$

$$\begin{aligned}
\min \quad & C^{D+} y^+ + C^{D-} y^- - \alpha + \sum_{v \in \mathcal{V}_c} \beta_v n_v \\
\text{s.t.} \quad & \sum_{v \in \mathcal{V}_c} L_v n_v + y^- - y^+ = D, \\
& l_v \leq n_v \leq u_v, \text{integer}, \quad \forall v \in \mathcal{V}_c, \\
& y^+ \geq 0, \\
& y^- \geq 0,
\end{aligned}$$

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Subproblem

- If $\beta_v - C^{D-}L_v \geq 0$, n_v can be set to l_v and removed
- If $\beta_v + C^{D+}L_v < 0$, n_v can be set to u_v and removed
- Let $\bar{\mathcal{V}}_c \subseteq \mathcal{V}_c$ be the set of vessels such that $\beta_v < -C^{D+}L_v$
- Define $\tilde{D}$ as $D - \sum_{v\in\mathcal{V}_c} L_v l_v$ - $\sum_{v\in\bar{\mathcal{V}}_c} L_v(u_v - l_v)$
- Let $\mathcal{V}_c^- \subseteq \mathcal{V}_c$ be the set of vessels such that $-C^{D+}L_v < \beta_v < C^{D-}L_v$
- Let $u'_v = u_v - l_v$

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Subproblem

$$
\begin{aligned}
\min \quad & K + (C^{D+} + C^{D-})y^+ + \sum_{v \in \mathcal{V}_c^-} (\beta_v - C^{D-} L_v) n_v \\
\text{s.t.} \quad & \sum_{v \in \mathcal{V}_c^-} L_v n_v \leq \tilde{D} + y^+, \\
& 0 \leq n_v \leq u'_v, \text{integer}, \quad \forall v \in \mathcal{V}_c^-, \\
& y^+ \geq 0.
\end{aligned}
$$

$$
K = C^{D-} D - \alpha + \textstyle\sum_{v \in \mathcal{V}_c} (\beta_v - C^{D-} L_v) l_v + \sum_{v \in \bar{\mathcal{V}}_c} (\beta_v - C^{D-} L_v) u'_v
$$

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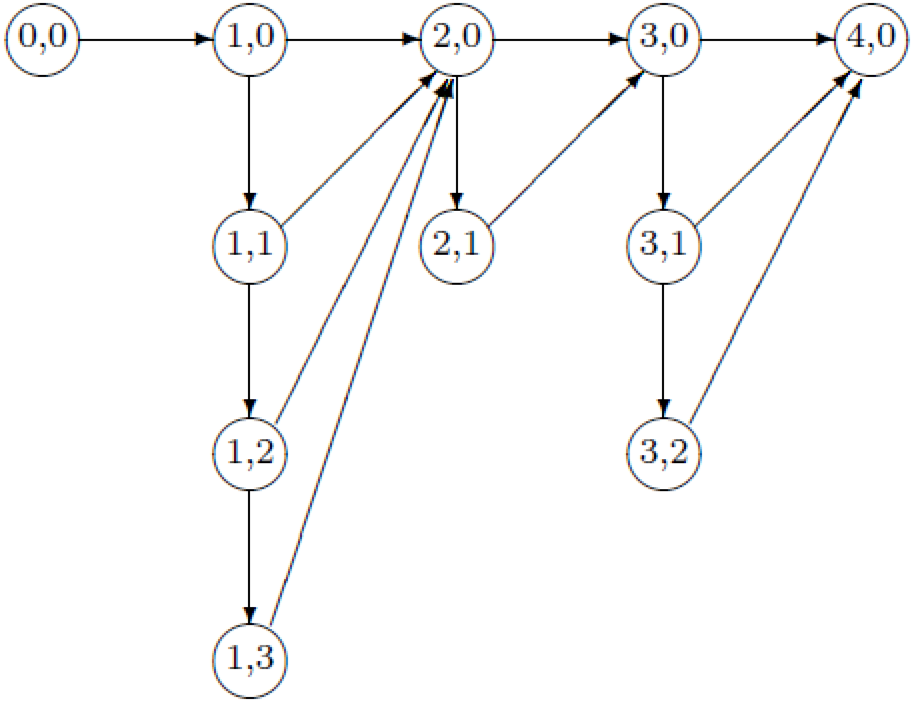

$$c_{ij} = \begin{cases} K & \text{if } i = (0,0) \\ k(\beta_v - C^{D-}L_v) & \text{if } (i,j) \text{ is a select arc and } i = (v,k) \\ 0 & \text{otherwise.} \end{cases}$$

$$q_{ij} = \begin{cases} kL_v & \text{if } (i,j) \text{ is a select arc and } i = (v,k) \\ 0 & \text{otherwise.} \end{cases}$$

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Labels

- A label contains two elements
  - The accumulated cost Z
  - The accumulated quantity Q
- By using arc (i, j) you update label $E_i = (Z_i, Q_i)$ to label $E_j = (Z_j, Q_j)$

$$Q_j = Q_i + q_{ij}$$

$$Z_j = Z_i + c_{ij} + z_{ij}(Q_i)$$

$$z_{ij}(Q_i) = \begin{cases} 0 & \text{if } Q_i + q_{ij} \leq \tilde{D} \\ \infty & \text{if } q_{ij} > 0 \text{ and } Q_i \geq \tilde{D} \\ (C^{D+} + C^{D-})(Q_i + q_{ij} - \tilde{D}) & \text{otherwise.} \end{cases}$$

Dominance

- Consider two labels

$$E_1 = (Z_1, Q_1)$$
$$E_2 = (Z_2, Q_2)$$

- E_1 dominates E_2 if:

$$Z_1 \leq Z_2 \text{ and } Q_1 \leq Q_2$$

- Given that the amount delivered is not constrained we can strengthen this dominance rule to:

$$Z_1 + (C^{D+} + C^{D-}) \max\{0, Q_1 - Q_2\} \leq Z_2$$

- Where we can discard label E_2 even if $Q_2 < Q_1$

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Cuts

- Timing cuts (similar to the ones presented in Engineer et al. (2012)
- Minimum number of loadings
- Maximum number of loadings

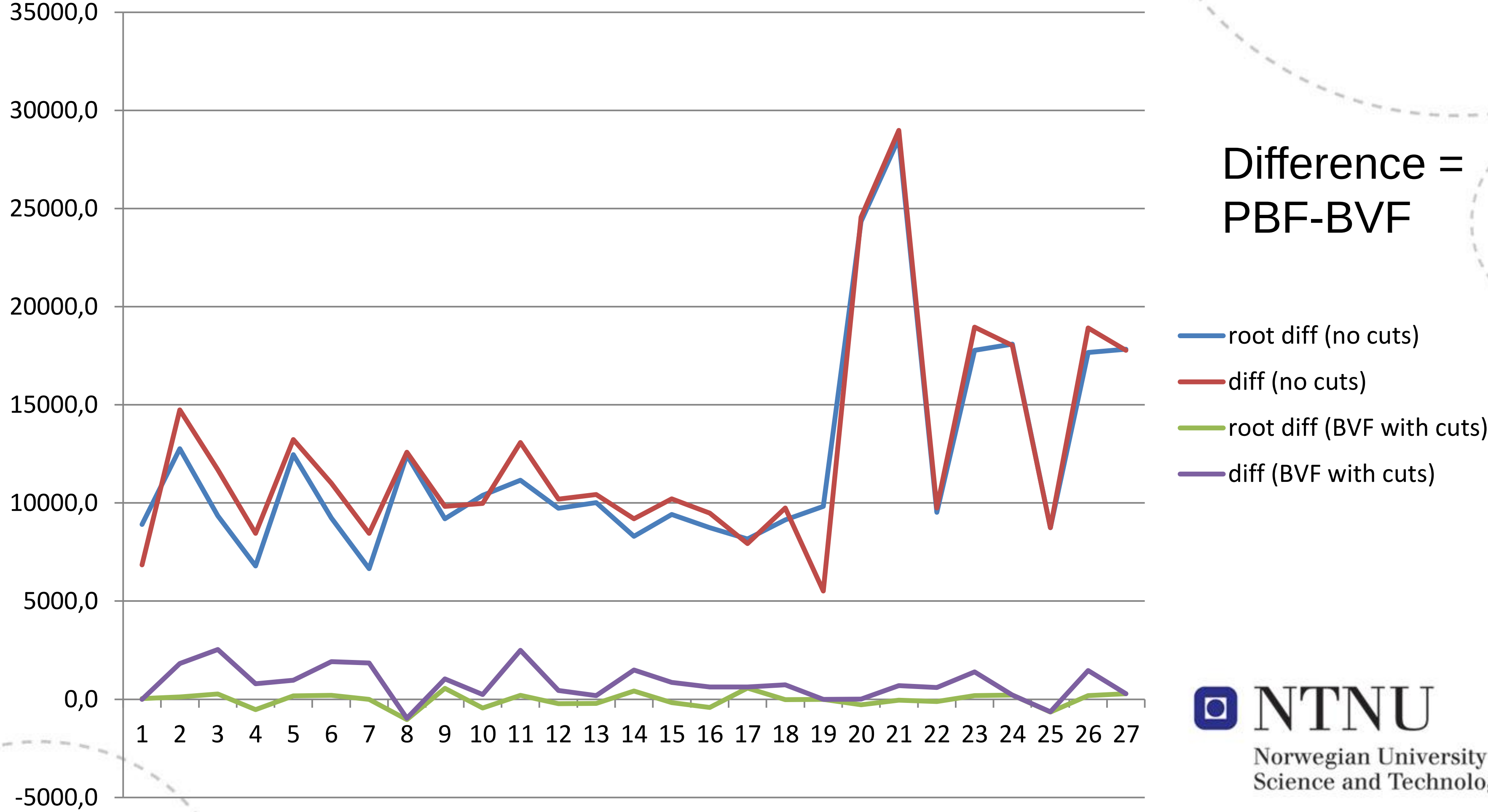
Solution method

- Branch-price-and-cut
  - Subproblem solved as a shortest path problem with resource constraints
  - The *n best* patterns priced in until no negative reduced cost pattern found
  - Generate only the *k* most violated cuts, until no violated cuts found

Tests

- Root node solutions of BVF vs. PBF
- Effect of cuts
- BVF vs. PBF

LB comparison PBF vs. BVF



Difference = PBF-BVF

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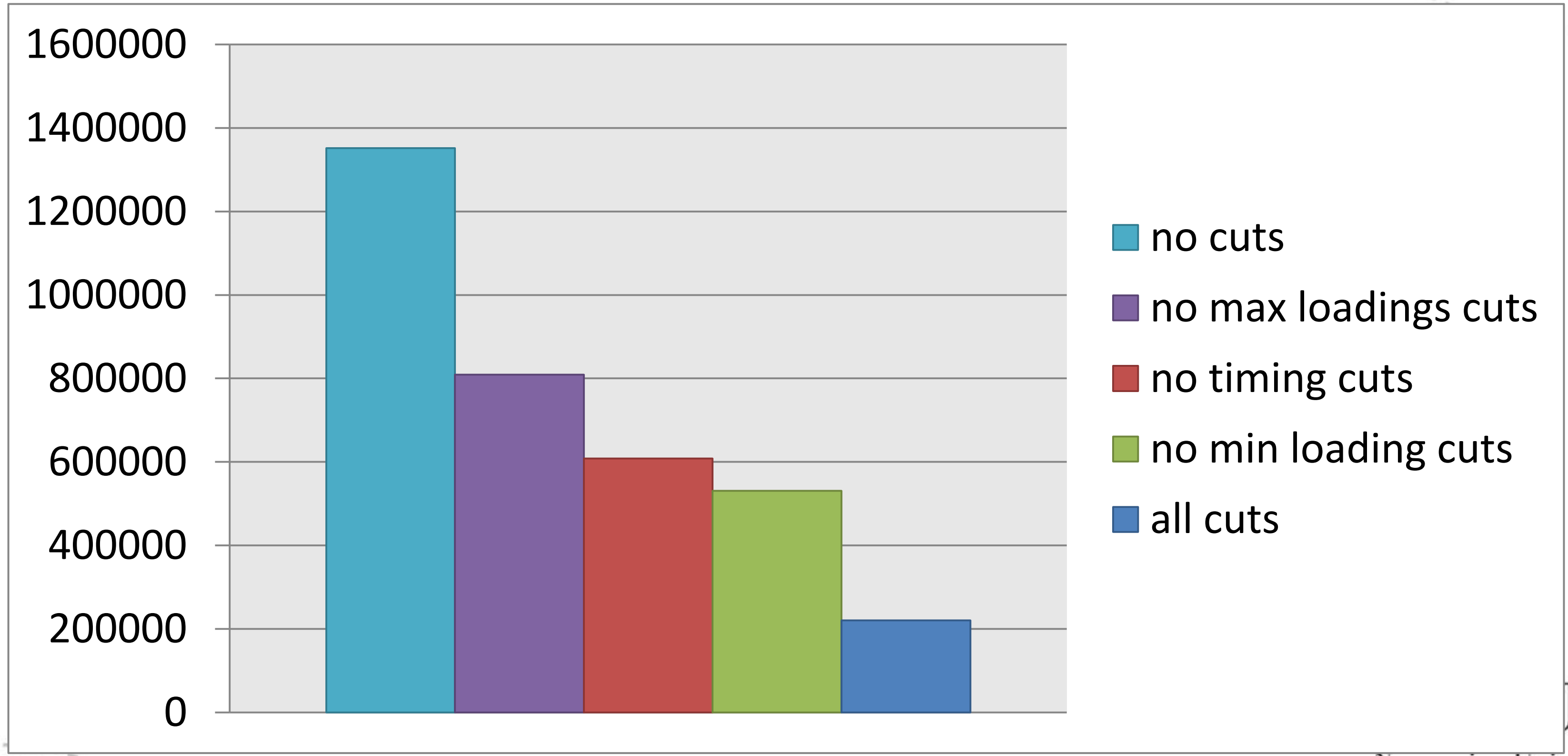
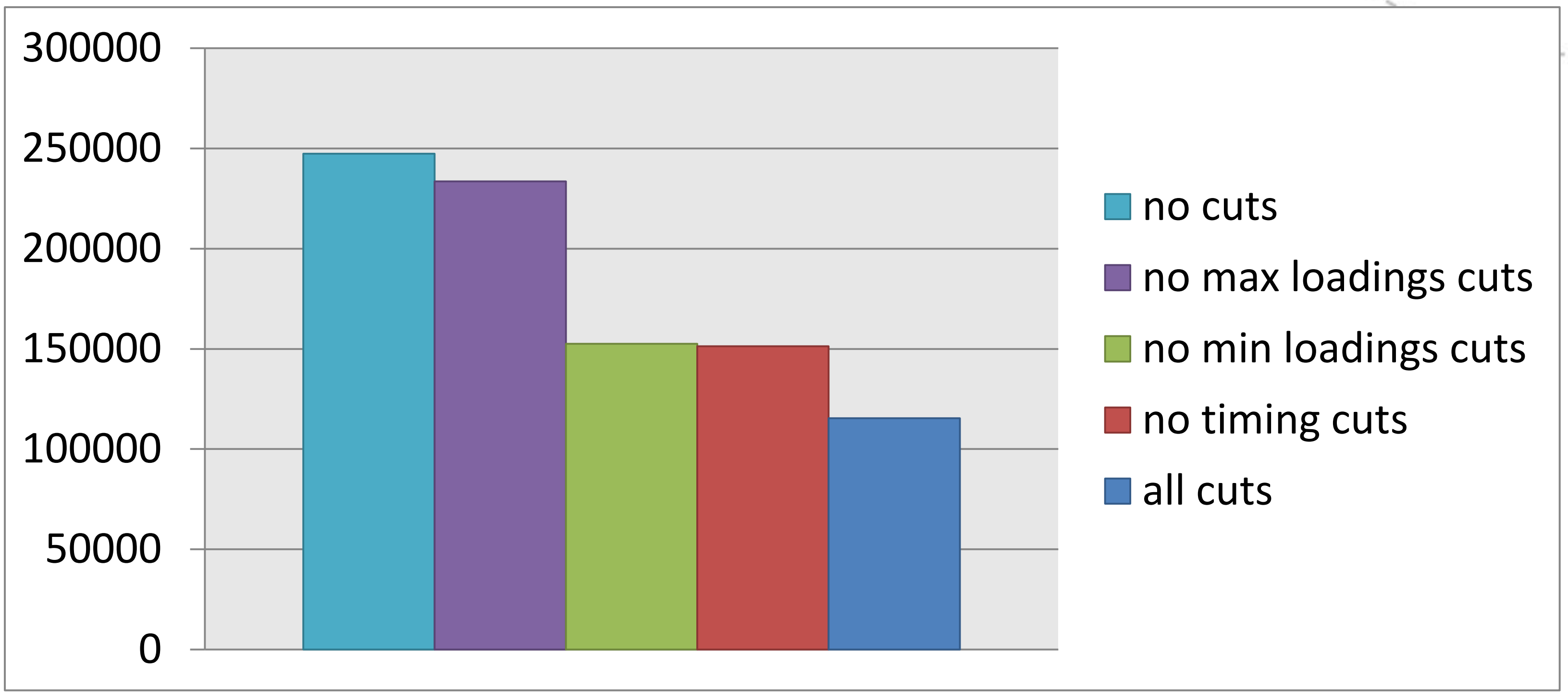
Effect of cuts for PBF

Number of nodes



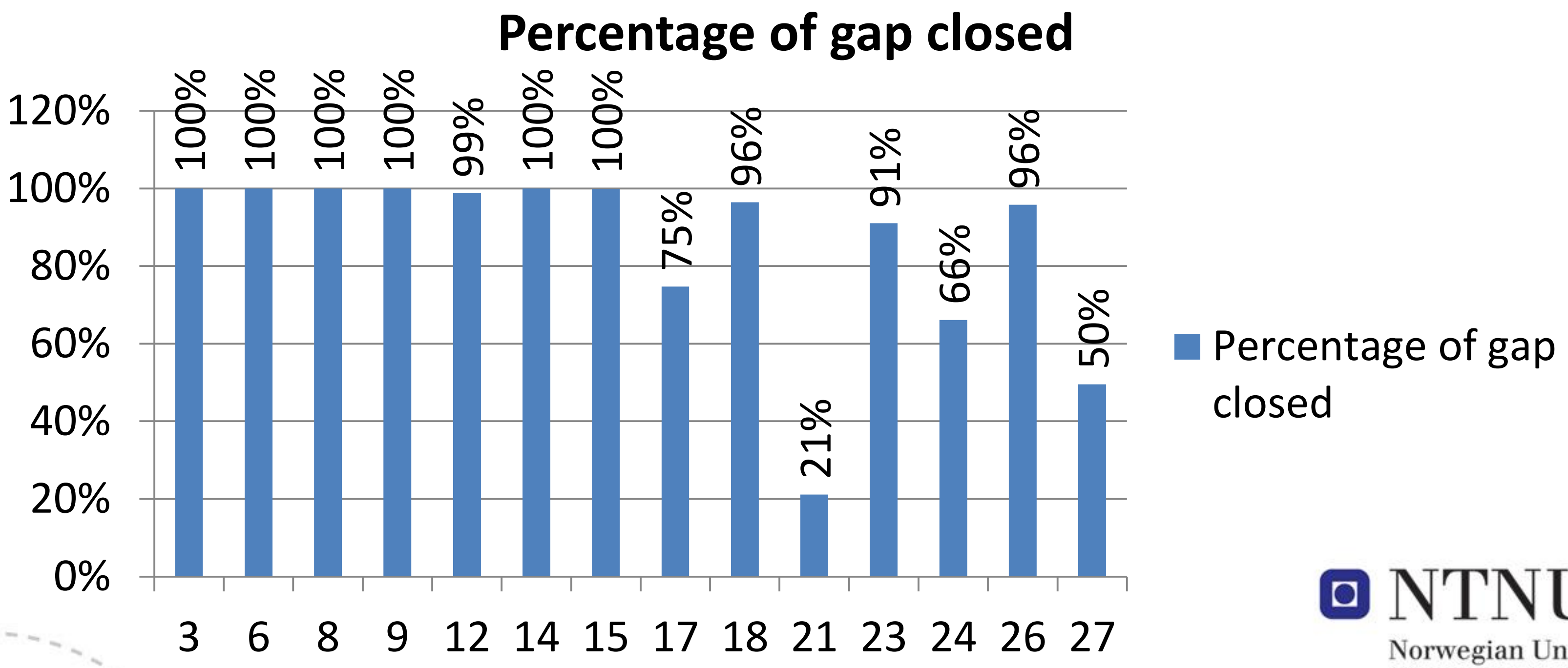
Effect of cuts for PBF

Solution time



Gap closed by the PBF

- 13 instances solved by both solution methods
  - Instances 1,2,4,5,7,10,11,13,16,19,20,22 and 25



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Concluding remarks

- Results show that the new formulation has a much stronger LP relaxation
- Solve more instances to optimality
- Close most of the gap for the remaining instances

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Jørgen Glomvik Rakke[a,b], Guy Desaulniers[c], Henrik Andersson[a], and Marielle Christiansen[a]

[a] Department of Industrial Economics and Technology Management, NTNU, Trondheim, Norway

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