

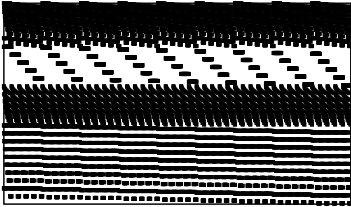
# Generic Branch-Price-and-Cut

## A Status Report

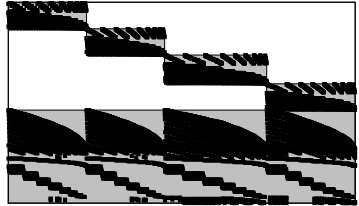


Column Generation · Bromont · June 12, 2012

# T-Shirts



can you spot the difference?



## Status Quo: On the Positive Side

- ▶ many exciting developments in branch-and-price
- ▶ some problems solvable *only* by B&P
- ▶ more and more experts (look around you!)
- ▶ more and more “standard” in IP solving
- ▶ many successful applications (and growing!)
- ▶ good frameworks for implementation available
- ▶ branch-and-price much easier to implement than years ago

# Status Quo: On the Negative Side

- ▶ implementation still non-trivial
- ▶ requires a good deal of work
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# Status Quo: On the Negative Side

- ▶ implementation still non-trivial
- ▶ requires a good deal of work
- ▶ need expert knowledge, experience, and “tricks”
- ▶ many implementations from scratch, little re-use of code
- ▶ implementations not done, because of lack of knowledge
- ▶ implementations not dared, because of unclear outcome
- ▶ many users have no access to a state-of-the-art methodology

# Goals of our Project

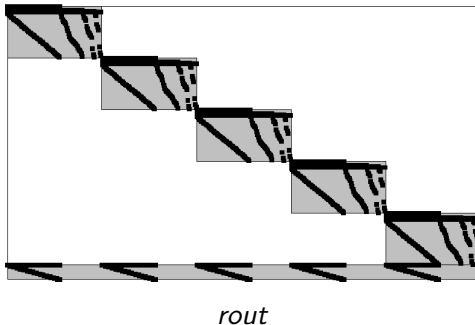
- ▶ we would like to make B&P accessible to a broad audience
- ▶ generic solver for *structured mixed integer programs*
- ▶ fully automatic branch-price-and-cut
- ▶ no user-interaction, usable for non-experts
- ▶ several related/similar projects: BaPCod, DIP, G12
- ▶ report only on our solver: GCG

# Our Generic BP&C Solver GCG

- ▶ based on state-of-the-art MIP solver SCIP
- ▶ automatic detection of decomposable structures
- ▶ alternatively: user provides a decomposition
- ▶ DW reformulation, column generation, branch-and-price
- ▶ synchronizes two trees, “original” and “extended” formulation
- ▶ convexification, discretization, aggregation
- ▶ parallelization of subproblems
- ▶ branching on original variables, Ryan/Foster, ...
- ▶ integration of generic (combinatorial) cutting planes
- ▶ primal heuristics particular to decompositions
- ▶ preprocessing, propagation, ... (inherited from SCIP)
- ▶

# Dantzig-Wolfe: Traditional Realm

- ▶ Dantzig-Wolfe works particularly well for bordered block-angular matrix structure

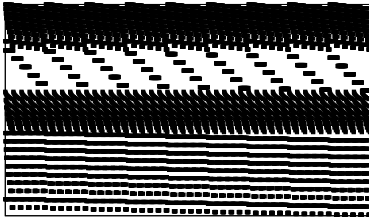


- ▶ vehicle routing, generalized assignment, cutting stock,  $p$ -median, bin packing, graph coloring, ...



# Block-Angular Structure?

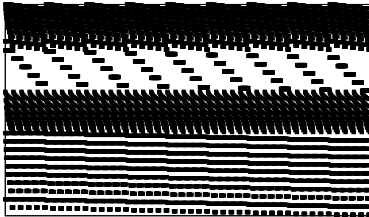
- ▶ key to automatic decomposition: detect “structure”



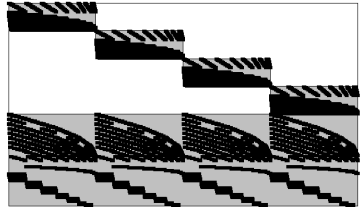
*10teams*

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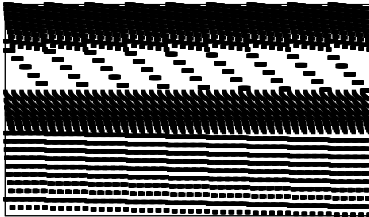
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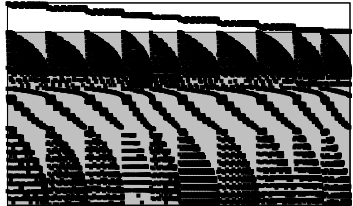
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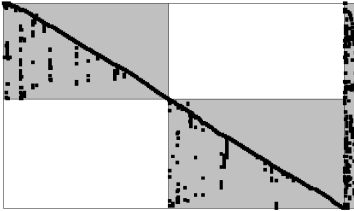


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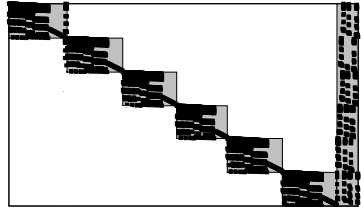


*10teams*

# Often: Linking Variables



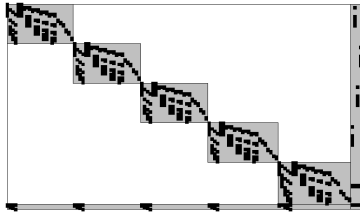
*timtab1*



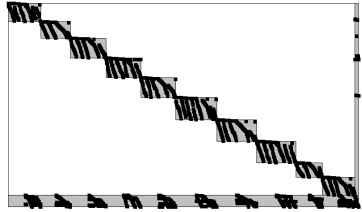
*gesa2*

- ▶ e.g., replace linking variable  $x$  by copies  $x^i$ , enforce  $x = x^i$
- ▶ alternatively, use DW reformulation  $x^i = \sum_{p \in P^i} \lambda_p p$

## More General: Arrowhead Structure



*noswt*



*aflow30a*

# Where do these Pictures come from?



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- ▶ “folklore” connection: (hyper-)graphs and matrices
  - ▶ several possibilities, e.g., (variant of Ferris/Horn, 1998)
  - ▶ vertex for every non-zero entry
  - ▶ hyperedge for each row, resp. each column
  - ▶ (different weights for different types of vars and conss)

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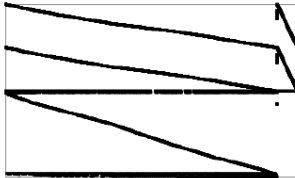
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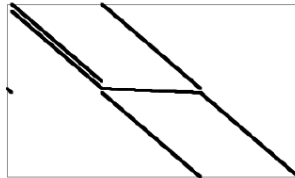
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  - ▶ (different weights for different types of vars and conss)
- ▶ arrowhead detection  $\equiv k$ -way hypergraph partitioning  
 $\equiv k$ -way vertex separation
- ▶ NP-hard problems, but effective heuristics available
- ▶ last year we used hMETIS 2.0pre1
- ▶ meanwhile we know about better algorithms

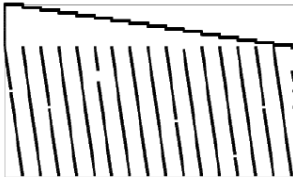
# Forcing MIPLIB2003 into Structure



(a) *p2756*



(b) *set1ch*

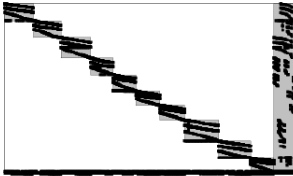


(c) *opt1217*

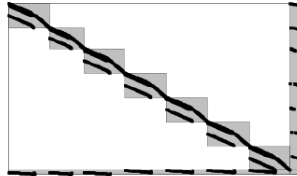


(d) *manna81*

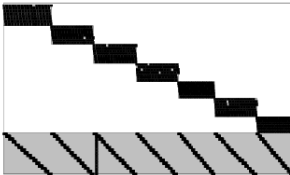
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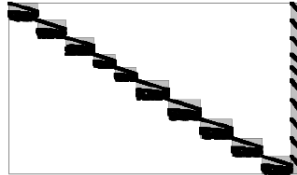
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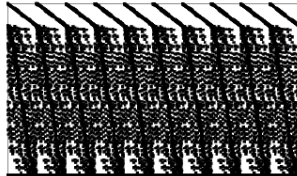


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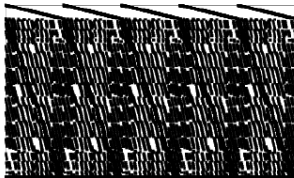
# Forcing MIPLIB2010 into Structure



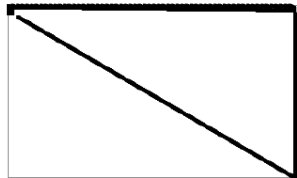
(a) *m100n500k4r1*



(b) *mine-90-10*

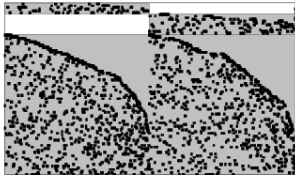


(c) *mine-166-5*



(d) *neos-686190*

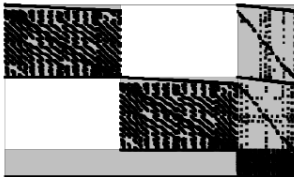
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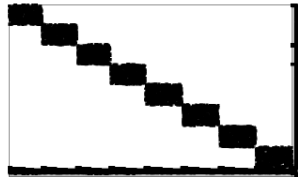
(a) *m100n500k4r1*



(b) *mine-90-10*



(c) *mine-166-5*

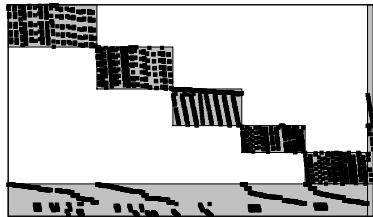
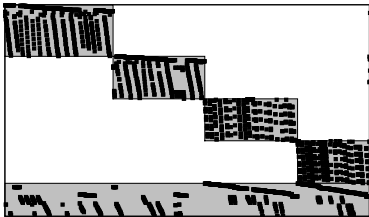
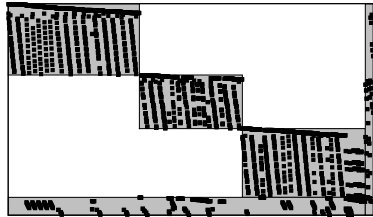
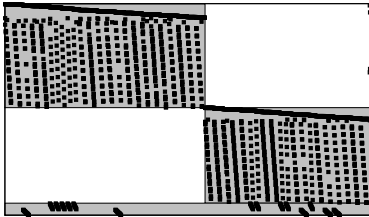


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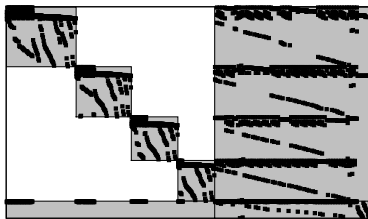
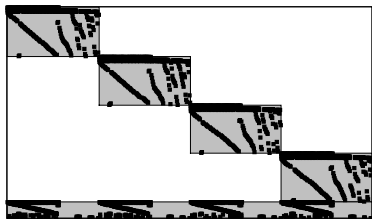
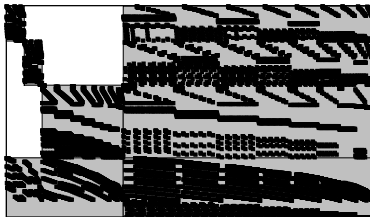
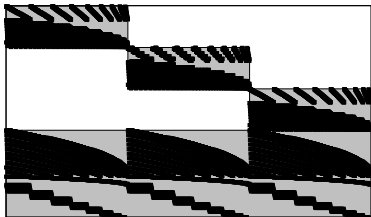
# What is a “good” Decomposition: Blocks

- influence of the number  $k$  of blocks on the “shape” (*fiber*)



# What is a “good” Decomposition: Visual

- ▶ visual differences for different decompositions (*10teams*, *roul*)



# Quality of a Decomposition

- ▶ caveat! don't think there is structure only b/c you see it
- ▶ a matrix can be permuted in many ways
- ▶ measure for the quality of a decomposition is needed
- ▶ this is a theoretically very challenging question
- ▶ here we used heuristic quality measures like “border area”

Selected MIPLIB 2003 and 2010 instances for which

- ▶ the optimum is known
- ▶ the density is between 0.05% and 5%
- ▶ the number of non-zeros is not larger than 20,000
- ▶ the percentage of discrete variables is at least 20%

# MIPLIB2003: Quality in Root Node

<i>instance</i>	rows	cols	<i>k</i>	<i>ℓ</i>	<i>c</i>	LP gap	DWR automatic gap	DWR automatic closed	DWR manual gap	DWR manual closed	CPLEX+cuts gap	CPLEX+cuts closed
<b>10teams</b>	2025	230	4	0	107	0.76	0.00	<b>100.00</b>	0.00	<b>100.00</b>	0.00	<b>100.00</b>
<i>afLOW30a</i>	842	479	2	0	28	15.10	14.71	2.59	10.23	32.27	5.35	<b>64.57</b>
<i>afLOW40b</i>	2728	1442	5	0	39	13.90	13.91	0.00	13.91	0.00	6.47	<b>53.44</b>
<b>fiber</b>	1298	363	2	2	21	61.55	1.07	<b>98.27</b>	1.07	<b>98.27</b>	1.77	97.13
<i>fixnet6</i>	878	478	4	3	14	69.85	18.89	72.96	18.89	72.96	6.06	<b>91.33</b>
<b>gesa2-o</b>	1224	1248	5	65	0	1.18	0.07	<b>94.17</b>	0.05	<b>96.16</b>	0.10	91.51
<b>gesa2</b>	1224	1392	3	65	0	1.18	0.06	<b>95.16</b>	0.03	<b>97.82</b>	0.21	82.43
<b>glass4</b>	322	396	3	16	0	33.33	25.12	<b>24.63</b>	25.12	<b>24.63</b>	33.33	0.00
<i>harp2</i>	2993	112	5	0	39	0.61	0.66	0.00	0.65	0.00	0.37	<b>39.61</b>
<i>manna81</i>	3321	6480	2	78	0	1.01	0.08	92.05	0.08	92.05	0.00	<b>100.00</b>
<b>mkc</b>	5325	3411	2	0	29	8.51	0.16	<b>98.13</b>	0.16	<b>98.13</b>	3.77	55.68
<b>modglob</b>	422	291	2	18	3	1.49	0.77	48.11	0.12	<b>91.74</b>	0.14	90.94
<b>noswot</b>	128	182	5	21	3	4.88	0.49	<b>90.00</b>	0.49	<b>90.00</b>	4.88	0.00
<i>opt1217</i>	769	64	4	0	16	25.13	25.13	0.00	25.13	0.00	0.00	<b>100.00</b>
<b>p2756</b>	2756	755	4	39	13	13.93	0.34	<b>97.59</b>	0.27	<b>98.07</b>	5.65	59.42
<b>pp08a</b>	240	136	2	16	0	62.61	2.27	<b>96.83</b>	2.27	<b>96.83</b>	2.53	95.97
<b>pp08aCUTS</b>	240	246	2	16	0	25.43	2.27	<b>91.09</b>	2.27	<b>91.09</b>	3.81	85.03
<b>rout</b>	556	291	5	0	16	8.88	0.68	<b>92.34</b>	0.68	<b>92.34</b>	8.86	0.26
<i>set1ch</i>	712	492	3	20	8	41.31	2.84	93.14	2.13	94.84	0.92	<b>97.78</b>
<b>timtab1</b>	397	171	2	13	0	96.25	14.60	<b>84.84</b>	14.60	<b>84.84</b>	39.05	59.43
<b>timtab2</b>	675	294	4	25	0	92.38	39.83	<b>56.89</b>	25.26	<b>72.66</b>	46.00	50.20
<i>tr12-30</i>	1080	750	3	24	0	89.12	93.47	0.00	2.83	96.83	0.68	<b>99.24</b>
<b>vpm2</b>	378	234	2	7	0	28.08	1.71	<b>93.92</b>	1.71	<b>93.92</b>	6.44	77.05
arithm. mean						30.28	11.27	66.18	<b>6.43</b>	<b>74.56</b>	7.67	69.17

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<i>instance</i>	<i>rows</i>	<i>cols</i>	<i>k</i>	<i>ℓ</i>	<i>c</i>	LP gap	DWR automatic gap	closed	DWR manual gap	closed	CPLEX+cuts gap	closed
<b>10teams</b>	2025	230	4	0	107	0.76	0.00	<b>100.00</b>	0.00	<b>100.00</b>	0.00	<b>100.00</b>
<i>afLOW30a</i>	842	479	2	0	28	15.10	14.71	2.59	10.23	32.27	5.35	<b>64.57</b>
<i>afLOW40b</i>	2728	1442	5	0	39	13.90	13.91	0.00	13.91	0.00	6.47	<b>53.44</b>
<b>fiber</b>	1298	363	2	2	21	61.55	1.07	<b>98.27</b>	1.07	<b>98.27</b>	1.77	97.13
<i>fixnet6</i>	878	478	4	3	14	69.85	18.89	72.96	18.89	72.96	6.06	<b>91.33</b>
<b>gesa2-o</b>	1224	1248	5	65	0	1.18	0.07	<b>94.17</b>	0.05	<b>96.16</b>	0.10	91.51
<b>gesa2</b>	1224	1392	3	65	0	1.18	0.06	<b>95.16</b>	0.03	<b>97.82</b>	0.21	82.43
<b>glass4</b>	322	396	3	16	0	33.33	25.12	<b>24.63</b>	25.12	<b>24.63</b>	33.33	0.00
<i>harp2</i>	2993	112	5	0	39	0.61	0.66	0.00	0.65	0.00	0.37	<b>39.61</b>
<i>manna81</i>	3321	6480	2	78	0	1.01	0.08	92.05	0.08	92.05	0.00	<b>100.00</b>
<b>mkc</b>	5325	3411	2	0	29	8.51	0.16	<b>98.13</b>	0.16	<b>98.13</b>	3.77	55.68
<b>modglob</b>	422	291	2	18	3	1.49	0.77	48.11	0.12	<b>91.74</b>	0.14	90.94
<b>noswot</b>	128	182	5	21	3	4.88	0.49	<b>90.00</b>	0.49	<b>90.00</b>	4.88	0.00
<i>opt1217</i>	769	64	4	0	16	25.13	25.13	0.00	25.13	0.00	0.00	<b>100.00</b>
<b>p2756</b>	2756	755	4	39	13	13.93	0.34	<b>97.59</b>	0.27	<b>98.07</b>	5.65	59.42
<b>pp08a</b>	240	136	2	16	0	62.61	2.27	<b>96.83</b>	2.27	<b>96.83</b>	2.53	95.97
<b>pp08aCUTS</b>	240	246	2	16	0	25.43	2.27	<b>91.09</b>	2.27	<b>91.09</b>	3.81	85.03
<b>rout</b>	556	291	5	0	16	8.88	0.68	<b>92.34</b>	0.68	<b>92.34</b>	8.86	0.26
<i>set1ch</i>	712	492	3	20	8	41.31	2.84	93.14	2.13	94.84	0.92	<b>97.78</b>
<b>timtab1</b>	397	171	2	13	0	96.25	14.60	<b>84.84</b>	14.60	<b>84.84</b>	39.05	59.43
<b>timtab2</b>	675	294	4	25	0	92.38	39.83	<b>56.89</b>	25.26	<b>72.66</b>	46.00	50.20
<i>tr12-30</i>	1080	750	3	24	0	89.12	93.47	0.00	2.83	96.83	0.68	<b>99.24</b>
<b>vpm2</b>	378	234	2	7	0	28.08	1.71	<b>93.92</b>	1.71	<b>93.92</b>	6.44	77.05
arithm. mean						30.28	11.27	66.18	<b>6.43</b>	<b>74.56</b>	7.67	69.17

# MIPLIB2003: Quality in Root Node

<i>instance</i>	<i>rows</i>	<i>cols</i>	<i>k</i>	<i>ℓ</i>	<i>c</i>	LP gap	DWR automatic gap	DWR automatic closed	DWR manual gap	DWR manual closed	CPLEX+cuts gap	CPLEX+cuts closed
<b>10teams</b>	2025	230	4	0	107	0.76	0.00	<b>100.00</b>	0.00	<b>100.00</b>	0.00	<b>100.00</b>
<i>afLOW30a</i>	842	479	2	0	28	15.10	14.71	2.59	10.23	32.27	5.35	<b>64.57</b>
<i>afLOW40b</i>	2728	1442	5	0	39	13.90	13.91	0.00	13.91	0.00	6.47	<b>53.44</b>
<b>fiber</b>	1298	363	2	2	21	61.55	1.07	<b>98.27</b>	1.07	<b>98.27</b>	1.77	97.13
<i>fixnet6</i>	878	478	4	3	14	69.85	18.89	72.96	18.89	72.96	6.06	<b>91.33</b>
<b>gesa2-o</b>	1224	1248	5	65	0	1.18	0.07	<b>94.17</b>	0.05	<b>96.16</b>	0.10	91.51
<b>gesa2</b>	1224	1392	3	65	0	1.18	0.06	<b>95.16</b>	0.03	<b>97.82</b>	0.21	82.43
<b>glass4</b>	322	396	3	16	0	33.33	25.12	<b>24.63</b>	25.12	<b>24.63</b>	33.33	0.00
<i>harp2</i>	2993	112	5	0	39	0.61	0.66	0.00	0.65	0.00	0.37	<b>39.61</b>
<i>manna81</i>	3321	6480	2	78	0	1.01	0.08	92.05	0.08	92.05	0.00	<b>100.00</b>
<b>mkc</b>	5325	3411	2	0	29	8.51	0.16	<b>98.13</b>	0.16	<b>98.13</b>	3.77	55.68
<b>modglob</b>	422	291	2	18	3	1.49	0.77	48.11	0.12	<b>91.74</b>	0.14	90.94
<b>noswot</b>	128	182	5	21	3	4.88	0.49	<b>90.00</b>	0.49	<b>90.00</b>	4.88	0.00
<i>opt1217</i>	769	64	4	0	16	25.13	25.13	0.00	25.13	0.00	0.00	<b>100.00</b>
<b>p2756</b>	2756	755	4	39	13	13.93	0.34	<b>97.59</b>	0.27	<b>98.07</b>	5.65	59.42
<b>pp08a</b>	240	136	2	16	0	62.61	2.27	<b>96.83</b>	2.27	<b>96.83</b>	2.53	95.97
<b>pp08aCUTS</b>	240	246	2	16	0	25.43	2.27	<b>91.09</b>	2.27	<b>91.09</b>	3.81	85.03
<b>rout</b>	556	291	5	0	16	8.88	0.68	<b>92.34</b>	0.68	<b>92.34</b>	8.86	0.26
<i>set1ch</i>	712	492	3	20	8	41.31	2.84	93.14	2.13	94.84	0.92	<b>97.78</b>
<b>timtab1</b>	397	171	2	13	0	96.25	14.60	<b>84.84</b>	14.60	<b>84.84</b>	39.05	59.43
<b>timtab2</b>	675	294	4	25	0	92.38	39.83	<b>56.89</b>	25.26	<b>72.66</b>	46.00	50.20
<i>tr12-30</i>	1080	750	3	24	0	89.12	93.47	0.00	2.83	96.83	0.68	<b>99.24</b>
<b>vpm2</b>	378	234	2	7	0	28.08	1.71	<b>93.92</b>	1.71	<b>93.92</b>	6.44	77.05
arithm. mean						30.28	11.27	66.18	6.43	74.56	7.67	69.17



# MIPLIB2003: Quality in Root Node

<i>instance</i>	<i>rows</i>	<i>cols</i>	<i>k</i>	<i>ℓ</i>	<i>c</i>	LP gap	DWR gap	automatic closed	DWR gap	manual closed	CPLEX+cuts gap	closed
<b>10teams</b>	2025	230	4	0	107	0.76	0.00	<b>100.00</b>	0.00	<b>100.00</b>	0.00	<b>100.00</b>
<i>afLOW30a</i>	842	479	2	0	28	15.10	14.71	2.59	10.23	32.27	5.35	<b>64.57</b>
<i>afLOW40b</i>	2728	1442	5	0	39	13.90	13.91	0.00	13.91	0.00	6.47	<b>53.44</b>
<b>fiber</b>	1298	363	2	2	21	61.55	1.07	<b>98.27</b>	1.07	<b>98.27</b>	1.77	97.13
<i>fixnet6</i>	878	478	4	3	14	69.85	18.89	72.96	18.89	72.96	6.06	<b>91.33</b>
<b>gesa2-o</b>	1224	1248	5	65	0	1.18	0.07	<b>94.17</b>	0.05	<b>96.16</b>	0.10	91.51
<b>gesa2</b>	1224	1392	3	65	0	1.18	0.06	<b>95.16</b>	0.03	<b>97.82</b>	0.21	82.43
<b>glass4</b>	322	396	3	16	0	33.33	25.12	<b>24.63</b>	25.12	<b>24.63</b>	33.33	0.00
<i>harp2</i>	2993	112	5	0	39	0.61	0.66	0.00	0.65	0.00	0.37	<b>39.61</b>
<i>manNA81</i>	3321	6480	2	78	0	1.01	0.08	92.05	0.08	92.05	0.00	<b>100.00</b>
<b>mkc</b>	5325	3411	2	0	29	8.51	0.16	<b>98.13</b>	0.16	<b>98.13</b>	3.77	55.68
<b>modglob</b>	422	291	2	18	3	1.49	0.77	48.11	0.12	<b>91.74</b>	0.14	90.94
<b>noswot</b>	128	182	5	21	3	4.88	0.49	<b>90.00</b>	0.49	<b>90.00</b>	4.88	0.00
<i>opt1217</i>	769	64	4	0	16	25.13	25.13	0.00	25.13	0.00	0.00	<b>100.00</b>
<b>p2756</b>	2756	755	4	39	13	13.93	0.34	<b>97.59</b>	0.27	<b>98.07</b>	5.65	59.42
<b>pp08a</b>	240	136	2	16	0	62.61	2.27	<b>96.83</b>	2.27	<b>96.83</b>	2.53	95.97
<b>pp08aCUTS</b>	240	246	2	16	0	25.43	2.27	<b>91.09</b>	2.27	<b>91.09</b>	3.81	85.03
<b>rout</b>	556	291	5	0	16	8.88	0.68	<b>92.34</b>	0.68	<b>92.34</b>	8.86	0.26
<i>set1ch</i>	712	492	3	20	8	41.31	2.84	93.14	2.13	94.84	0.92	<b>97.78</b>
<b>timtab1</b>	397	171	2	13	0	96.25	14.60	<b>84.84</b>	14.60	<b>84.84</b>	39.05	59.43
<b>timtab2</b>	675	294	4	25	0	92.38	39.83	<b>56.89</b>	25.26	<b>72.66</b>	46.00	50.20
<i>tr12-30</i>	1080	750	3	24	0	89.12	93.47	0.00	2.83	96.83	0.68	<b>99.24</b>
<b>vpm2</b>	378	234	2	7	0	28.08	1.71	<b>93.92</b>	1.71	<b>93.92</b>	6.44	77.05
arithm. mean						30.28	11.27	66.18	6.43	74.56	7.67	69.17

# MIPLIB2010: Quality in Root Node

instance	rows	cols	k	$\ell$	c	LP	DWR automatic		DWR manual		CPLEX+cuts	
						gap	gap	closed	gap	closed	gap	closed
<i>beasleyC3</i>	1750	2500	3	52	5	94.64	20.95	77.86	20.95	77.86	0.08	<b>91.29</b>
<i>csched010</i>	351	1758	2	1	55	18.52	7.91	<b>57.28</b>	7.91	<b>57.28</b>	12.62	31.86
<i>enlight13</i>	169	338	2	25	0	100.00	57.55	<b>42.45</b>	57.55	<b>42.45</b>	80.66	19.34
<i>gmu-35-40</i>	424	1205	2	0	9	0.01	0.03	0.00	0.03	0.00	0.01	<b>2.54</b>
<i>m100n500k4r1</i>	100	500	2	11	86	4.17	4.17	<b>0.00</b>	4.17	<b>0.00</b>	4.17	<b>0.00</b>
<i>macrophage</i>	3164	2260	2	7	0	100.00	0.37	<b>99.63</b>	0.05	<b>99.95</b>	11.48	88.52
<i>mcsched</i>	2107	1747	2	69	2	8.56	1.03	<b>87.91</b>	1.03	<b>87.91</b>	8.54	0.23
<i>mine-166-5</i>	8429	830	2	166	1320	45.09	9.78	<b>78.30</b>	0.00	<b>100.00</b>	22.79	49.44
<i>mine-90-10</i>	6270	900	2	0	1264	13.12	6.90	<b>47.39</b>	6.71	<b>48.81</b>	8.73	33.41
<i>neos-686190</i>	3664	3660	3	59	179	23.70	100.00	0.00	100.00	0.00	21.54	<b>9.11</b>
<i>pigeon-10</i>	931	490	2	96	40	11.11	11.11	<b>0.00</b>	11.11	<b>0.00</b>	11.11	<b>0.00</b>
<i>pw-myciel4</i>	8164	1059	2	24	49	100.00	60.00	40.00	0.00	<b>100.00</b>	53.54	46.46
<i>ran16x16</i>	288	512	2	0	16	18.48	10.86	41.27	10.82	41.44	6.93	<b>62.51</b>
<i>reblock67</i>	2523	670	2	67	112	13.61	1.87	<b>86.23</b>	1.87	<b>86.23</b>	8.87	34.86
<i>rmine6</i>	7078	1096	2	186	926	1.12	3.22	0.00	1.89	0.00	0.98	<b>12.26</b>
<i>rococoC10-001000</i>	1293	3117	2	0	82	34.42	34.43	0.00	34.42	0.00	11.62	<b>66.25</b>
arithm. mean						36.66	20.64	41.14	16.16	46.37	16.48	34.26

# MIPLIB2010: Quality in Root Node

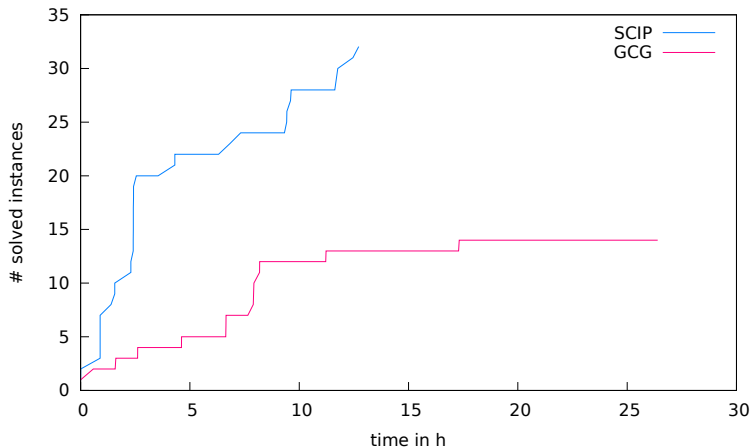
instance	rows	cols	k	$\ell$	c	LP	DWR automatic		DWR manual		CPLEX+cuts	
						gap	gap	closed	gap	closed	gap	closed
<i>beasleyC3</i>	1750	2500	3	52	5	94.64	20.95	77.86	20.95	77.86	0.08	<b>91.29</b>
<i>csched010</i>	351	1758	2	1	55	18.52	7.91	<b>57.28</b>	7.91	<b>57.28</b>	12.62	31.86
<i>enlight13</i>	169	338	2	25	0	100.00	57.55	<b>42.45</b>	57.55	<b>42.45</b>	80.66	19.34
<i>gmu-35-40</i>	424	1205	2	0	9	0.01	0.03	0.00	0.03	0.00	0.01	<b>2.54</b>
<i>m100n500k4r1</i>	100	500	2	11	86	4.17	4.17	<b>0.00</b>	4.17	<b>0.00</b>	4.17	<b>0.00</b>
<i>macrophage</i>	3164	2260	2	7	0	100.00	0.37	<b>99.63</b>	0.05	<b>99.95</b>	11.48	88.52
<i>mcsched</i>	2107	1747	2	69	2	8.56	1.03	<b>87.91</b>	1.03	<b>87.91</b>	8.54	0.23
<i>mine-166-5</i>	8429	830	2	166	1320	45.09	9.78	<b>78.30</b>	0.00	<b>100.00</b>	22.79	49.44
<i>mine-90-10</i>	6270	900	2	0	1264	13.12	6.90	<b>47.39</b>	6.71	<b>48.81</b>	8.73	33.41
<i>neos-686190</i>	3664	3660	3	59	179	23.70	100.00	0.00	100.00	0.00	21.54	<b>9.11</b>
<i>pigeon-10</i>	931	490	2	96	40	11.11	11.11	<b>0.00</b>	11.11	<b>0.00</b>	11.11	<b>0.00</b>
<i>pw-myciel4</i>	8164	1059	2	24	49	100.00	60.00	40.00	0.00	<b>100.00</b>	53.54	46.46
<i>ran16x16</i>	288	512	2	0	16	18.48	10.86	41.27	10.82	41.44	6.93	<b>62.51</b>
<i>reblock67</i>	2523	670	2	67	112	13.61	1.87	<b>86.23</b>	1.87	<b>86.23</b>	8.87	34.86
<i>rmine6</i>	7078	1096	2	186	926	1.12	3.22	0.00	1.89	0.00	0.98	<b>12.26</b>
<i>rococoC10-001000</i>	1293	3117	2	0	82	34.42	34.43	0.00	34.42	0.00	11.62	<b>66.25</b>
arithm. mean						36.66	20.64	41.14	16.16	46.37	16.48	34.26

# MIPLIB2010: Quality in Root Node

instance	rows	cols	k	$\ell$	c	LP	DWR automatic		DWR manual		CPLEX+cuts	
						gap	gap	closed	gap	closed	gap	closed
<i>beasleyC3</i>	1750	2500	3	52	5	94.64	20.95	77.86	20.95	77.86	0.08	<b>91.29</b>
<i>csched010</i>	351	1758	2	1	55	18.52	7.91	<b>57.28</b>	7.91	<b>57.28</b>	12.62	31.86
<i>enlight13</i>	169	338	2	25	0	100.00	57.55	<b>42.45</b>	57.55	<b>42.45</b>	80.66	19.34
<i>gmu-35-40</i>	424	1205	2	0	9	0.01	0.03	0.00	0.03	0.00	0.01	<b>2.54</b>
<i>m100n500k4r1</i>	100	500	2	11	86	4.17	4.17	<b>0.00</b>	4.17	<b>0.00</b>	4.17	<b>0.00</b>
<i>macrophage</i>	3164	2260	2	7	0	100.00	0.37	<b>99.63</b>	0.05	<b>99.95</b>	11.48	88.52
<i>mcsched</i>	2107	1747	2	69	2	8.56	1.03	<b>87.91</b>	1.03	<b>87.91</b>	8.54	0.23
<i>mine-166-5</i>	8429	830	2	166	1320	45.09	9.78	<b>78.30</b>	0.00	<b>100.00</b>	22.79	49.44
<i>mine-90-10</i>	6270	900	2	0	1264	13.12	6.90	<b>47.39</b>	6.71	<b>48.81</b>	8.73	33.41
<i>neos-686190</i>	3664	3660	3	59	179	23.70	100.00	0.00	100.00	0.00	21.54	<b>9.11</b>
<i>pigeon-10</i>	931	490	2	96	40	11.11	11.11	<b>0.00</b>	11.11	<b>0.00</b>	11.11	<b>0.00</b>
<i>pw-myciel4</i>	8164	1059	2	24	49	100.00	60.00	40.00	0.00	<b>100.00</b>	53.54	46.46
<i>ran16x16</i>	288	512	2	0	16	18.48	10.86	41.27	10.82	41.44	6.93	<b>62.51</b>
<i>reblock67</i>	2523	670	2	67	112	13.61	1.87	<b>86.23</b>	1.87	<b>86.23</b>	8.87	34.86
<i>rmine6</i>	7078	1096	2	186	926	1.12	3.22	0.00	1.89	0.00	0.98	<b>12.26</b>
<i>rococoC10-001000</i>	1293	3117	2	0	82	34.42	34.43	0.00	34.42	0.00	11.62	<b>66.25</b>
arithm. mean						36.66	20.64	41.14	16.16	46.37	16.48	34.26

# MIPLIBs: Computation Time

- # instances solved to integer optimality, **SCIP** vs. **GCG**



## General IPs: These are the wrong Instances!

- ▶ we wish to exploit *structure* in IPs,  
so we need to consider structured IPs!

## General IPs: These are the wrong Instances!

- ▶ we wish to exploit *structure* in IPs,  
so we need to consider structured IPs!
- ▶ to this end, we are currently building the `strIPlib`
- ▶ planned as a web portal as source of information and sandbox
- ▶ bin packing, cutting stock, vertex coloring,  $p$ -median,  
generalized assignment, vehicle routing, machine scheduling,  
*your instances here*, ...

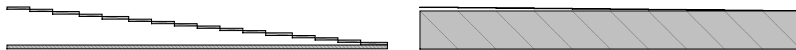
# General IPs: These are the wrong Instances!

- ▶ we wish to exploit *structure* in IPs,  
so we need to consider structured IPs!
- ▶ to this end, we are currently building the strIPlib
- ▶ planned as a web portal as source of information and sandbox
- ▶ bin packing, cutting stock, vertex coloring,  $p$ -median,  
generalized assignment, vehicle routing, machine scheduling,  
your instances here, ...
- ▶ rather small subset for this talk:
  - ▶ all N1\*, N2\* instances from bison 1 binpacking set (360 instances)
  - ▶ selected instances from
    - ▶ GAP: orlibrary and Yagiura testset (16 instances)
    - ▶ cut. stock: Schwerin, Waescher and hard28 (12 instances)
    - ▶ cap.  $p$ -median: optlab and orlibrary (6 instances)
    - ▶ vertex coloring: r-set (4 instances)
  - ▶ criterion: solved in less than 600s, varying in nodes and size



# Detecting Structure in Structured IPs

- ▶ application of hypergraph approach fails surprisingly often!



- ▶ # blocks for *gapa\_3*: hmetis: 16, “natural/best:” 10

# Detecting Structure in Structured IPs

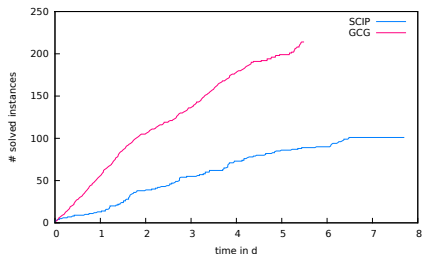
- ▶ better, easier, faster detection based on constraint types  
e.g., SPP constraints → master, block detection on remainder

	blocks	hmetis		blocks	best	
		nodes	time		nodes	time
gapa_3	16	1	0.8	10	1	6.0
gapc_5	16	1217	57.8	20	27	6.8
N1C3W4_E	16	≈ 26k	5.41%	50	1	9.2
N1C3W4_L	16	≈ 25k	6.20%	50	22	14.3
out15-1	—	—	—	100	271	212.8
out18-1	—	—	—	100	503	264.9
R50_1g	9	1	47.5	9	6580	7.7
R75_1gb	13	1	411.6	13	10426	228.9
TEST0005	—	—	—	57	59	553.0
TEST0084	—	—	—	46	454	137.1

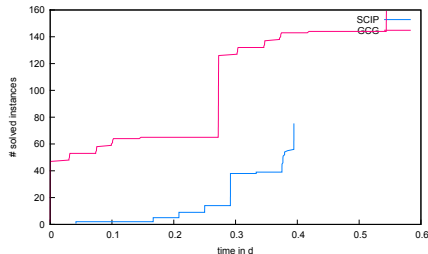
- ▶ ample room for (heuristic) improvements

# Computation Times for Structured Problems

- # instances solved to integer optimality, GCG vs. SCIP



cutting stock



generalized assignment

# Benefits from this Approach

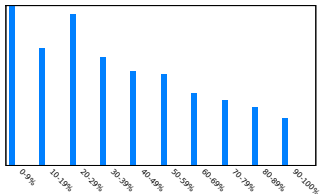
- ▶ use a *single* generic implementation on a large number of classical and non-classical applications
- ▶ collect all kinds of data and compare in unified way

- ▶ caveat!

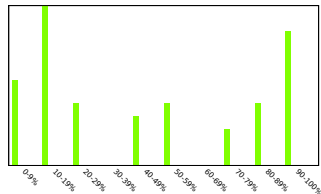
the following figures only serve the purpose of illustration!

# When do we Generate the “Important Columns”?

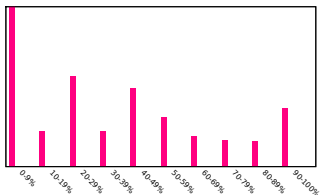
- columns that contribute to the best IP solution



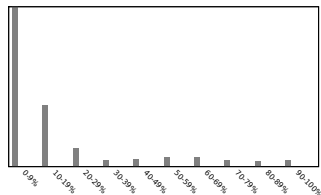
bin packing



vertex coloring



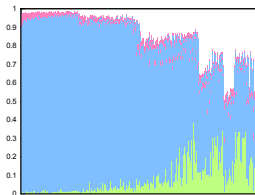
cutting stock



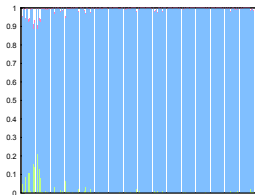
capacitated  $p$ -median

# Where do we Spend the Computation Time?

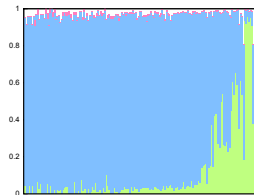
- instances sorted according to computation time



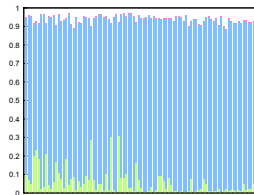
bin packing



cutting stock



generalized assignment



capacitated  $p$ -median

# Much more is Collected

- ▶ degree of degeneracy
- ▶ behavior of dual variables
- ▶ how do the primal/dual bounds develop in the tree
- ▶ which heuristics work well
- ▶ effectiveness of features like pseudo-costs, propagation, ...
- ▶ pictures/statistics about different decompositions
- ▶  $\vdots$
- ▶ this information available about *your instance* via `strIPlib`

# Benefits from Building on a State-of-the-Art Solver

- ▶ e.g., presolve (−19% time) and cutting planes (−3% time)

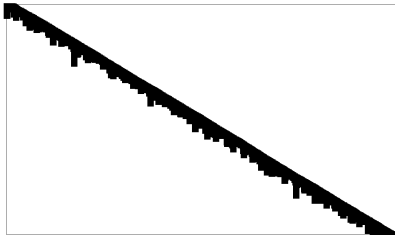
	GCG default settings		no presolve		no cuts	
	Nodes	Time	Nodes	Time	Nodes	Time
10teams	180	32.1	537	170.5	180	37.4
afLOW30a	33 246	2064.2	>490	<12.0%	>65 105	<7.3%
afLOW40b	>2 521	<41.9%	>4 400	<22.1%	>9 554	>3600.0
fiber	19	95.6	44	151.4	19	104.5
fixnet6	>543	<4.1%	>751	<5.5%	2 111	1006.6
gesa2	1	4.5	1	7.3	1	4.4
gesa2-o	>578	<1.2%	>475	<0.8%	>238	<2.9%
glass4	>67	>3615.7	—	—	>67	>3951.3
harp2	>7	>3672.0	>11 645	<0.5%	>7	>3672.0
manna81	>1	<445.6%	>8	<68.0%	>1	<445.6%
mkc	>100	<5.7%	>100	<40.2%	>485	<7.8%
modglob	7	48.0	—	—	23	60.0
noswot	1	4.1	1	1.2	1	2.1
opt1217	>207 599	<25.0%	>71 917	<25.1%	>253 938	<25.0%
p2756	4 942	892.5	>9 855	<0.1%	>42 006	<0.1%
pp08a	549	50.5	425	42.6	398	36.8
pp08aCUTS	409	41.5	609	58.8	465	50.7
rout	2 111	932.6	2 493	1565.5	2 449	944.2
set1ch	69	17.0	84	31.9	69	26.6
timtab1	>207	>3600.9	>271	>3700.2	>199	>3600.4
timtab2	>21	>3604.0	>15	>3600.7	>32	>3600.2
tr12-30	—	—	>1	>3673.7	—	—
vpM2	131	55.4	371	380.1	131	88.2

— did not finish Farkas pricing, had numerical problems, or a bug



# More to Detect!

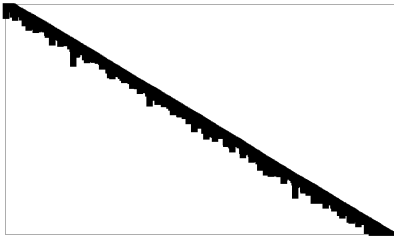
- ▶ e.g., problems “indexed over time”



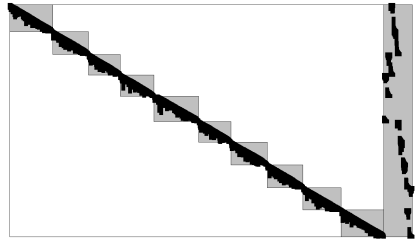
canonical

# More to Detect!

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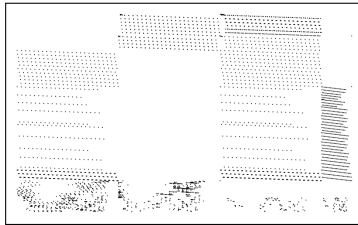
canonical



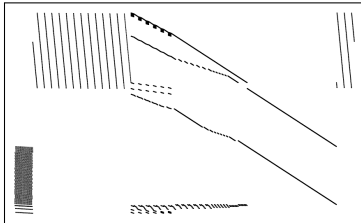
“detected”

- ▶ detection  $\sim$  “structure enforcement”
- ▶ reformulation essentially convexifies all constraints
- ▶ gives very tight relaxations

# More to Detect!

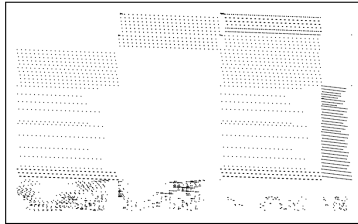


*bg512142* original

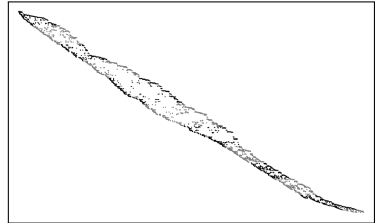


*a1c1s1* original

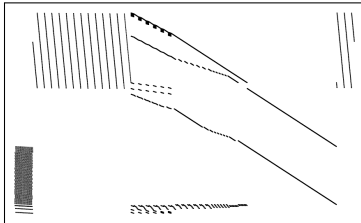
# More to Detect!



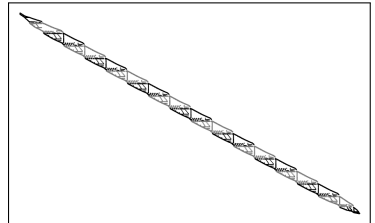
*bg512142* original



staircase detected



*a1c1s1* original




staircase detected

# Feature Request and Call for Instances

- ▶ if you were to use a generic BP&C solver:  
what features would you like to see?
- ▶ if you like the solver to work well on your problems:  
send us your instances!

- ▶ automatic decomposition and branch-price-and-cut doable
- ▶ lucky for “unstructured” IPs, successful on “structured”
- ▶ challenge: tell promising from unpromising instances
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- ▶ here are hiring!  
research grant (postdoc) sponsored by 



**AUGUST 19-24**

[www.ismp2012.org](http://www.ismp2012.org)



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