The fire and rescue vehicle location problem

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Introduction

- The Swedish Civil Contingencies Agency is enhancing and supporting societal capacities for preparedness for and prevention of emergencies and crises

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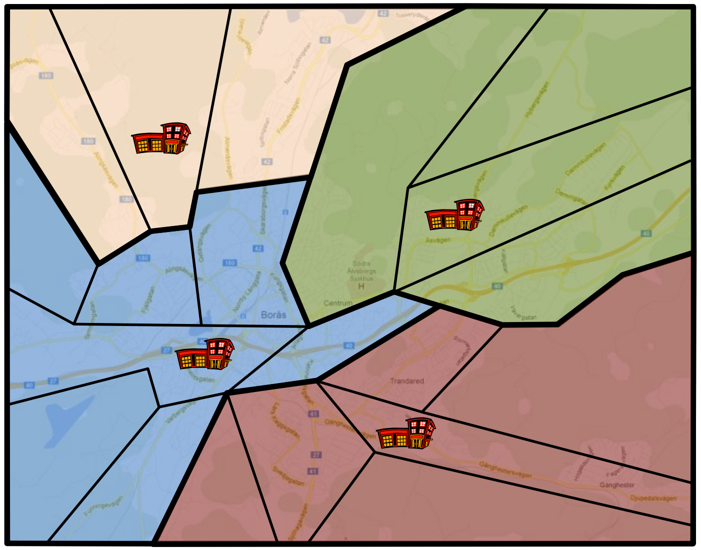
- How can this be done better using column generation?

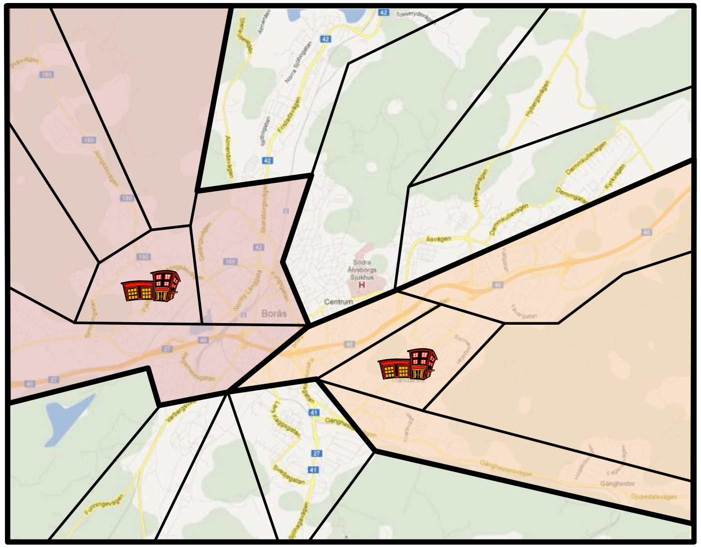
Introduction

- Strategic decision support for locating emergency units
- New ways of thinking about emergency response

Where to locate the units

- Location set covering problem
  - Define a response requirement
  - Minimize the number of units needed to fulfill the requirements
- Maximal covering location problem
  - Define a covering criterion
  - Maximize the population covered by a fixed fleet





Traditionally

- Station location
- Homogeneous fleet
  - Only one type of vehicle
- Homogeneous demand
  - One type of events
  - No or simple demand
- Only first response counts
  - Or not even this

Traditionally vs More realism

- Station location
- Homogeneous fleet
  - Only one type of vehicle
- Homogeneous demand
  - One type of events
  - No or simple demand
- Only first response counts
  - Or not even this

- Locating individual units
- Heterogeneous fleet
  - Multiple types of vehicles
- Heterogeneous demand
  - Multiple types of events
  - Demand based on statistics
- First and full response

Heterogeneous fleet and demand

- Different vehicles
  - Base unit
  - Ladder unit
  - Small unit
  - Different manning
- Different events
  - Fire in low-rise building
  - Fire in high-rise building
  - Traffic accident

Heterogeneous fleet and demand

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  - Ladder unit
  - Small unit
  - Different manning
- Different events
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  - Fire in high-rise building
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Alarm plan				
	Base	Ladder	Small	# People
Low building	1	0	0	5
High building	1	1	0	6
Traffic	1	0	0	5

Preparedness

- Ideally we would like to minimize equivalence time

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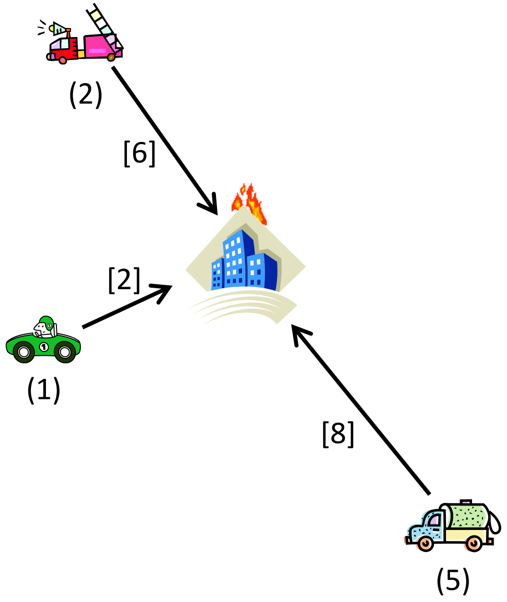
  - Fire in high-rise building
    - Base unit
    - Ladder unit
    - 6 fire fighters



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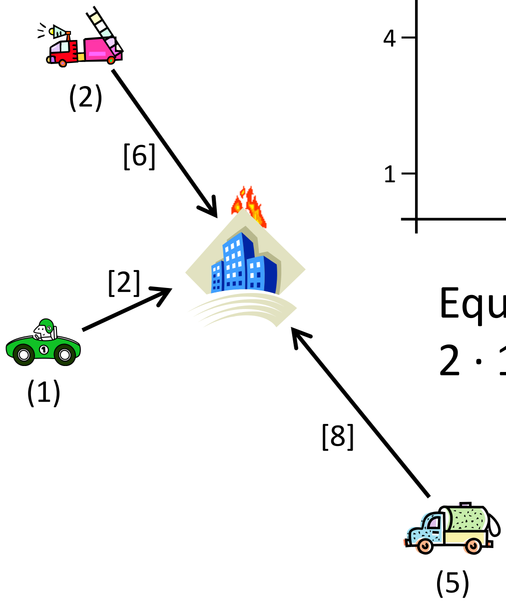
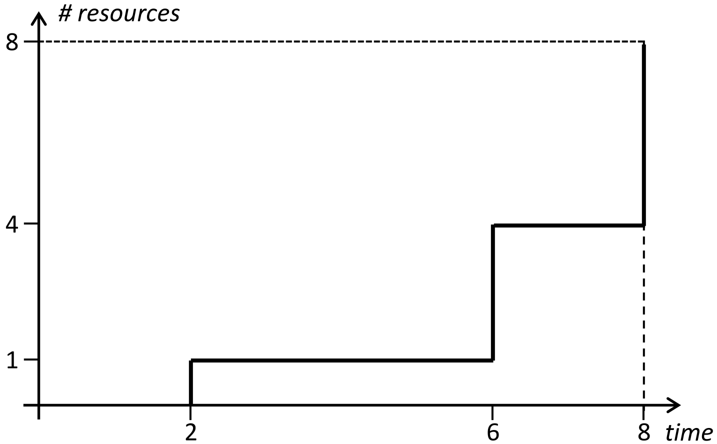
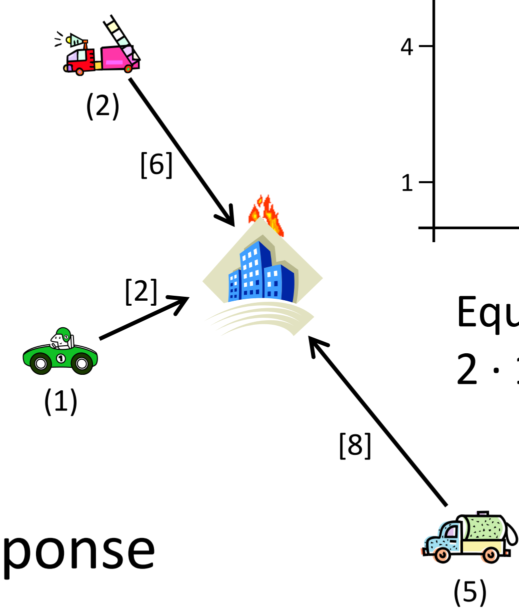


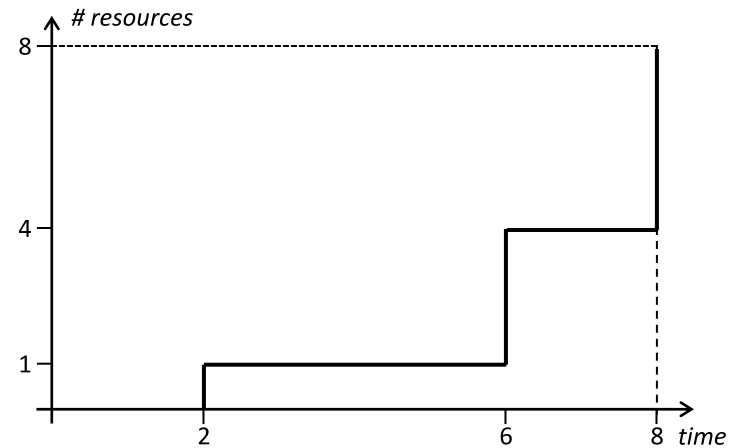


Equivalence time =
$2 \cdot 1 + 6 \cdot 3 + 8 \cdot 4 = 52 / 8 = 6.5$

Preparedness

- Ideally we would like to minimize equivalence time
  - Fire in high-rise building
    - Base unit
    - Ladder unit
    - 6 fire fighters
  - Too complicated
    - First and full response





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$2 \cdot 1 + 6 \cdot 3 + 8 \cdot 4 = 52 / 8 = 6.5$

First and full response

- First response
  – The time it takes for the first unit to get on site
- Full response
  – The time until all resources defined in the alarm plan are on site

- Minimize the total demand weighted

  $\alpha \cdot \text{First response} + (1 - \alpha) \cdot \text{Full response}$

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Sets, parameters and variables

I	Zones,	$I = I^H \cup I^L$	[High , Low]
O	Accidents,	$O = O^F \cup O^T$	[Fire , Traffic]
V	Vehicles,	$V = V^B \cup V^L \cup V^S$	[Base , Ladder , Small]

D_{oi} Expected number of accidents of type *o* in zone *i*
H_{oi} Number of people needed for accident *o* in zone *i*
P_v Number of people in vehicle *v*
A_v Call out time for vehicle *v*
T_{ij} Travel time between zone *i* and zone *j*
W Weight factor for first response

x_{vi} 1 if vehicle *v* is located in zone *i*
y_{voi} 1 if vehicle *v* responds to an accident of type *o* in zone *i*
z_{voi} 1 if vehicle *v* is the first response to an accident of type *o* in zone *i*
t_{oi} Response time for the full response to an accident of type *o* in zone *i*
f_{oi} Response time for the first response to an accident of type *o* in zone *i*

Original formulation

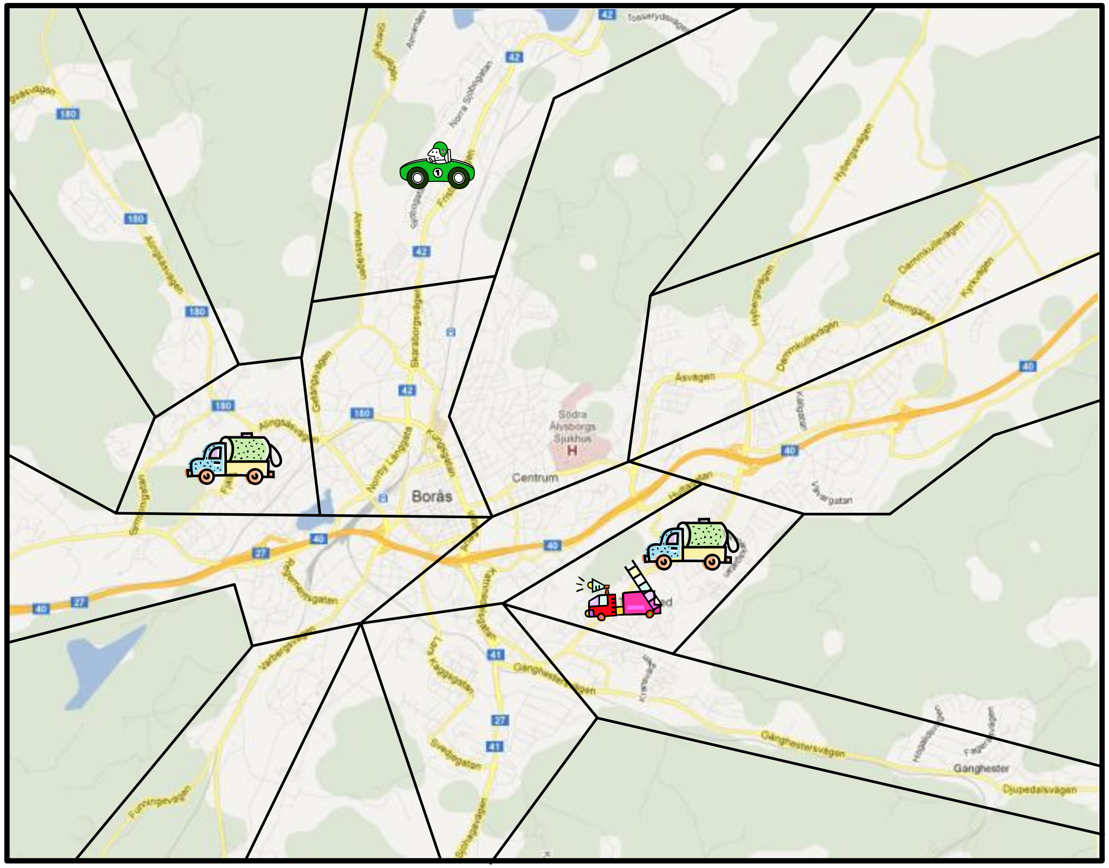
$$\min \sum_{o \in O} \sum_{i \in I} D_{oi} \left(W \cdot f_{oi} + (1-W) \cdot t_{oi} \right)$$

$\sum_{i \in I} x_{vi} = 1$	$v \in V$	Each vehicle must be located once
$\sum_{v \in V} z_{voi} = 1$	$o \in O, i \in I$	Each accident must have a first response
$\sum_{v \in V^B} y_{voi} \geq 1$	$o \in O, i \in I$	A base unit is needed for each accident
$\sum_{v \in V^L} y_{voi} \geq 1$	$o \in O^F, i \in I^H$	A ladder unit is needed for fires in high buildings
$\sum_{v \in V} P_v y_{voi} \geq H_{oi}$	$o \in O, i \in I$	Number of people needed for each accident
$\sum_{v \in V^L} x_{vi} - \sum_{v \in V^B} x_{vi} \leq 0$	$i \in I$	A ladder unit must be located in tandem with a base unit
$t_{oj} \geq \sum_{i \in I} (T_{ij} + A_v) x_{vi} - M(1 - y_{voj})$	$v \in V, o \in O, j \in I$	Full response time
$f_{oj} \geq \sum_{i \in I} (T_{ij} + A_v) x_{vi} - M(1 - z_{voj})$	$v \in V, o \in O, j \in I$	First response time
$x_{vi}, y_{voi}, z_{voi} \in \{0,1\}$	$v \in V, o \in O, i \in I$	
t_{oi}, f_{oi}	$v \in V, o \in O, i \in I$	

Comments

- The LP bound is very weak
- The time constraints are challenging
- There are continuous variables
- The problem is easy if the location is fixed
- Only preliminary testing has been done

Solution structure



- Base unit
- Ladder unit
- Small unit

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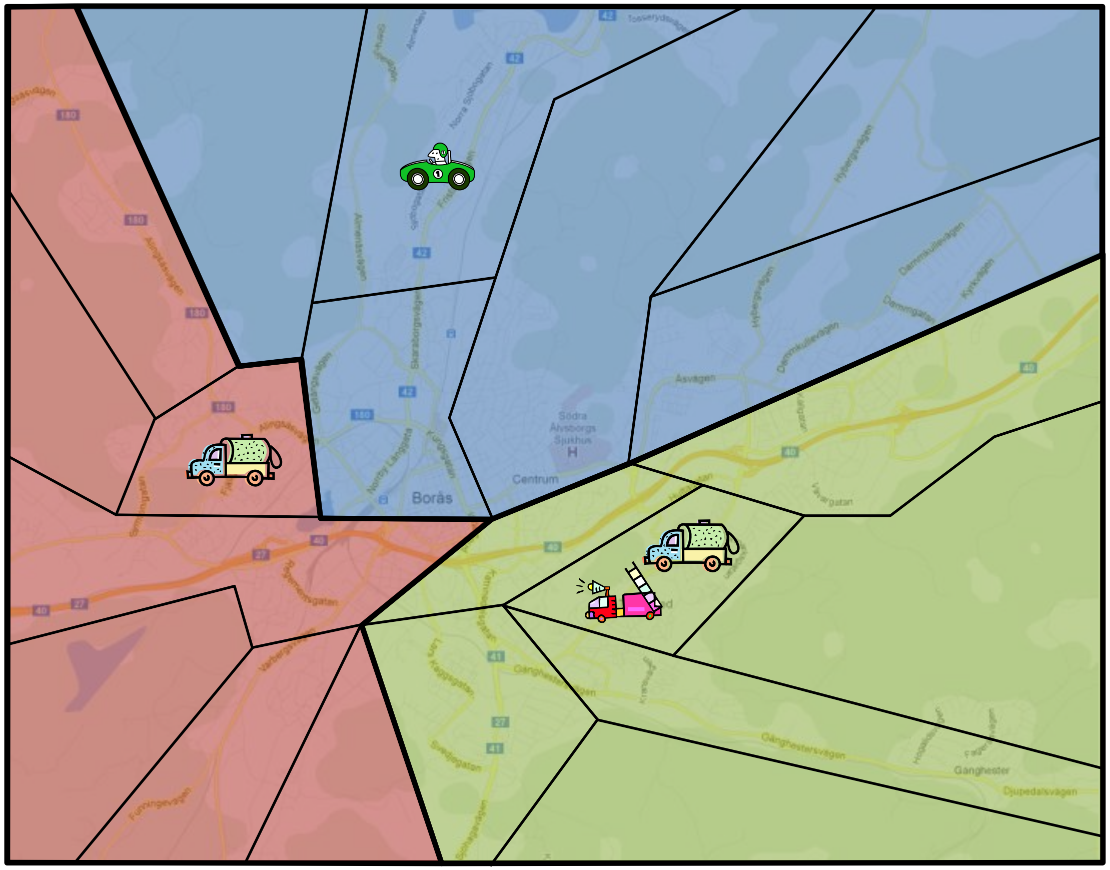
Solution structure



Base unit

Ladder unit

Small unit

- First response

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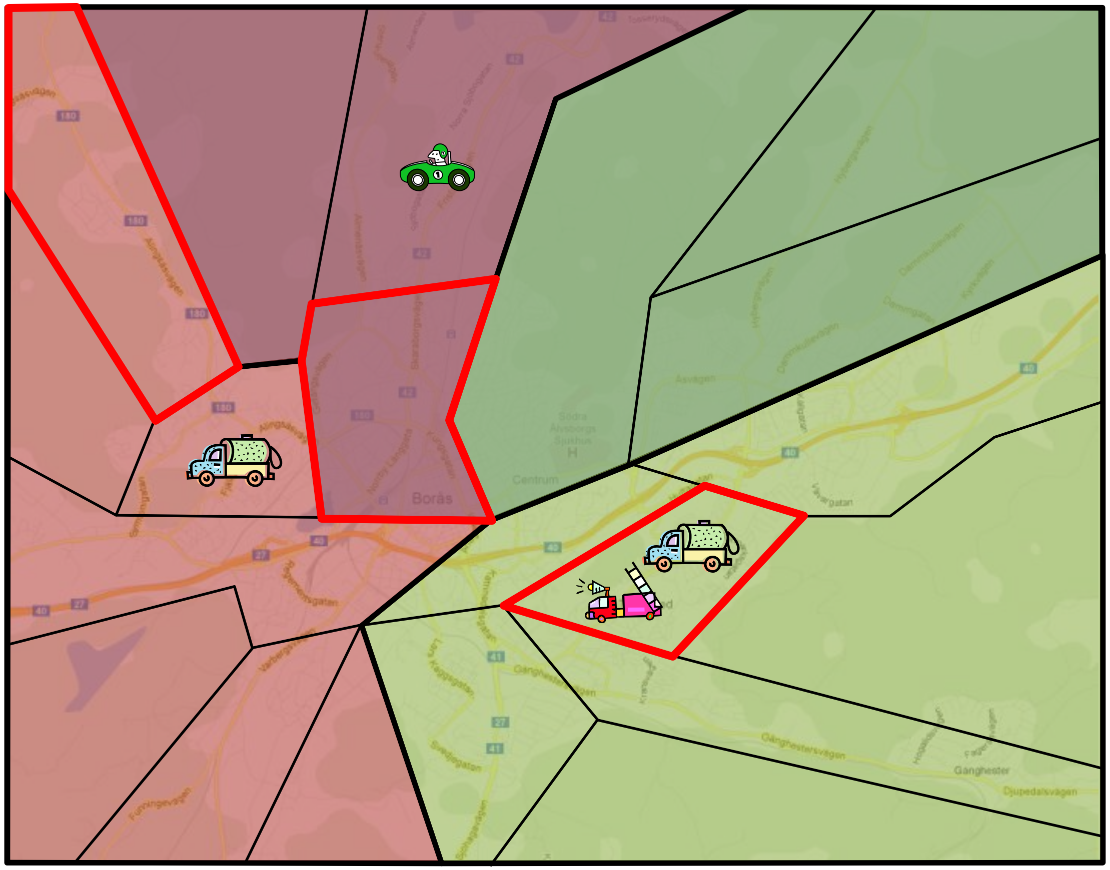
Solution structure



- Base unit
- Ladder unit
- Small unit

- Coverage
  - Base
  - Demand
  - Ladder

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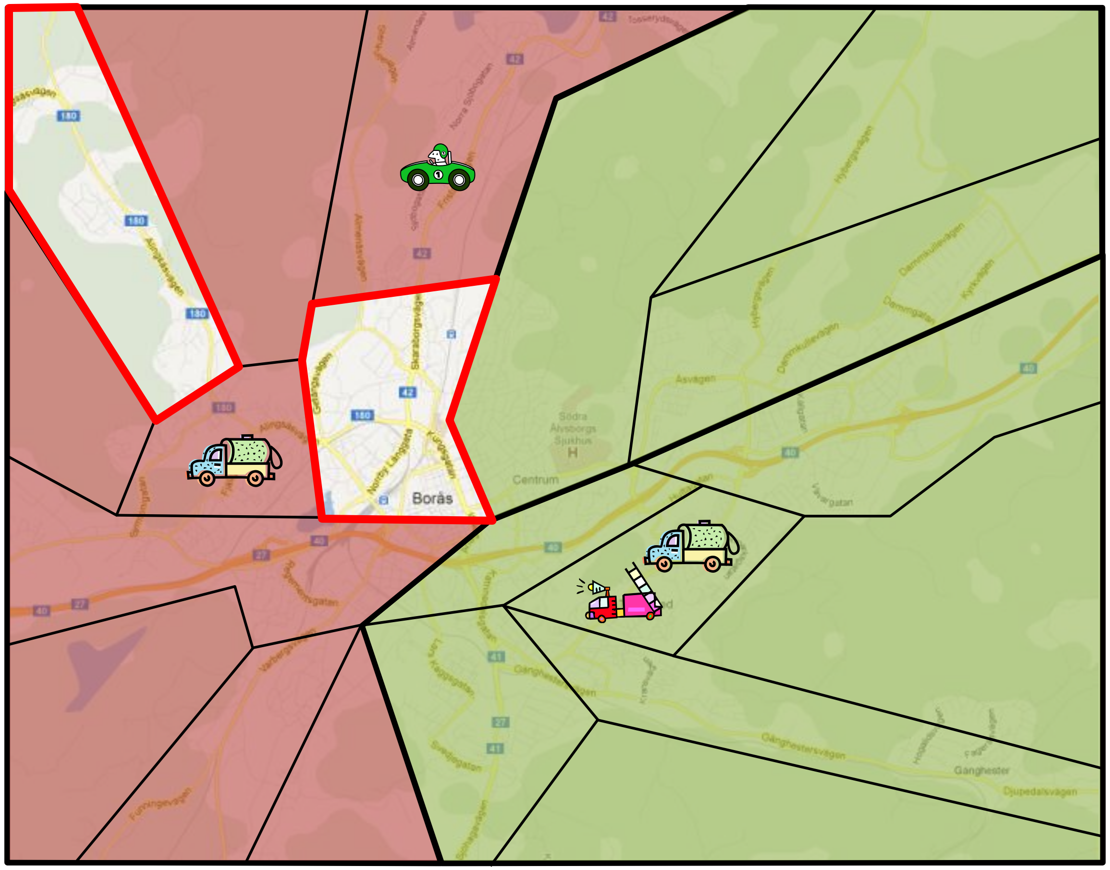
Solution structure



- Base unit
- Ladder unit
- Small unit

- Full response

New variable

- Define a variable to which we can assign a cost
- Create a structure that captures
  - Where the vehicle is located
  - Which zones the vehicle covers
  - How these zones are covered
- We call this a cover

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Cover

E_v Covers for vehicle v

B_{vei} 1 if vehicle v is located in zone i in cover e
G_{veoi} 1 if vehicle v responds to an accident of type o in zone i in cover e
L_{veoi} 1 if vehicle v is the last response to an accident of type o in zone i in cover e
F_{veoi} 1 if vehicle v is the first response to an accident of type o in zone i in cover e
C_{ve} The cost of cover e for vehicle v

w_{ve} 1 if vehicle v is assigned cover e

$$C_{ve} = \sum_{o \in O} \sum_{i \in I} D_{oi} \cdot \left(\sum_{j \in I} T_{(i|B_{vei}=1),j} + A_v \right) \cdot \left(W \cdot F_{veoi} + (1-W) \cdot L_{veoi} \right) \quad v \in V, e \in E_v$$

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New formulation

$$\min \sum_{v \in V} \sum_{e \in E_v} C_{ve} w_{ve}$$

$\sum_{e \in E_v} w_{ve} = 1$	$v \in V$	Each vehicle must be assigned one cover
$\sum_{v \in V} \sum_{e \in E_v} F_{veoi} w_{ve} \geq 1$	$o \in O, i \in I$	Each accident must have a first response
$\sum_{v \in V} \sum_{e \in E_v} L_{veoi} w_{ve} \geq 1$	$o \in O, i \in I$	Each accident must have a last response
$\sum_{v \in V^B} \sum_{e \in E_v} G_{veoi} w_{ve} \geq 1$	$o \in O, i \in I$	A base unit is needed for each accident
$\sum_{v \in V^L} G_{veoi} w_{ve} \geq 1$	$o \in O^F, i \in I^H$	A ladder unit is needed for fires in high buildings
$\sum_{v \in V} P_v G_{veoi} w_{ve} \geq H_{oi}$	$o \in O, i \in I$	Number of people needed for each accident
$\sum_{v \in V^L} \sum_{e \in E_v} B_{vei} w_{ve} - \sum_{v \in V^B} \sum_{e \in E_v} B_{vei} w_{ve} \leq 0$	$i \in I$	A ladder unit must be located in tandem with a base unit
$\sum_{f \in V} \sum_{e \in E_f} \sum_{i \in I} (T_{ij} + A_f) \cdot B_{fei} \cdot L_{feoj} w_{fe} \geq \sum_{e \in E_v} \sum_{i \in I} (T_{ij} + A_v) \cdot B_{vei} \cdot G_{veoj} w_{ve}$	$v \in V, o \in O, j \in I$	The last response must be the last
$w_{ve} \in \{0,1\}$	$v \in V, e \in E_v$	

Comments

- No complicating time constraints
- A pure binary problem
- Too many feasible covers

Solution approach

- Solve the LP relaxation with a small number of covers
  - This is the restricted master problem (RMP)
- Price the covers and add promising candidates to the RMP
- Embed this in a branch-and-bound framework to get integer feasibility

The pricing problem

- The pricing problem decomposes into one problem for
  - each vehicle and
  - each accident type and
  - each pair of zones

- One zone is the possible location of the vehicle
- The other is the zone to be covered

Pricing

- Contributions to the reduced cost
  - Location [Assignment ; Ladder/Base]
  - Cover [Alarm plan ; Last being last]
  - First [Real cost ; First]
  - Last [Real cost ; Last ; Last being last]

- Restrictions
  - Cannot be first or last and not cover
  - Cannot be both first and last if too small capacity

Reduced cost

- The reduced cost of locating a vehicle in a given zone is the sum of the contributions from all subproblems where the vehicle is located in that zone
- If a vehicle/zone combination has negative reduced cost a cover can be created based on the information from each subproblem

Branching

- Branching is needed to guarantee integer feasible solutions

- The branching strategy is vehicle/zone
  - If a given vehicle is located in a given zone or not
  - Start with the base units
  - Corresponds to the original variable x_{vi}
  - Easy to handle in the pricing problem

Evaluating a location

- Branching on vehicle/zone is not enough
  - Different covers can be used
  - The original variables y_{voi} and z_{voi} can be fractional
  - But we have a fixed location

- Solve the original model to evaluate the location
  - Forbid the solution by branching $x_{vi} = 1$ to 0

Computational study

- A straight forward implementation in Mosel/Xpress with little engineering

- One case
  - 300 zones, 17 with high buildings
  - 3 accident types
  - 37 vehicles; 22 base , 6 ladder , 9 small

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Instances

- Extremely hard to solve
  - Not even close to solve the original case

- Smaller instances where created

	# Zones	# High	# Base	# Ladder	# Small
I15	15	3	2	2	2
I30	30	4	3	2	2
I45	45	6	4	2	3
I60	60	7	4	2	3

  - The small instances are extended to larger

Results

	Root node			After 4 hours						
	LP	IP	# Iter	Bound	IP	# Nodes	Bound	IP	# Nodes	Time (s)
I15	46952.3	54359.1	163	50844.5	50844.5	198	–	–	–	–
I30	82663	105712	608	84415.4	93426.4	114	86682.9	92176.9	363	10^6
I45	118767	162192	1200	118835	162192	17	118937	162192	69	10^6
I60	198167	261933	3493	–	–	–	–	–	–	–

- I15 : Solved to optimality
  - Total time: 915.92 Master: 884.93 Sub: 11.48
- I30 : After 4 hours
  - Total time: 14412.98 Master: 14265.01 Sub: 104.74
- I45 : After 4 hours
  - Total time: 15733.07 Master: 15449.40 Sub: 259.61
- I60 : Root node solved after 283729 seconds

Future work

- Reduce the time spent in the master problem
  - Dual stabilization
  - More careful cover generation
- Strengthen the formulation
  - Valid inequalities from the demand constraints
  - Symmetry breaking
- Branching
  - More balanced branching

Thank you all for listening

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