Branch-and-Price for Vehicle Routing Problems with Multiple Synchronization Constraints

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Objectives of talk:

- Introduce 'Vehicle routing problems with multiple synchronization constraints'
- Point out difficulties of a branch-and-price approach
- Present approaches for overcoming these difficulties

Three abbreviations:

VRP(TW):
Vehicle routing problem (with time windows)

VRPMS:
VRP with multiple synchronization constraints

VRPTT:
VRP with trailers and transshipments

Agenda

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The VRPTT

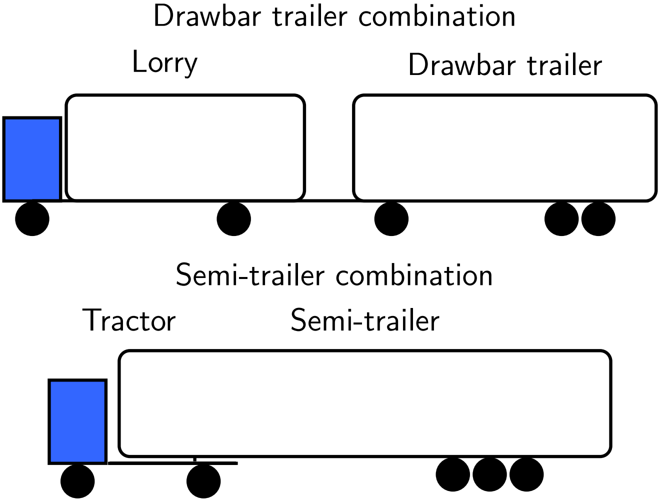
VRP with Trailers and Transshipments (VRPTT):

Two extensions to basic VRP:

1. Heterogeneous locations: Four different types of location
2. Heterogeneous fleet: Four different types of vehicle

The VRPTT

VRPTT: Relevant types of vehicle



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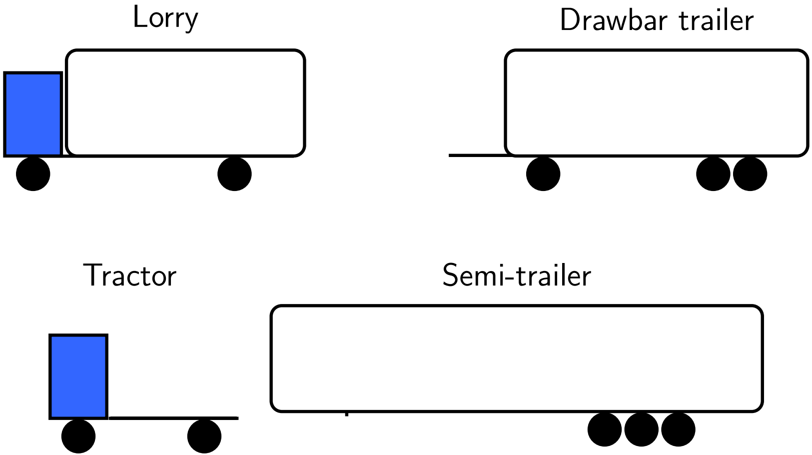


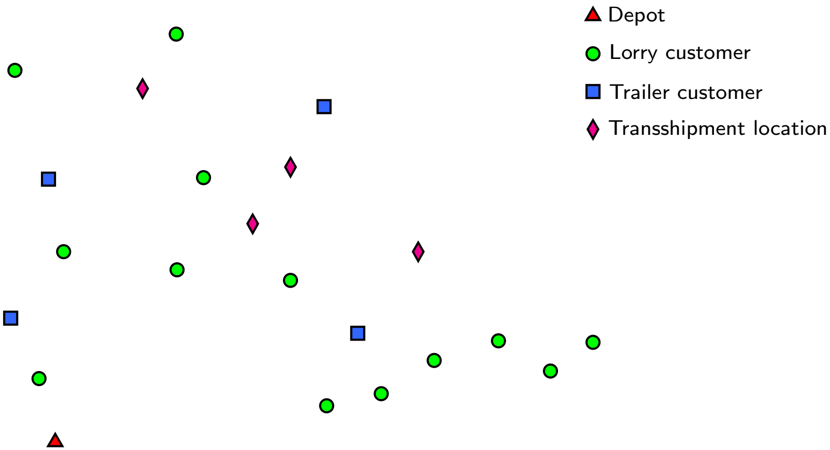
The VRPTT

VRPTT: Locations

- Depot
- Lorry customer
- Trailer customer
- Transshipment location

The VRPTT

VRPTT: Locations and example route plan



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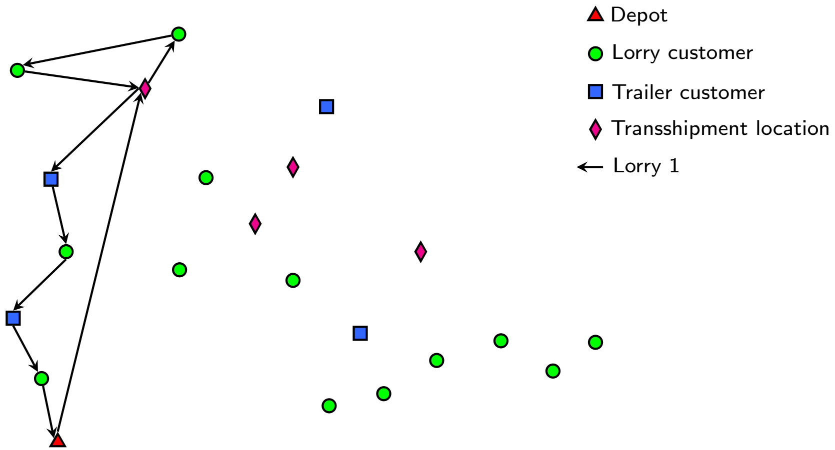

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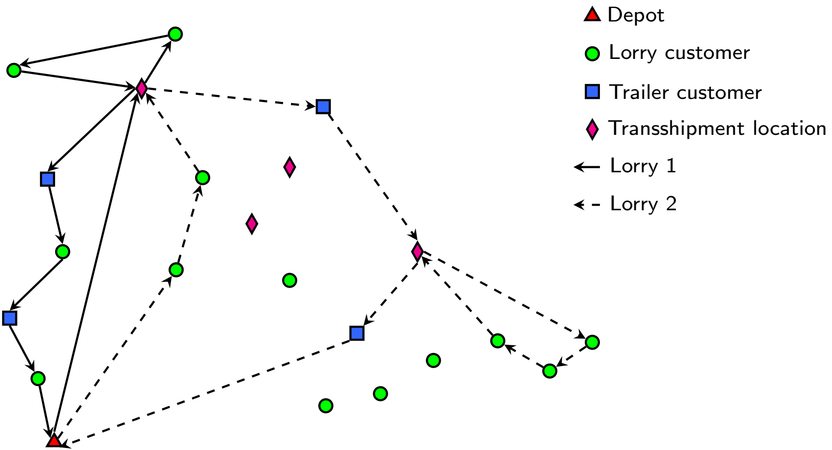

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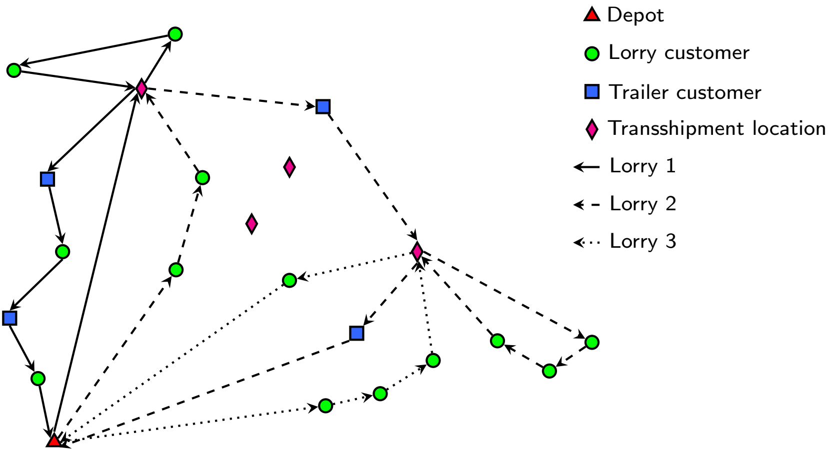

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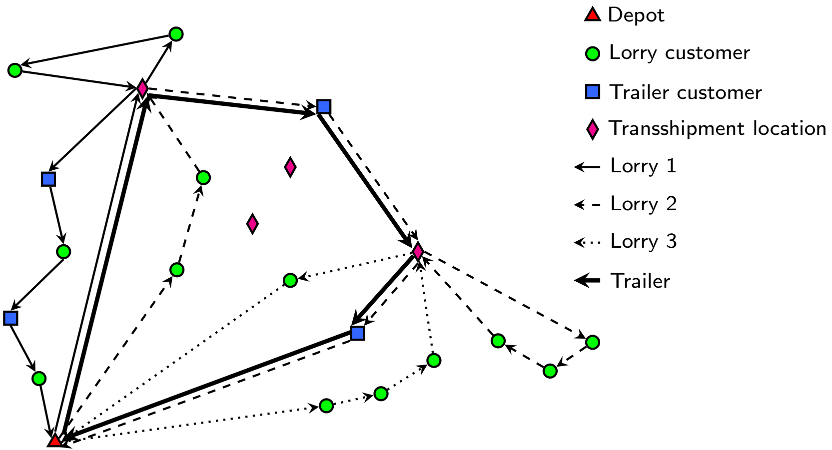

The VRPTT

VRP with Trailers and Transshipments (VRPTT):

Crux of problem:

Close interdependency between vehicles!

Requires five-fold synchronization of vehicles:

1. Task synchronization (customer covering)
2. Operation synchronization (time and locations of transshipments)
3. Movement synchronization (of lorry pulling a trailer)
4. Load synchronization (quantity transshipped)
   Note: Duration of transfer depends on quantity transshipped
5. Resource synchronization (use of transshipment locations)

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Examples of VRPMSs

Real-world applications:

- Raw milk collection
- Food distribution to supermarkets
- Fuel oil delivery to private households
- Garbage collection
- City logistics
- Bitumen and concrete delivery
- Forest management
- Mid-air refuelling of aircraft
- Letter mail or parcel delivery
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VRPMSs

Generic characteristics:

- More than one (type of) vehicle may or must be used to fulfil tasks
- Possibility or requirement of transshipments
- Collection and/or transshipment quantities not fixed
- Common scarce resources

Important problem classes:

- Simultaneous vehicle and crew routing and scheduling
- Pickup-and-delivery with transshipments
- Single- and multi-echelon location-routing

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Decomposition Approach

Basic decomposition approach for *VRPTW:*

Master problem:

Coupling constraints:

- Customer covering synchronization

One pricing problem:

Non-coupling constraints (individual routes/vehicles):

- Flow conservation
- Vehicle capacity
- Static time windows

Decomposition Approach

Basic decomposition approach for *VRPTT:*

Master problem (coupling constraints):

- Customer covering synchronization
- Operation synchronization
- Movement synchronization
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- Resource synchronization

Several pricing problems (individual routes/vehicles):

- Flow conversation
- Vehicle capacity
- Static time windows

Pricing Problem

Pricing problem:

- (Elementary) shortest path problem with resource constraints ((E)SPPRC)
- Traditionally solved by dynamic-programming based labelling algorithm
- Uses resources and resource extension functions (REFs)

Resources and REFs in VRPTW pricing problem:

- Cost c_i: $f_{ij}^{cost}(c_i) = c_i + \tilde{c}_{ij}$
- Time t_i: $f_{ij}^{time}(t_i) = \max(a_j, t_i + t_{ij}^{travel})$
- Load l_i: $f_{ij}^{load}(l_i) = l_i + s_i$

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Pricing Problem

Two desirable properties of REFs (Desaulniers et al. 1998):

1. *All REFs for an arc (i, j) should depend only on the resource vector at i.*
   → Intermediate resource values can be computed;
   yield lower bounds for values of resource variables
2. *All REFs should be non-decreasing.*
   → Lowest cost at j always obtained for smallest possible resource values

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Pricing Problem Issues

REFs in VRPTT pricing problems:

- Load and time influenced by other vehicles
  $\rightarrow$ REFs not only dependent on resource vector at i
- Two trade-offs:
  - Load: gain capacity or save time
  - Time: provide capacity early or avoid binding lorries

  $\rightarrow$ REFs not non-decreasing

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Approaches for Solving the Pricing Problems

Approaches for dealing with synchronization requirements:

- *Solution by dynamic programming/labelling:*
  - Discretization (Desrosiers 2005)
  - Branching (Dohn, Rasmussen, and Larsen 2011)
  - Non-pairwise dominance between sets of functions (Ioachim, Gélinas, Soumis, and Desrosiers 1998)
  - Point-in-polyhedron tests used in computational geometry (O'Rourke 1998)
- *Solution as MIP:*
  - Branch-and-cut (Jepsen, Petersen, Spoorendonk, and Pisinger 2011)
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DP Approaches for Solving the Pricing Problems

Discretization (Desrosiers 2005):

- *Space-time-vehicle-load network*
- One vertex for each combination of
  - Location
  - Point in time
  - Passive vehicle
  - Load transfer quantity
- Allows using standard labelling algorithms
- Trade-off between granularity of discretization and network size
- Important special cases: discrete load quantities by nature (swap-body platforms, garages)
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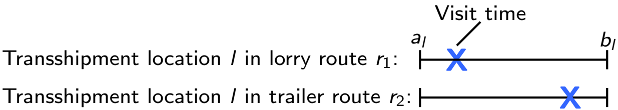

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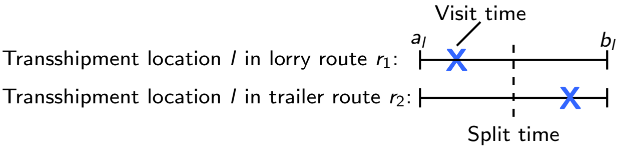

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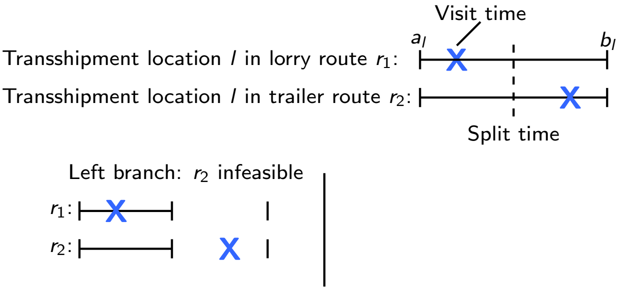

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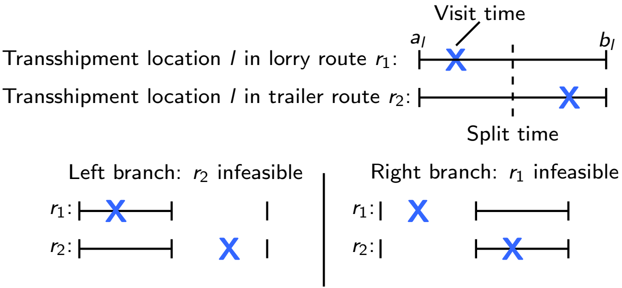

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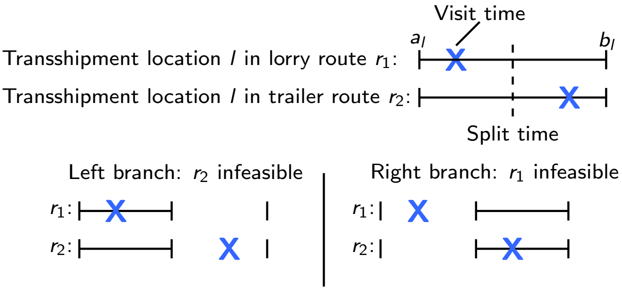

- VRPTT requires branching on both time and load

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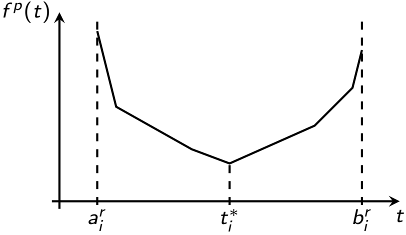
Non-pairwise dominance between sets of functions (Ioachim, Gélinas, Soumis, and Desrosiers 1998):

- Pricing problem with *linear time costs at vertices*



- Vertex cost function $f^p(t)$ for given path p:
  - Piecewise linear
  - Convex
  - Finite number of linear pieces
  - Increasing pieces can be ignored

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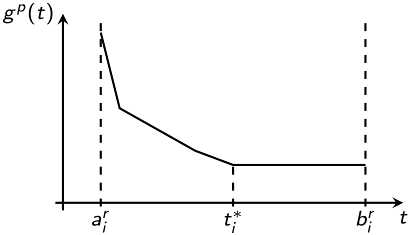

- Modified vertex cost function $g^p(t)$ for given path p:
  - Piecewise linear
  - Convex
  - Nonincreasing
  - At most as many linear pieces as there are vertices in p

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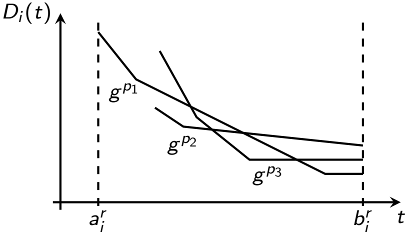

- Dominance function $D_i(t)$:
  - Piecewise linear
  - Nonincreasing
  - Not necessarily convex or continuous
- Labels: Breakpoints of $D_i(t)$: $(t_i^k, D_i(t_i^k), s_i^k)$

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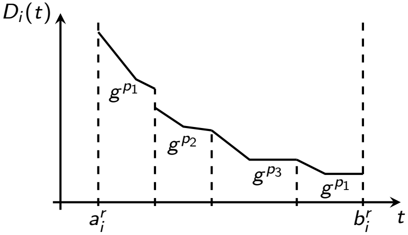

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DP Approaches for Solving the Pricing Problems

Point-in-polyhedron tests used in computational geometry (O'Rourke 1998):

- Extension of algorithm of Ioachim et al. to higher dimensions
- More difficult for non-convex point sets
- Fast and numerically robust implementation appears non-trivial (K. Mehlhorn, 2006)

MIP Approaches for Solving the Pricing Problems

Branch-and-cut (Jepsen, Petersen, Spoorendonk, and Pisinger 2011):

- Promising results for ESPP with a capacity constraint
- Ten types of valid inequalities used
- Behaviour for VRPTT pricing problem unclear

MIP Approaches for Solving the Pricing Problems

Branch-and-price (Hennig, Nygreen, and Lübbecke 2010):

- *'Nested column generation for the crude oil tanker routing and scheduling problem with split pickup and split delivery'*
- Issue of loading/unloading quantities similar to VRPTT
- Two-level approach:

Level 1 master problem:
Binary ship routing and continuous cargo pattern variables
↓
Level 2 master problem (= level 1 pricing problem):
Receives dual information on supply constraints,
returns ship routing variables
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Level 2 pricing problem (ESPP with time windows):
Receives dual information on load and time restrictions along arcs, generates ship routing variables

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Outlook

Open questions:

- How do presented approaches compare with each other?
- Which one(s) is/are best
  - for which problem types?
  - for which data?
- What about assumption of one pricing problem per vehicle class?
- Which other (CG-based) solution approaches are there?

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