Branch-and-Price for Vehicle Routing Problems with Multiple Synchronization Constraints

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Objectives of talk:

- Introduce ‘Vehicle routing problems with multiple synchronization constraints’
- Point out difficulties of a branch-and-price approach
- Present approaches for overcoming these difficulties
Three abbreviations:

**VRP(TW):**
Vehicle routing problem (with time windows)

**VRPMS:**
VRP with multiple synchronization constraints

**VRPTT:**
VRP with trailers and transshipments
Agenda

1 The VRPTT

2 VRPMSs

3 Solving VRPMSs by Branch-and-Price

4 Summary and Outlook
Agenda

1. The VRPTT
2. VRPMSs
3. Solving VRPMSs by Branch-and-Price
4. Summary and Outlook
VRP with Trailers and Transshipments (VRPTT):

Two extensions to basic VRP:

1. Heterogeneous locations: Four different types of location
2. Heterogeneous fleet: Four different types of vehicle
**VRPTT:** Relevant types of vehicle

- **Drawbar trailer combination**
  - Lorry
  - Drawbar trailer

- **Semi-trailer combination**
  - Tractor
  - Semi-trailer
VRPTT: Relevant types of vehicle

- Lorry
- Drawbar trailer
- Tractor
- Semi-trailer
The VRPTT

**VRPTT**: Locations

- ▲ Depot
- ● Lorry customer
- □ Trailer customer
- ♣ Transshipment location
The VRPTT

**VRPTT**: Locations and example route plan

- **Depot**
- **Lorry customer**
- **Trailer customer**
- **Transshipment location**
VRPTT: Locations and example route plan
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- Depot
- Lorry customer
- Trailer customer
- Transshipment location
- Lorry 1
- Lorry 2
**The VRPTT**

**VRPTT:** Locations and example route plan

- **Depot**
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- **Lorry 3**
VRPTT: Locations and example route plan

- Depot
- Lorry customer
- Trailer customer
- Transshipment location
- Lorry 1
- Lorry 2
- Lorry 3
- Trailer
The VRPTT

VRP with Trailers and Transshipments (VRPTT):

Crux of problem:

Close interdependency between vehicles!

Requires five-fold synchronization of vehicles:
1. Task synchronization (customer covering)
2. Operation synchronization (time and locations of transshipments)
3. Movement synchronization (of lorry pulling a trailer)
4. Load synchronization (quantity transshipped)
   Note: Duration of transfer depends on quantity transshipped
5. Resource synchronization (use of transshipment locations)
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Examples of VRPMSs

*Real-world applications:*

- Raw milk collection
- Food distribution to supermarkets
- Fuel oil delivery to private households
- Garbage collection
- City logistics
- Bitumen and concrete delivery
- Forest management
- Mid-air refuelling of aircraft
- Letter mail or parcel delivery
- Field service and homecare personnel dispatching
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Generic characteristics:

- More than one (type of) vehicle may or must be used to fulfil tasks
- Possibility or requirement of transshipments
- Collection and/or transshipment quantities not fixed
- Common scarce resources

Important problem classes:

- Simultaneous vehicle and crew routing and scheduling
- Pickup-and-delivery with transshipments
- Single- and multi-echelon location-routing
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Decomposition Approach

Basic decomposition approach for VRPTW:

*Master problem:*
Coupling constraints:
- Customer covering synchronization

*One pricing problem:*
Non-coupling constraints (individual routes/vehicles):
- Flow conservation
- Vehicle capacity
- Static time windows
Decomposition Approach

Basic decomposition approach for *VRPTT*:

*Master problem* (coupling constraints):
- Customer covering synchronization
- Operation synchronization
- Movement synchronization
- Load synchronization
- Resource synchronization

*Several pricing problems* (individual routes/vehicles):
- Flow conversation
- Vehicle capacity
- Static time windows
Pricing Problem

**Pricing problem:**

- (Elementary) shortest path problem with resource constraints ((E)SPPRC)
- Traditionally solved by dynamic-programming based labelling algorithm
- Uses resources and resource extension functions (REFs)

**Resources and REFs in VRPTW pricing problem:**

- Cost $c_i$: $f_{ij}^{cost}(c_i) = c_i + \tilde{c}_{ij}$
- Time $t_i$: $f_{ij}^{time}(t_i) = \max(a_j, t_i + t_{ij}^{travel})$
- Load $l_i$: $f_{ij}^{load}(l_i) = l_i + s_i$
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Pricing Problem

Two desirable properties of REFs (Desaulniers et al. 1998):

1. **All REFs for an arc \((i, j)\) should depend only on the resource vector at \(i\).**
   → Intermediate resource values can be computed; yield lower bounds for values of resource variables

2. **All REFs should be non-decreasing.**
   → Lowest cost at \(j\) always obtained for smallest possible resource values

REFs in VRPTW pricing problem possess both properties:

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Pricing Problem Issues

REFs in VRPTT pricing problems:
- Load and time influenced by other vehicles
  → REFs not only dependent on resource vector at \( i \)
- Two trade-offs:
  - Load: gain capacity or save time
  - Time: provide capacity early or avoid binding lorries
  → REFs not non-decreasing
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Essentially, the determination of a cost-optimal schedule and load plan for a fixed path becomes an optimization problem in itself.
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Approaches for Solving the Pricing Problems

Approaches for dealing with synchronization requirements:

■ **Solution by dynamic programming/labelling:**
  ■ Discretization (Desrosiers 2005)
  ■ Branching (Dohn, Rasmussen, and Larsen 2011)
  ■ Non-pairwise dominance between sets of functions (Ioachim, Gélinas, Soumis, and Desrosiers 1998)
  ■ Point-in-polyhedron tests used in computational geometry (O’Rourke 1998)

■ **Solution as MIP:**
  ■ Branch-and-cut (Jepsen, Petersen, Spoorendonk, and Pisinger 2011)
  ■ Branch-and-price (Hennig, Nygreen, and Lübbecke 2010)
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DP Approaches for Solving the Pricing Problems

Discretization (Desrosiers 2005):

- **Space-time-vehicle-load network**
- One vertex for each combination of
  - Location
  - Point in time
  - Passive vehicle
  - Load transfer quantity
- Allows using standard labelling algorithms
- Trade-off between granularity of discretization and network size
- Important special cases: discrete load quantities by nature (swap-body platforms, garages)
- Partial discretization possible (only load, only time)
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![Diagram showing visit times and splitting decision in lorry and trailer routes.](image_url)
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Transshipment location $l$ in lorry route $r_1$:

Transshipment location $l$ in trailer route $r_2$: 

Visit time

Split time
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![Diagram showing branching on resource variables with transshipment locations and routes](image)

Transshipment location $l$ in lorry route $r_1$: 

- Left branch: $r_2$ infeasible

Transshipment location $l$ in trailer route $r_2$:

- Right branch: $r_1$ infeasible
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VRPTT requires branching on both time and load
Non-pairwise dominance between sets of functions (Ioachim, Gélinas, Soumis, and Desrosiers 1998):

- Pricing problem with *linear time costs at vertices*

- Vertex cost function $f^p(t)$ for given path $p$:
  - Piecewise linear
  - Convex
  - Finite number of linear pieces
  - Increasing pieces can be ignored
DP Approaches for Solving the Pricing Problems

Non-pairwise dominance between sets of functions (Ioachim, Gélinas, Soumis, and Desrosiers 1998):

- Pricing problem with *linear time costs at vertices*

Modified vertex cost function $g^p(t)$ for given path $p$:

- Piecewise linear
- Convex
- Nonincreasing
- At most as many linear pieces as there are vertices in $p$
Non-pairwise dominance between sets of functions (Ioachim, Gélinas, Soumis, and Desrosiers 1998):

- **Non-pairwise dominance**:

\[
D_{i}(t) = \begin{cases} 
\text{piecewise linear} \\
\text{nonincreasing} \\
\text{not necessarily convex or continuous} \\
\text{labels: Breakpoints of } D_{i}(t): (t_{i}^{k}, D_{i}(t_{i}^{k}), s_{i}^{k})
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- Non-pairwise dominance:

\[ D_i(t) \]

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- Labels: Breakpoints of \( D_i(t) \): \((t_i^k, D_i(t_i^k), s_i^k)\)
Point-in-polyhedron tests used in computational geometry (O’Rourke 1998):

- Extension of algorithm of Ioachim et al. to higher dimensions
- More difficult for non-convex point sets
- Fast and numerically robust implementation appears non-trivial (K. Mehlhorn, 2006)
MIP Approaches for Solving the Pricing Problems

Branch-and-cut (Jepsen, Petersen, Spoorendonk, and Pisinger 2011):

- Promising results for ESPP with a capacity constraint
- Ten types of valid inequalities used
- Behaviour for VRPTT pricing problem unclear
MIP Approaches for Solving the Pricing Problems

Branch-and-price (Hennig, Nygreen, and Lübbecke 2010):

- ‘Nested column generation for the crude oil tanker routing and scheduling problem with split pickup and split delivery’
- Issue of loading/unloading quantities similar to VRPTT
- Two-level approach:

  \[\begin{align*}
  \text{Level 1 master problem:} \\
  \text{Binary ship routing and continuous cargo pattern variables} \\
  \downarrow \\
  \text{Level 2 master problem (= level 1 pricing problem):} \\
  \text{Receives dual information on supply constraints, returns ship routing variables} \\
  \downarrow \\
  \text{Level 2 pricing problem (ESPP with time windows):} \\
  \text{Receives dual information on load and time restrictions along arcs, generates ship routing variables}
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Outlook

*Open questions:*

- How do presented approaches compare with each other?
- Which one(s) is/are best
  - for which problem types?
  - for which data?
- What about assumption of one pricing problem per vehicle class?
- Which other (CG-based) solution approaches are there?
Branch-and-Price for Vehicle Routing Problems with Multiple Synchronization Constraints

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