LOCATION AND ROUTING PROBLEMS: A UNIFIED APPROACH

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Introduction: Location and Routing Problems

Optimization Problems involving

Location aspects, like in P-Median Problems.

$+$

Routing features, like in Vehicle Routing Problems.

Traditional view:

Location

- Strategic decisions.
- No modification.

Routing

- Tactical decisions.
- Subject to medium-term changes.

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Traditional view:

Location

- Strategic decisions.
- No modification.

Routing

- Tactical decisions.
- Subject to medium-term changes.

But sometimes ...

Introduction: Motivation

We propose a general framework:

- for location and routing problems
- handling both costs and profits
- modeling with extended formulations
- solving with branch-and-cut-and-price

Modeling Profitable Location and Distribution

No locations (pure routing)

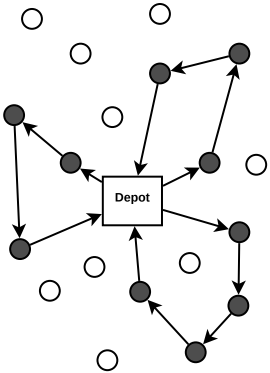
$$\max \sum_{r \in \Omega} p_r z_r$$

$$\text{s.t. } s_i + \sum_{r \in \Omega} a_{ir} z_r = 1 \ \forall i \in \mathcal{N}$$

$$\sum_{r \in \Omega} z_r \leq K$$

$$s_i \in \{0,1\} \quad \forall i \in \mathcal{N}$$

$$z_r \in \{0,1\} \quad \forall r \in \Omega$$



Modeling Profitable Location and Distribution

Passive locations (depots)

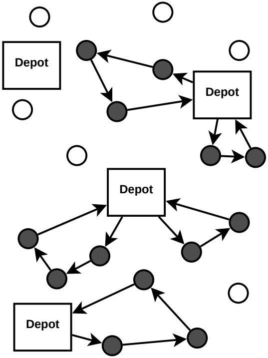
$$\max \sum_{l\in\mathcal{L}} \sum_{r\in\Omega_l} p_r z_r$$

$$\text{s.t.} \quad s_i + \sum_{l\in\mathcal{L}} \sum_{r\in\Omega_l} a_{ir} z_r = 1 \quad \forall i \in \mathcal{N}$$

$$\sum_{r\in\Omega_l} z_r \le K_l x_l \quad \forall l \in \mathcal{L}$$

$$\sum_{l\in\mathcal{L}} \sum_{r\in\Omega_l} z_r \le K$$

$$\sum_{l\in\mathcal{L}} x_l \le B$$



$$s_i \in \{0,1\} \quad \forall i \in \mathcal{N}$$

$$z_r \in \{0,1\} \quad \forall l \in \mathcal{L}, \forall r \in \Omega_l$$

$$x_l \in \{0,1\} \quad \forall l \in \mathcal{L}$$

Modeling Profitable Location and Distribution

Active locations (distribution centers)

$$\max \sum_{l\in\mathcal{L}} \sum_{r\in\Omega_l} p_r z_r + \sum_{l\in\mathcal{L}} \sum_{w\in\Theta_l} q_w y_w$$

$$\text{s.t.} \quad s_i + \sum_{l\in\mathcal{L}} \sum_{r\in\Omega_l} a_{ir} z_r + \sum_{l\in\mathcal{L}} \sum_{w\in\Theta_l} b_{iw} y_w = 1 \quad \forall i \in \mathcal{N}$$

$$\sum_{r\in\Omega_l} z_r + \sum_{w\in\Theta_l} K_l y_w \le K_l x_l \qquad \forall l \in \mathcal{L}$$

$$\sum_{l\in\mathcal{L}} \sum_{r\in\Omega_l} z_r \le K$$

$$\sum_{l\in\mathcal{L}} x_l \le B$$

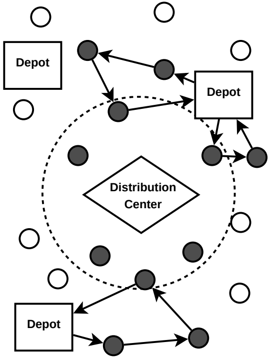
$$\sum_{l\in\mathcal{L}} \sum_{w\in\Theta_l} y_w \le M$$

$$s_i \in \{0,1\} \qquad \forall i \in \mathcal{N}$$

$$z_r \in \{0,1\} \qquad \forall l \in \mathcal{L}, \forall r \in \Omega_l$$

$$x_l \in \{0,1\} \qquad \forall l \in \mathcal{L}$$

$$y_w \in \{0,1\} \qquad \forall l \in \mathcal{L}, \forall w \in \Theta_l$$



Modeling Profitable Location and Distribution

Active locations (distribution centers)

$$\max \sum_{l\in\mathcal{L}} \sum_{r\in\Omega_l} p_r z_r + \sum_{l\in\mathcal{L}} \sum_{w\in\Theta_l} q_w y_w$$

$$\text{s.t. } s_i + \sum_{l\in\mathcal{L}} \sum_{r\in\Omega_l} a_{ir} z_r + \sum_{l\in\mathcal{L}} \sum_{w\in\Theta_l} b_{iw} y_w = 1 \ \forall i \in \mathcal{N} \qquad (\lambda)$$

$$\sum_{r\in\Omega_l} z_r + \sum_{w\in\Theta_l} K_l y_w \leq K_l x_l \qquad \forall l \in \mathcal{L} \qquad (\mu)$$

$$\sum_{l\in\mathcal{L}} \sum_{r\in\Omega_l} z_r \leq K \qquad (\eta)$$

$$\sum_{l\in\mathcal{L}} x_l \leq B$$

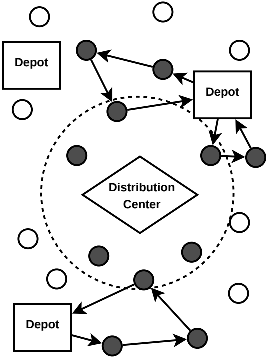
$$\sum_{l\in\mathcal{L}} \sum_{w\in\Theta_l} y_w \leq M \qquad (\tau)$$

$$0 \leq s_i \leq 1 \qquad \forall i \in \mathcal{N}$$

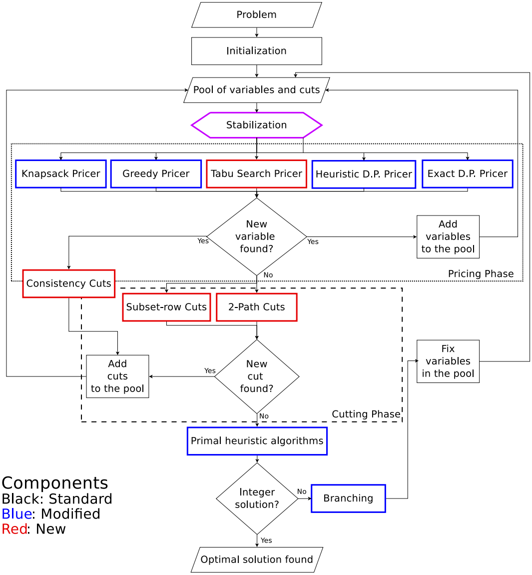
$$0 \leq z_r \leq 1 \qquad \forall l \in \mathcal{L}, \forall r \in \Omega_l$$

$$0 \leq x_l \leq 1 \qquad \forall l \in \mathcal{L}$$

$$0 \leq y_w \leq 1 \qquad \forall l \in \mathcal{L}, \forall w \in \Theta_l$$



Unified Approach: Branch-and-Cut-and-Price



Components
Black: Standard
Blue: Modified
Red: New

Pricing sub-problems - DCs

DCs pricing: find, for every potential location, the variable y_w of maximum reduced cost

$$rc_w = q_w - \sum_{i \in \mathcal{N}} b_{iw} \lambda_i - K_l \cdot \mu_l - \tau$$

such that:

- maximum distance from the depot;
- capacity constraint.

i.e. solve a 0-1 Knapsack Problem for each potential location.

Pricing sub-problems - routes

Route pricing: find, for every vehicle type and for every potential location, the variable z_r of maximum reduced cost

$$rc_r = p_r - \sum_{i \in \mathcal{N}} a_{ir} \lambda_i - \mu_l - \eta$$

such that:

- the route is elementary;
- the route starts from a depot;
- length constraints;
- capacity constraints.

i.e. solve a Resource Constrained Elementary Shortest Path Problem for each vehicle type and for each potential location.

Exact pricing algorithm

Dynamic programming algorithm: labels $(\mathcal{S}, g, h, v, i)$ encoding a path:

- of cost v,
- starting from the depot and ending in node i,
- visiting all sites in $\mathcal{S}$,
- using g time units,
- serving a total demand equal to h.

Implementation:

- $\mathcal{S}$ includes unreachable sites (Feillet et al., 2004),
- bi-directional DP,
- DSSR.

Subset-Row inequalities

Taken from Jepsen et al ('08):

- $\mathcal{C}$: set of all triplets of sites
- for each $C \in \mathcal{C}$:
    - $\mathcal{K}(C)$: set of routes including at least two of the sites in C
    - $\mathcal{W}(C)$: set of clusters including at least two of the sites in C

then:

$$\sum_{r \in \mathcal{K}(C)} z_r + \sum_{w \in \mathcal{W}(C)} y_w \leq 1 \qquad \forall C \in \mathcal{C}.$$

Adapting the pricing algorithms

New resources: $m_C \forall C \in \mathcal{C}$ whose inequality is in the master (ψ_C)

Extension:

- $m''_C = |C \cap \mathcal{S}''|$, for each $C \in \mathcal{C}$,
- $v'' = v' + c_{i'i''} + \frac{\lambda_{i'}}{2} + \frac{\lambda_{i''}}{2} + \sum_{C \in \bar{\mathcal{C}}} \psi_C$

where $\bar{\mathcal{C}} = \{C \in \mathcal{C} : m''_C = 2 \ \& \ m'_C = 1\}$.

Dominance:

$$v' + \sum_{C \in \tilde{\mathcal{C}}} \psi_C \leq v'',$$

for the DP pricer:

$$\tilde{\mathcal{C}} = \{C \in \mathcal{C} : C \setminus \mathcal{S}' \neq \emptyset \text{ and } m'_C = 1 \text{ and } m''_C \in \{0, 2\}\}$$

for the route pricer:

$$\begin{aligned} \tilde{\mathcal{C}} = \{C \in \mathcal{C} : & C \setminus \mathcal{S}' \neq \emptyset \text{ and} \\ & (\ (m'_C = 1 \text{ and } m''_C \in \{0, 2, 3\}) \text{ or} \\ & (m'_C = 0 \text{ and } m''_C \in \{2, 3\})\ ) \\ & \}. \end{aligned}$$

Adapting the pricing algorithms

condition	rationale
$\lvert C \cap \mathcal{S}' \rvert \geq 2$	v' already includes ψ_C
$\lvert C \cap \mathcal{S}' \rvert \leq \lvert C \cap \mathcal{S}'' \rvert \leq 1$	any extension of ℓ' triggering C triggers the same in ℓ''
$m''_C = 2$ and $m'_C = 0$ and $S'' = S'$	at most 1 element of C is in $C \setminus \mathcal{S}''$: if $S'' \subseteq S'$, C will never be triggered in ℓ'
$m''_C = 3$ and $m'_C \leq 1$ and $S'' = S'$	same rationale

2-path inequalities

Taken from Kohl et al ('99), but handled in *strong form*:

- $Q \subseteq \mathcal{N}$: any subset of clients that cannot be served by a single route or cluster,
- $\mathcal{K}(Q)$ and $\mathcal{W}(Q)$: respectively the set of routes and clusters including at least one site in Q

then

$$\sum_{i \in Q} s_i + \sum_{r \in \mathcal{K}(Q)} e_r^Q z_r + \sum_{w \in \mathcal{W}(Q)} f_w^Q y_w \geq 2 \qquad \forall Q \subseteq \mathcal{N}$$

where e_r^Q and f_w^Q take value

- two if the variables y_w or z_r encode respectively a cluster or a route containing all sites in Q,
- one if a cluster or route serves Q only partially,
- zero otherwise.

Separation heuristic similar to Kohl et al ('99), but in three steps: identification, filtering, coefficients setting.

Adapting the pricing algorithm

New resources: o_Q (initially = 0), one for each Q corresponding to an inequality in the master (γ_Q)

Extension:

- set $o_Q = 1$ as soon as a site in Q is visited, and subtract γ_Q in the partial cost
- set $o_Q = 2$ as soon as all sites in Q are visited, and subtract again γ_Q from the partial cost

Dominance:

$$v' + \sum_{C \in \tilde{\mathcal{C}}} \psi_C \leq v'' - \sum_{Q \in \tilde{\mathcal{Q}}'} \gamma_Q - \sum_{Q \in \tilde{\mathcal{Q}}''} \gamma_Q, .$$

where

$$\tilde{\mathcal{Q}}' = \{Q \in \mathcal{Q} : Q \setminus \mathcal{S}'' \neq \emptyset \text{ and } o''_Q = 0 \text{ and } o'_Q > 0\}$$

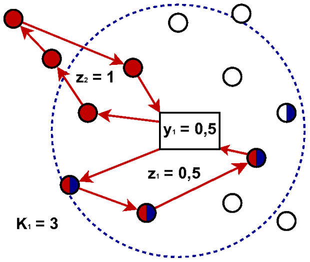
and

$$\tilde{\mathcal{Q}}'' = \{Q \in \mathcal{Q} : o''_Q < 2 \text{ and } |Q \cap \mathcal{S}''| = o''_Q \text{ and } |Q \cap \mathcal{S}'| > o'_Q\}.$$

Adapting the pricing algorithm

- ℓ'' may include for the first time an element of Q, while ℓ' cannot; this can happen only if (a) elements of Q can still be included in ℓ'' (b) no element of Q was already included in ℓ'' (c) at least one element of Q was already included in ℓ' (set $\tilde{\mathcal{Q}}'$);
- ℓ'' may include all the elements of Q, while ℓ' cannot; this can happen only if all the elements of $\mathcal{S}''$ belonging to Q were included in ℓ'', while some of the elements of $\mathcal{S}'$ belonging to Q were not included in ℓ'. (set $\tilde{\mathcal{Q}}''$);

Consistency Cuts



The Issue

Many fractional solutions arise since it is often convenient to partially use a location as both DC and Depot.

$$\sum_{r \in \Omega_l} z_r + \sum_{w \in \Theta_l} K_l y_w \leq K_l$$

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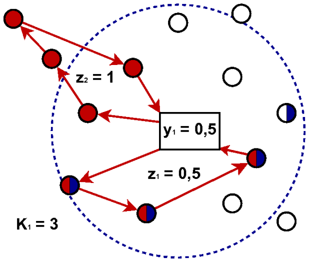


$$\sum_{r\in\Omega_l} z_r + \sum_{w\in\Theta_l} K_l y_w \leq K_l$$

$$z_r + \sum_{w\in\Theta_l} y_w \leq 1 \quad \forall l \in \mathcal{L}, \forall r \in \Omega_l \quad (\pi_r).$$

Simultaneous Columns and Rows Generation If there is a route starting from a location all DCs variables associated with the same location must be equal to 0. Every time a route column is generated its corresponding consistency cut is inserted in the MP.

Adapting the pricing algorithms

Change in the objective function of the pricing subproblem.

$$rc_r = p_r - \sum_{i \in \mathcal{N}} a_{ir}\lambda_i - \mu_l - \eta - \pi_r$$

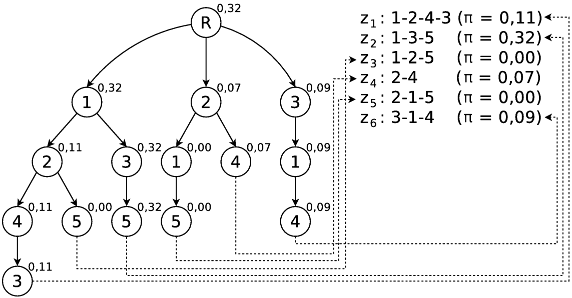
π_r is required during the generation of r but can be exactly determined only after the route r is generated!
$\rightarrow$ custom data-structure for quick estimation (UB).

Init : $O(1)$.

Insert : $O(|\mathcal{N}| \log |\mathcal{N}|)$.

Update : $O(Cols \times |\mathcal{N}|)$.

Find : $O(|r| \log |\mathcal{N}|)$.



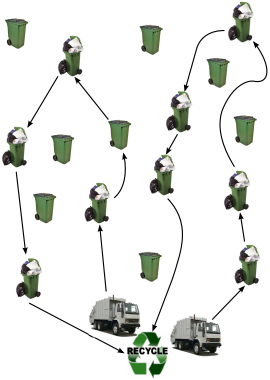
price-and-cut loop

- Subset-row and 2-path inequalities are separated when no useful column is found during pricing
- Consistency Cuts are inserted as soon as the corresponding route is generated.

Waste Collection

VRP with Different Service Constraints

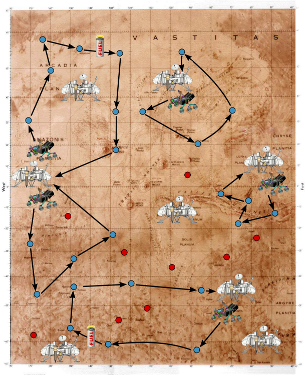
- Proposed by Macedo et al ('09).
- CVRP with additional constraints.
- Optional and Mandatory Clients; all Full Bins are to be imperatively collected.
- Minimum Filling Constraints.
- Minimum Daily Collection Requirements.



Planetary Surface Exploration

Generalized Location Routing Problem with Profits:

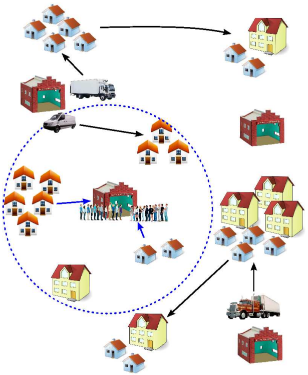
- Proposed and named by Ahn et al. ('08).
- Local and global resources to be taken into account in building routes.
- Strategies are associated with the bases used.
- Tactics influence the limitations on the routes.
- Budget constraints.



Drugs Distribution in Case of Emergency

The Location and Distribution Problem is a new problem inspired by Ordonez et al ('09):

- Mixed distribution system: routes and distribution centers.
- Multi-depot multi-vehicle VRP with profits as subproblem.
- Time-limit.



Choosing components

Components	VRPDSC	GLRPP	GLDP
Greedy Pricer	X	X	X
Heur DP Pricer	X	X	X
Tabu Pricer		X	
Exact DP Pricer	X	X	X
Knapsack Pricer			X
Subset-row Cuts	X	X	X
2-path Cuts	X		X
Consistency Cuts		X	X
Stabilization	X		X
Primal Heur.	X	X	X

Overview of Computational Results

Waste Collection:

- Optimally solved 56 over 75 instances (53 within 1000 seconds);
- average residual gap $\simeq$ 1.3%;
- Macedo et al. ('09) provided optimal results for only 27 instances.

Planetary Surface Exploration:

- 146 over 216 instances solved to optimality;
- average gap in the other instances $\simeq$ 3.5%;
- Ahn et al. ('08) do not provide an exact algorithm.

Drugs Distribution:

- Proven optimal solutions in 434 over 440 instances;
- large residual gap ($\geq$ 10%) in 3 instances;
- running time comparable to the tabu search of Ordoñez (but a priori optimality guarantee)

MANY THANKS FOR YOUR ATTENTION!! :)