The integrated airline recovery problem on a minimal disruption neighbourhood

Stephen J Maher

School of Mathematics and Statistics
University of New South Wales
Sydney, Australia

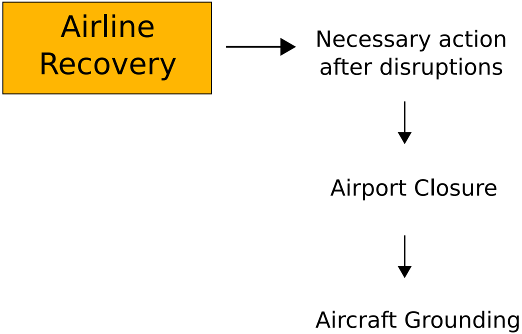
11th June 2012

Outline

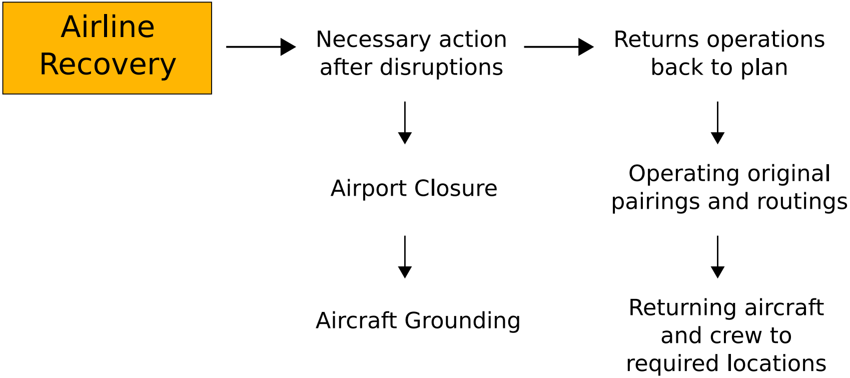
Airline Recovery

Airline Recovery

Airline Recovery



Airline Recovery



Airline Recovery

- Large problem with many stages.
- Needs to be solvable in real time.
- Interaction between each stage in the problem.

Airline Recovery

Airline Recovery Process

Schedule	→	Flight cancellation and delays
Aircraft	→	Rerouting and delay decisions
Crew	→	Pairing repair and deadhead
Passengers	→	Reallocation or lost passengers

Airline Recovery

Airline Recovery Process

Schedule	→	Flight cancellation and delays
Aircraft	→	Rerouting and delay decisions
Crew	→	Pairing repair and deadhead
Passengers	→	Reallocation or lost passengers

Previous Approaches

Problem Size

- Time-band network - Bard *et. al.* (2001), Eggenberg *et. al.* (2007).
- Preprocessing selection of resources, both crew and aircraft - Abdelghany *et. al.* (2004, 2008), Petersen *et. al.* (2010).
- Fixed flight schedules - Wei *et. al.* (1997), Stojković *et. al.* (1998), Medhard and Sawhney (2007).

Previous Approaches

Complexity

- Selection of recovery policies - Johnson *et. al.* (1994), Stojković and Soumis (2001 and 2005), Abdelghany *et. al.* (2004).
- Decomposition techniques - Lettosvky (1997), Petersen *et. al.* (2010), Abdelghany *et. al.* (2004, 2008)

Outline

Integrated Airline Recovery

Considerations

- Time allowed to return back to schedule.
- Size of the recovery network - restricted by recovery window.
- Equipment included in the recovery problem - using all available crew and aircraft.

Integrated Airline Recovery

- Integrating crew, aircraft and passenger reallocation.
- Recovery policies of cancellation and delays.
- Crew - rescheduling, reserve crew and deadhead.
- Aircraft - rerouting.
- Integration through short and restricted connections, and delay consistency.

Crew Recovery

$$\text{Min} \quad \sum_{k \in K} \sum_{p \in P^k} c_p^k x_p^k + \sum_{k \in K} g^{DH} \nu_k + \sum_{r \in R} \sum_{p \in P^r} c_p^r y_p^r + \sum_{j \in N^D} \sum_{p \in P^j} d_{jp} z_{jp},$$

$$\sum_{k \in K} \sum_{p \in P^k} a_{jp}^k x_p^k + \sum_{p \in P^j} z_{jp} = 1 \quad \forall j \in N^D,$$

$$\sum_{k \in K} \sum_{p \in P^k} a_{jp}^k x_p^k = 1 \quad \forall j \in N_{out}^K,$$

$$\sum_{p \in P^k} x_p^k + \nu_k = 1 \quad \forall k \in K,$$

$$x_p^k \in \{0, 1\} \; \forall k \in K, \forall p \in P^k, \quad \nu_k \in \{0, 1\} \; \forall k \in K.$$

Aircraft and Passengers

$$\sum_{r \in R} \sum_{p \in P^r} a^r_{jp} y^r_p + \sum_{p \in P^j} z_{jp} = 1 \quad \forall j \in N^D,$$

$$\sum_{r \in R} \sum_{p \in P^r} a^r_{jp} y^r_p = 1 \quad \forall j \in N^R_{out},$$

$$\sum_{r \in R} o^r_{tp} y^r_p \geq M^D_t \quad \forall t \in T,$$

$$\sum_{r \in R} y^r_p \leq 1 \quad \forall r \in R,$$

$$\sum_{p \in P^j} z_{jp} \leq 1 \quad \forall j \in N^D,$$

$$y^r_p \in \{0,1\} \; \forall r \in R, \forall p \in P^r, \quad z_{jp} \in \{0,1\} \; \forall j \in N^D, \forall p \in P^j.$$

Integration Constraints

$$\sum_{k \in K} \sum_{p \in P^k} b^k_{ijp} x^k_p - \sum_{r \in R} \sum_{p \in P^r} b^r_{ijp} y^r_p \leq 0 \quad \forall (i,j) \in E^D,$$

$$\sum_{k \in K} \sum_{p \in P^k} a^{ku}_{jp} x^k_p - \sum_{r \in R} \sum_{p \in P^r} a^{ru}_{jp} y^r_p = 0 \quad \forall j \in N^D, \forall u \in U_j,$$

$$\sum_{k \in K} \sum_{p \in P^k} a^{ku}_{jp} (Maxcap - Pax(j)) x^k_p - \sum_{i \in N^j} \sum_{p \in P^i} h^u_{ijp} z_{ip} \geq 0 \quad \forall j \in N^D, \forall u \in U_j,$$

Column Generation - Crew and Aircraft

- Solved using a multi-label shortest path algorithm.
- Delays are incorporated in the subproblem through the use of flight copies.

$$\hat{c}_p^k = c_p^k - \sum_{j \in N^D \cup N_{out}^K} a_{jp}^k \alpha_j^K - \sum_{(i,j) \in E^D} b_{ijp}^k \rho_{ij} - \sum_{j \in N^D} \sum_{u \in U_j} a_{jp}^{ku} \gamma_j^u$$
$$- \sum_{j \in N^D} \sum_{u \in U_j} a_{jp}^{ku} (Maxcap - Pax(j)) \lambda_j^u - \delta^k,$$
$$\hat{c}_p^r = c_p^r - \sum_{j \in N^D \cup N_{out}^R} a_{jp}^r \alpha_j^R - \sum_{t \in T} o_{tp}^r \epsilon_t + \sum_{(i,j) \in E^D} b_{ijp}^r \rho_{ij} + \sum_{j \in N^D} \sum_{u \in U_j} a_{jp}^{ru} \gamma_j^u - \delta^r.$$

Column Generation - Cancellation variables

- Column generation subproblem formulated as a knapsack problem.
- A column is generated for each flight in the recovery window.

$$\hat{d}_{ip} = g^{CAN} Pax(i) - \alpha_i^K - \alpha_i^R + \sum_{j \in N_i^{post}} \sum_{u \in U_j} h_{ijp}^u \left(\lambda_j^u - g^{CAN} + d_{ij} \right) - \delta^i .$$

Cancellation Variables Subproblem

$$
\begin{aligned}
\max \quad & \sum_{j \in N_i^{post}} \sum_{u \in U_j} \left(g^{CAN} - d_{ij} - \lambda_{ij}^u \right) h_{ij}^u, \\
\text{s.t.} \quad & \sum_{j \in N_i^{post}} \sum_{u \in U_j} h_{ij}^u \leq Pax(i), \\
& \sum_{u \in U_j} h_{ij}^u \leq (Maxcap - Pax(j)) a_{ij}^u \quad \forall j \in N_i^{post}, \\
& \sum_{u \in U_j} a_{ij}^u = 1 \quad \forall j \in N_i^{post}, \\
& h_{ij}^u \in \mathbb{Z}, \forall j \in N_j^{post}, \forall u \in U_j, \\
& a_{ij}^u \in \{0, 1\}, \forall j \in N_j^{post}, \forall u \in U_j.
\end{aligned}
$$

Cancellation Variables Subproblem

Scale Cancellation Same Schedule constraints

$$\sum_{k \in K} \sum_{p \in P^k} a_{jp}^{ku} x_p^k - \sum_{i \in N^j} \sum_{p \in P^i} \bar{h}_{ijp}^u z_{ip} \geq 0 \quad \forall j \in N^D, \forall u \in U_j$$

Relax the knapsack subproblem

$$\begin{aligned} \max \quad & \sum_{j \in N_i^{post}} \max_{u \in U_j} \left(\left(g^{CAN} - d_{ij} \right) (Maxcap - Pax(j)) - \lambda_{ij}^u \right) \bar{h}_{ij}, \\ \text{s.t.} \quad & \sum_{j \in N_i^{post}} \bar{h}_{ij} (Maxcap - Pax(j)) \leq Pax(i), \\ & \bar{h}_{ij}^u \in [0, 1], \forall j \in N_j^{post}. \end{aligned}$$

Row Generation

- Shorter C-G Master problem $\implies$ rows are added as needed.
- Restricted Connection constraints - assume $\rho_{ij} = 0, \forall (i,j) \in E^D \backslash \bar{E}^D$.
- Same Schedule constraints - use a row generation procedure.

Row Generation - Same Schedule Constraints

Row generation occurs when no reduced cost columns exist for current set of rows.
Assume that $\hat{c}_p^r < 0$ and $\hat{d}_{ip} < 0$, so

$$-\gamma_j^{u'} > c_p^r - \sum_{j \in N^D \cup N_{out}^R} a_{jp}^r \beta_j - \sum_{t \in T} o_{tp}^r \epsilon_t + \sum_{(i,j) \in \bar{E}^D} b_{ijp}^r \rho_{ij} + \sum_{j \in N^D} \sum_{u \in \bar{U}_j} a_{jp}^{ru} \gamma_j^u - \delta^r, \quad \forall j \in N^D, \forall u' \in U_j \backslash \bar{U}_j,$$

$$-\lambda_j^{u'} > g^{CAN} Pax(i) - \alpha_i^K - \alpha_i^R - \delta^i + \sum_{j \in N_i^{post}} \sum_{u \in U_j} \bar{h}_{ijp}^u \left(\lambda_j^u + \left(d_{ij} - g^{CAN} \right) (Maxcap - Pax(j)) \right), \quad \forall j \in N^D, \forall u' \in U_j \backslash \bar{U}_j.$$

Solving the Row Generation Problem

- Solve shortest path problem for each aircraft enforcing the use of $(j, u'), \forall j \in N^D, \forall u' \in U_j \backslash \bar{U}_j$, to find $\gamma_j^{u'}$,
- Solve knapsack problem for each flight enforcing the reallocation to $(j, u'), \forall j \in N^D, \forall u' \in U_j \backslash \bar{U}_j$, to find $\lambda_j^{u'}$,
- Solve the shortest path problem for each crew $k \in K$, using the estimated duals for $\gamma_j^{u'}$ and $\lambda_j^{u'}, \forall j \in N^D, \forall u' \in U_j \backslash \bar{U}_j$.
- If any copies, $(j, u'), \forall j \in N^D, \forall u' \in U_j \backslash \bar{U}_j$, are used in a negative reduced cost path, add the row to the master problem.

Outline

Data and Scenarios

Data

- Planning stage consists of 262 flights, serviced by 79 crew groups and 48 aircraft.
- Flight network has 11 overnight bases for aircraft and 4 crew bases.

Scenarios

- 16 scenarios based on airport closures at different times and different lengths.
- 4 major airports in the network.
- Each major airport closed for 3 or 5 hours at either 6am or 1pm.
- Recovery window = 4 hours.

Recovery Cost

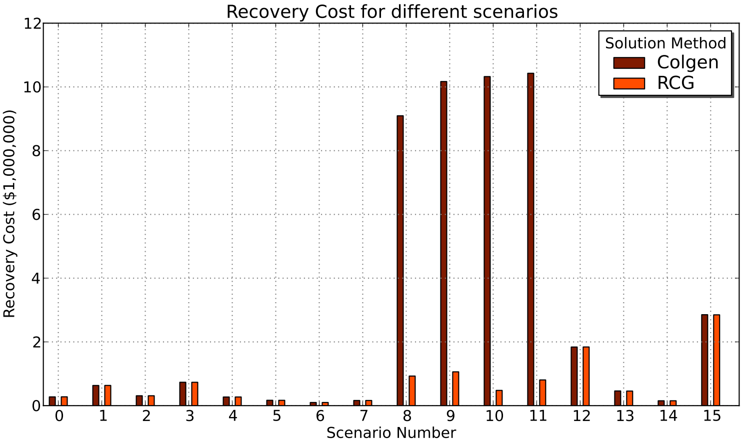


Figure: 3 hour closure: scenarios 0-7. 5 hour closure: scenarios 8-15. Morning: scenarios 0-3, 8-11. Afternoon: scenarios 4-7, 12-15.

Runtime

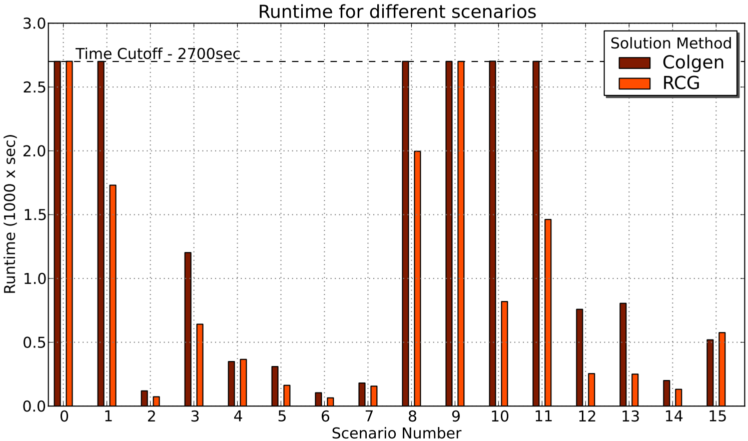


Figure: 3 hour closure: scenarios 0-7. 5 hour closure: scenarios 8-15. Morning: scenarios 0-3, 8-11. Afternoon: scenarios 4-7, 12-15.

Analysis of Reallocation Model - Runtime

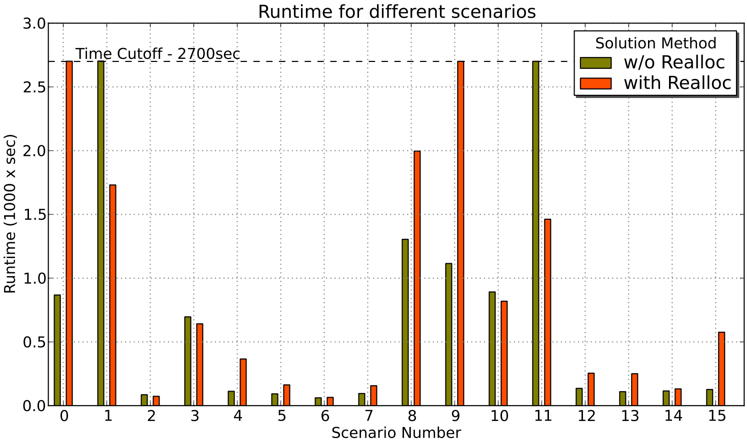


Figure: 3 hour closure: scenarios 0-7. 5 hour closure: scenarios 8-15. Morning: scenarios 0-3, 8-11. Afternoon: scenarios 4-7, 12-15.

Analysis of Reallocation Model - Improvement

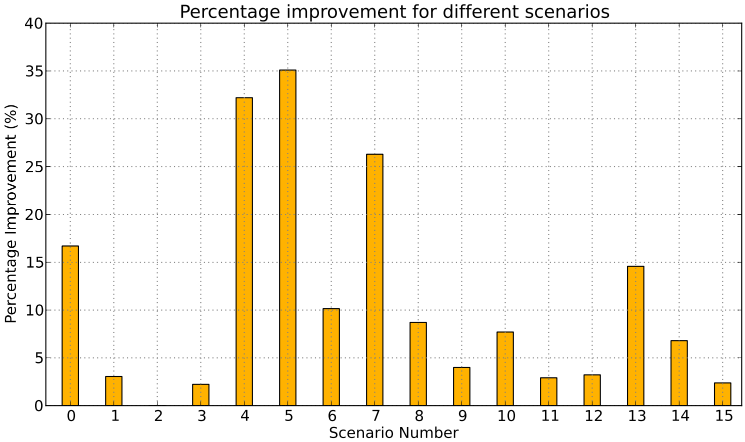


Figure: 3 hour closure: scenarios 0-7. 5 hour closure: scenarios 8-15. Morning: scenarios 0-3, 8-11. Afternoon: scenarios 4-7, 12-15.

Conclusions

- Row-and-column generation is quite efficient in solving the integrated airline recovery problem.
- Row-and-column generation finds a solution with a smaller optimality gap when terminated early.
- Significant cost reductions by reallocation of passengers to flights through cancellation variables.