Challenging Column Generation Models for the Design of Survivable IP-over-WDM Networks

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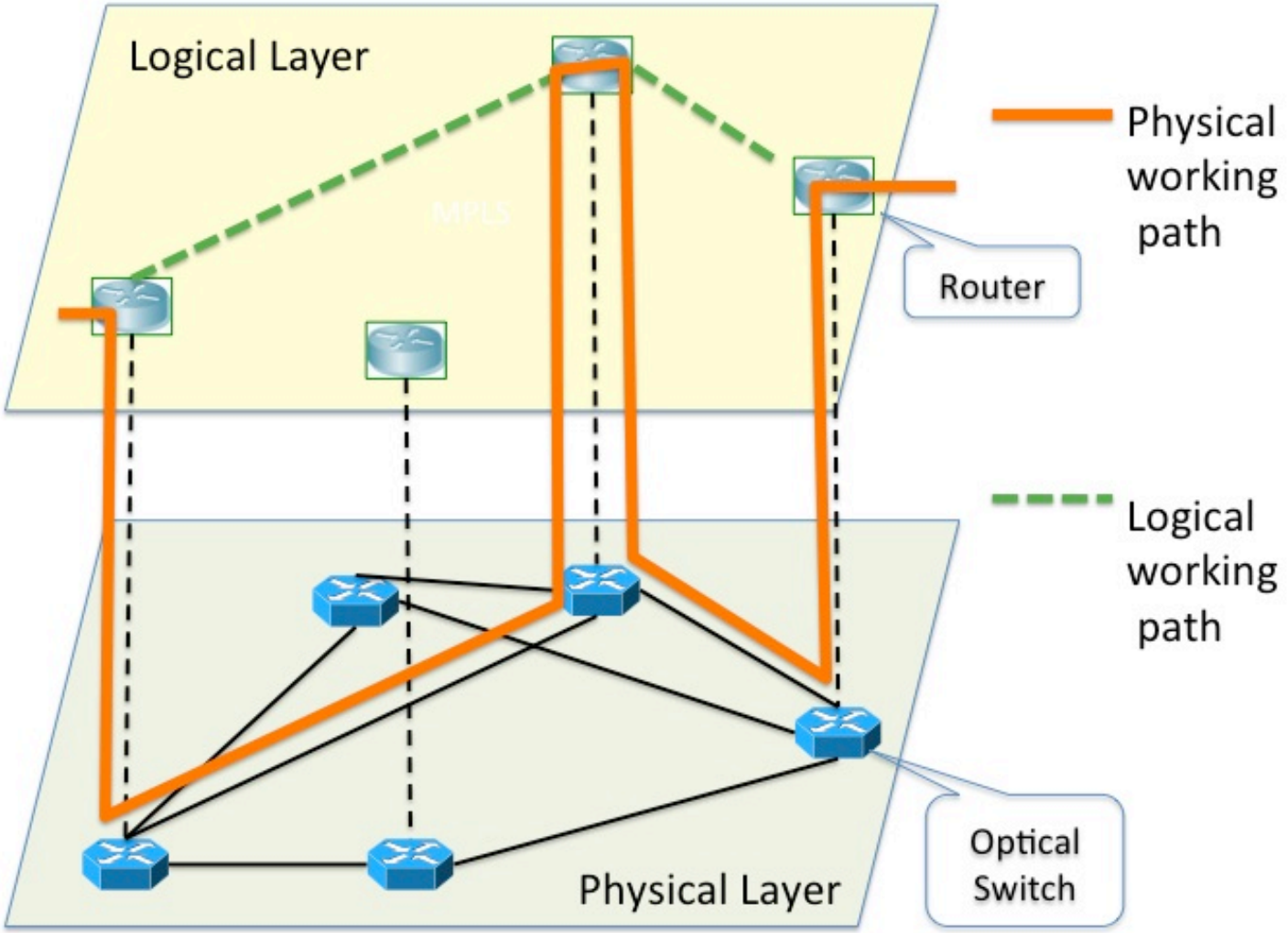
Cross layer optimization (1/5)

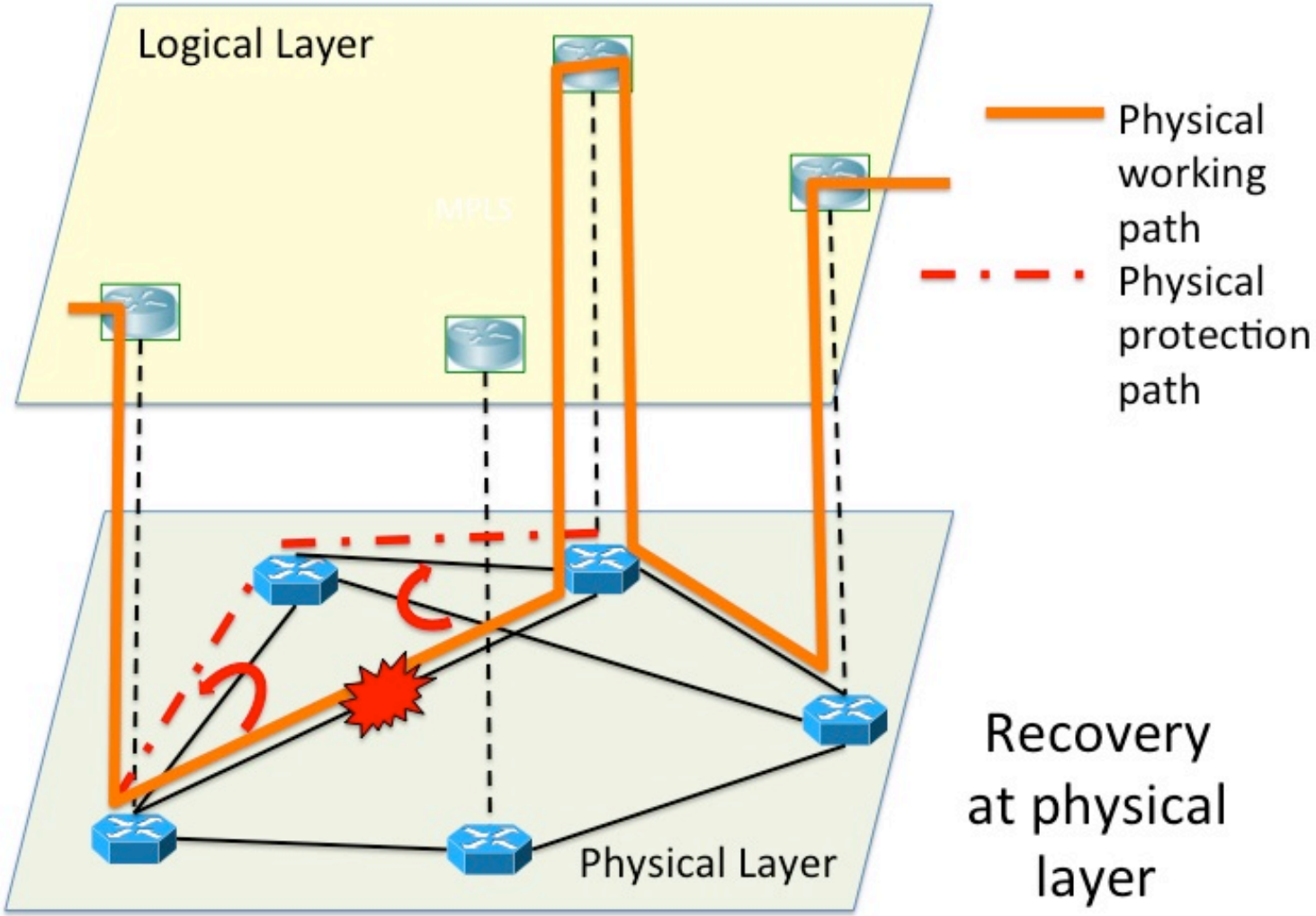


IP Restoration vs. Optical Protection?

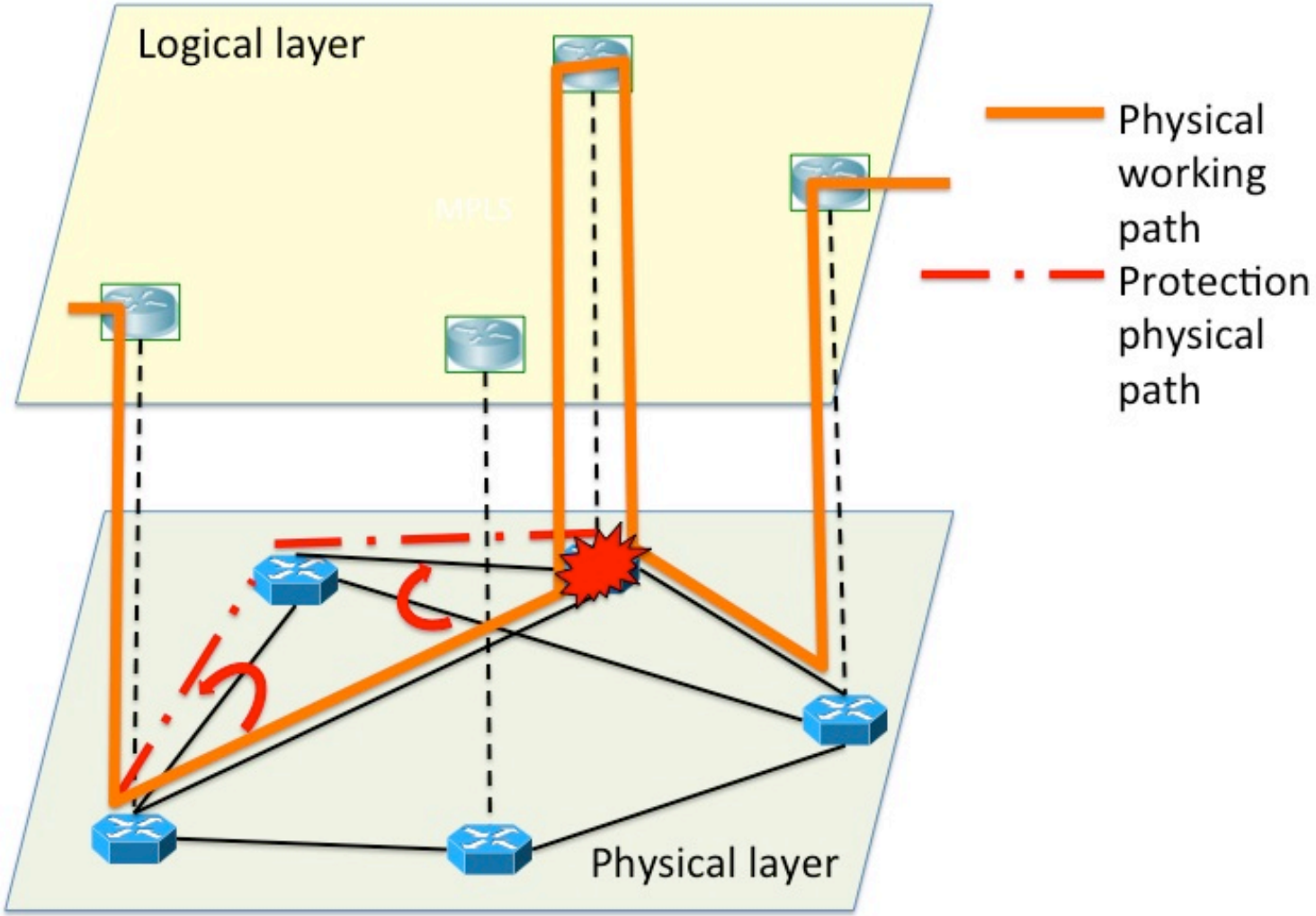
- Multi-layer restoration is a critical point in current optical survivability research.
- Logical layer and optical (physical) layer must be resilient to network failures (physical link or node failures)
- Backup mechanisms: restoration in the logical layer, protection in the optical layer
- Joint IP/optical restoration mechanism is the trend in next generation optical network
  - Reduce the energy consumption:Energy bottleneck in IP routers is looming
  - Guarantee the Service Level Agreements (SLA) with bandwidth greedy applications (video services, IPTV...)

Optical Protection

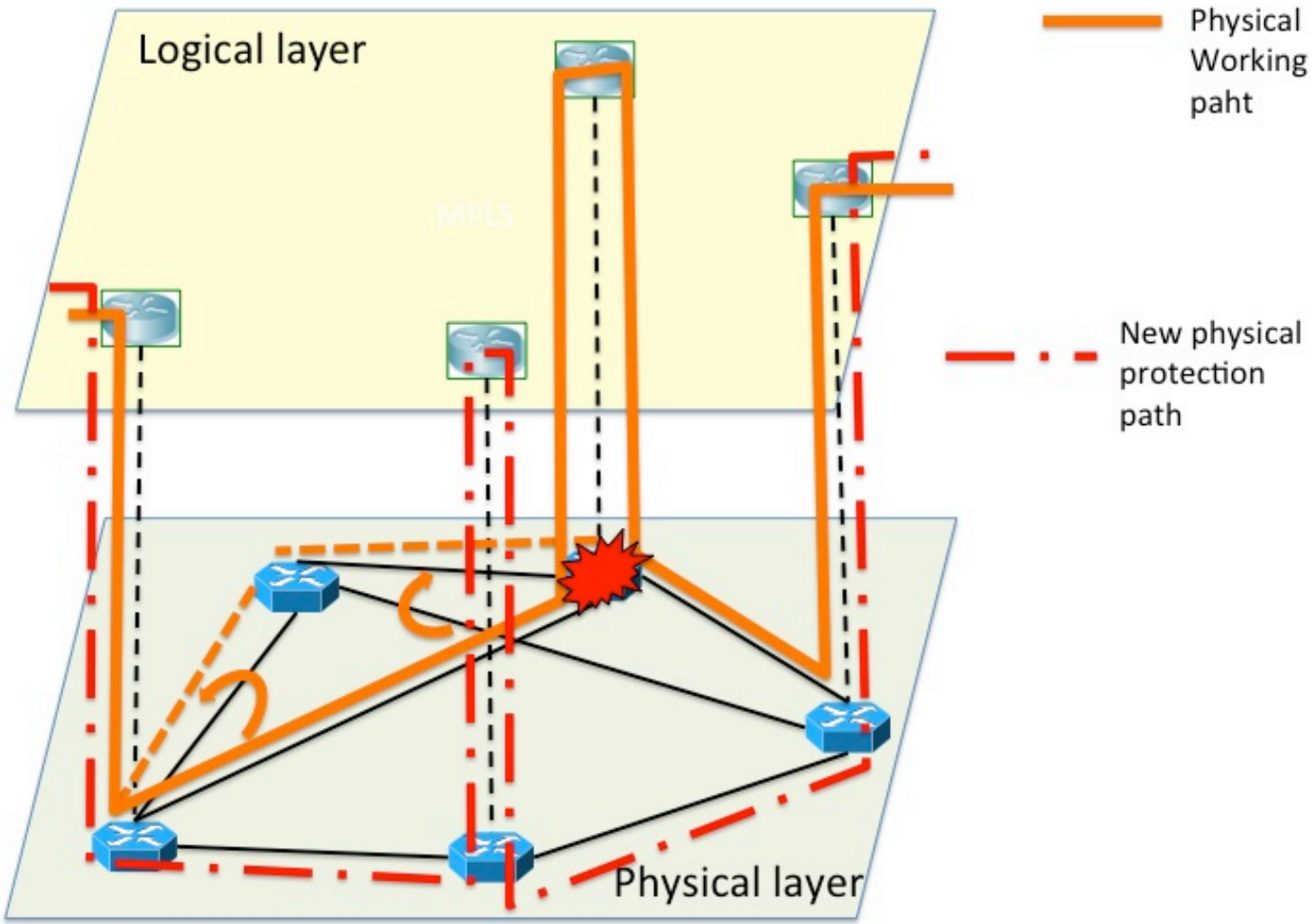
Cross layer optimization (2/5)



Cross layer optimization (3/5)

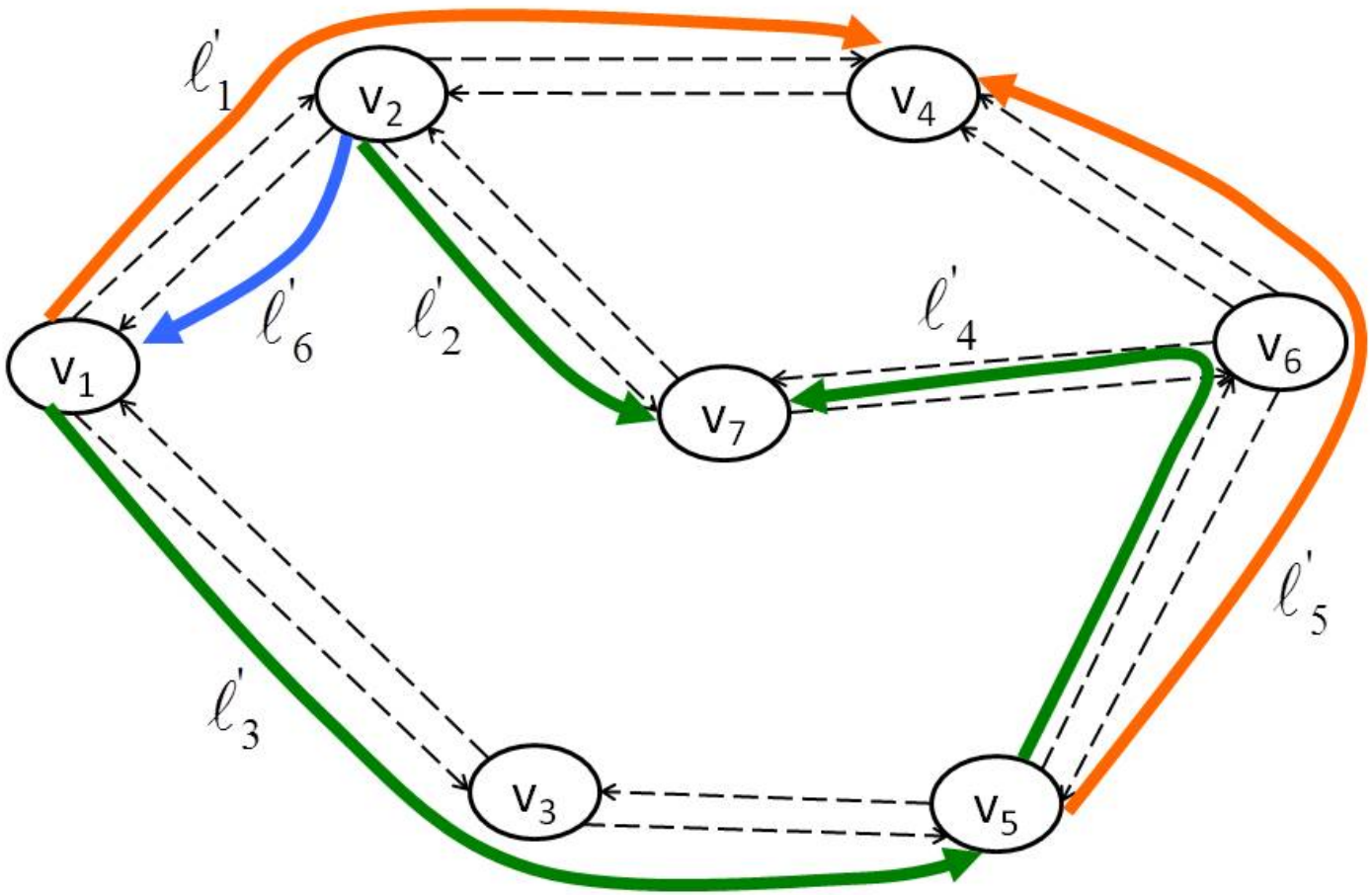


Cross layer optimization (4/5)



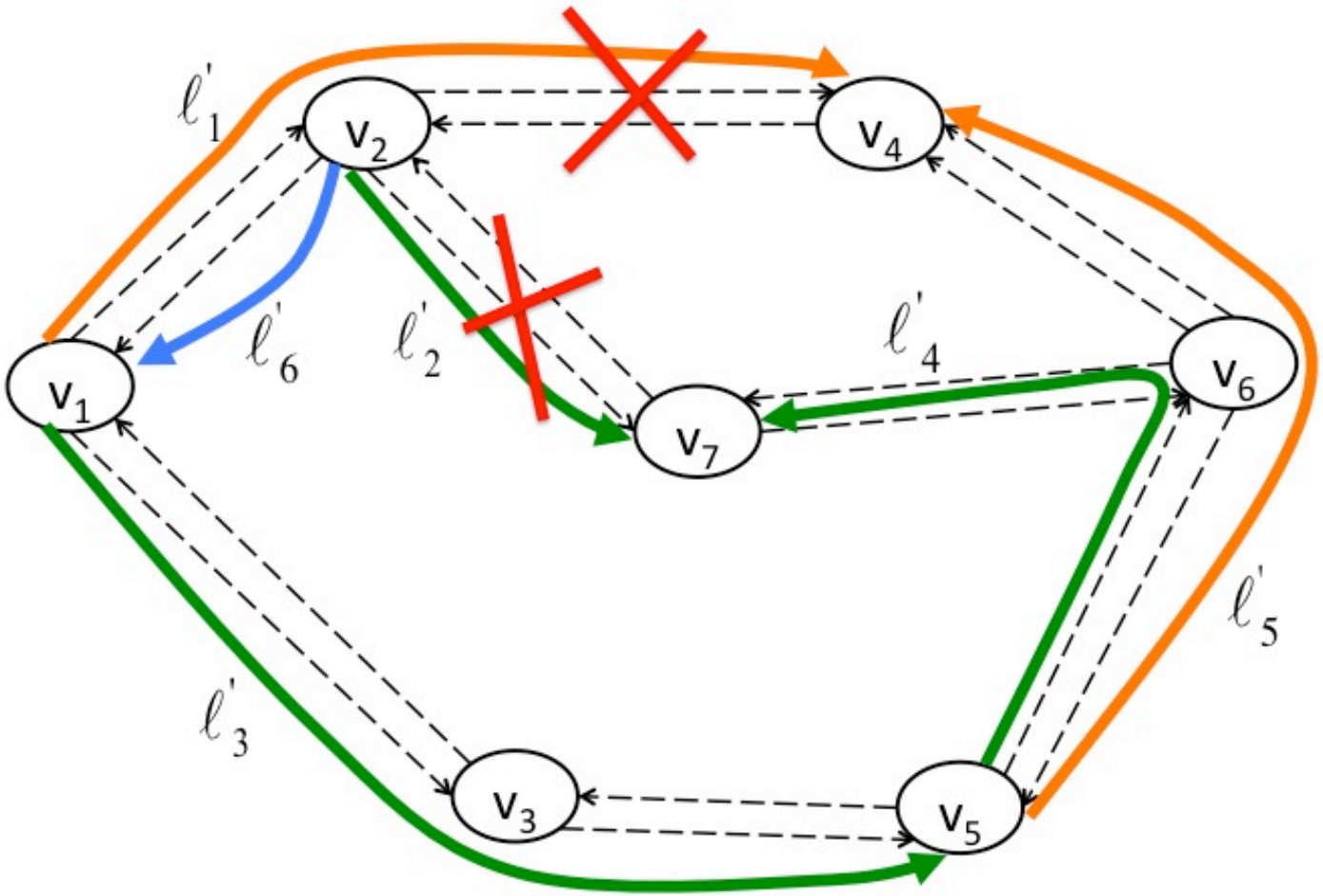
IP Restoration

Physical vs. logical topology: Connectivity Issue



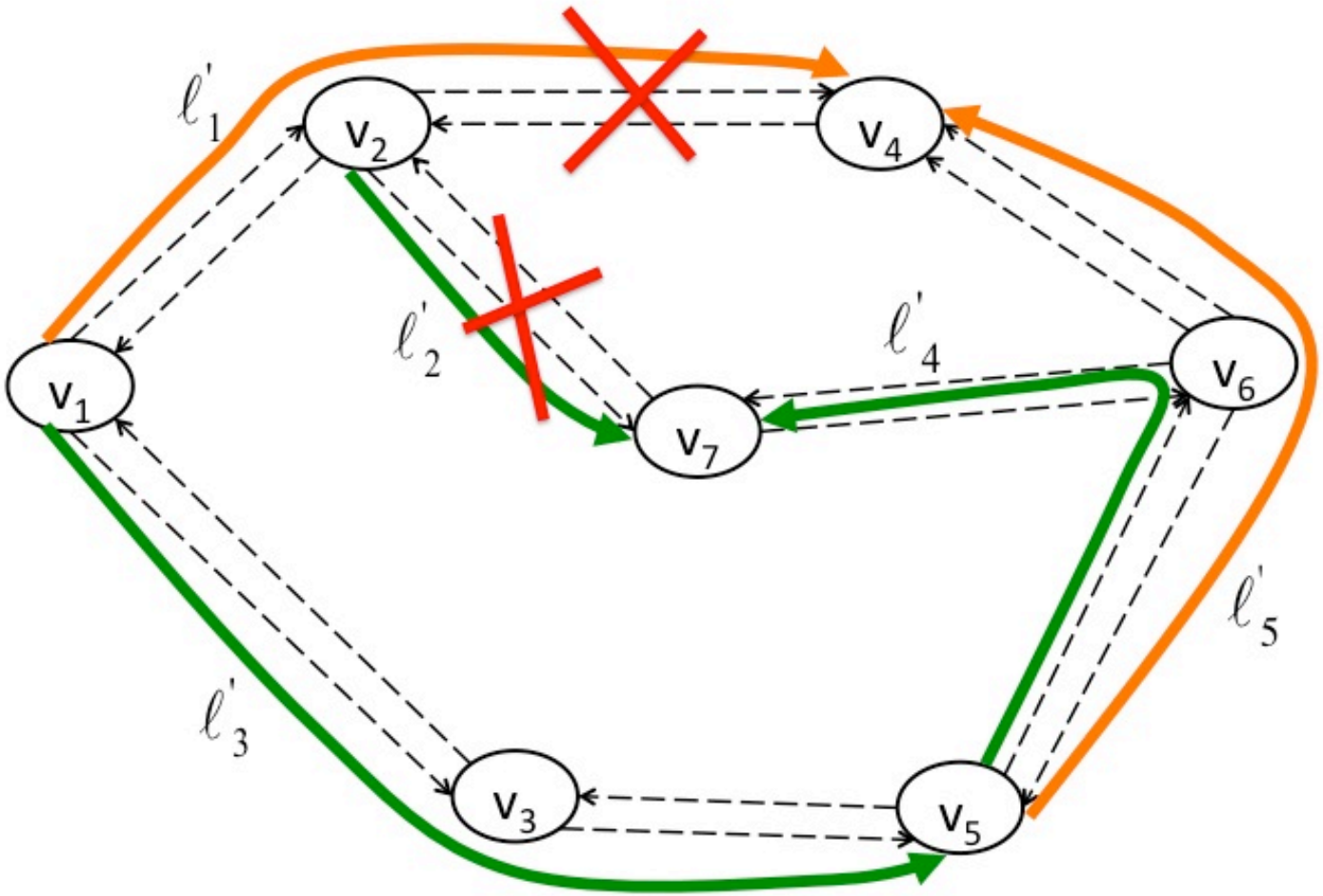
Physical topology and Logical topology: Dual Failure

A survivable topology

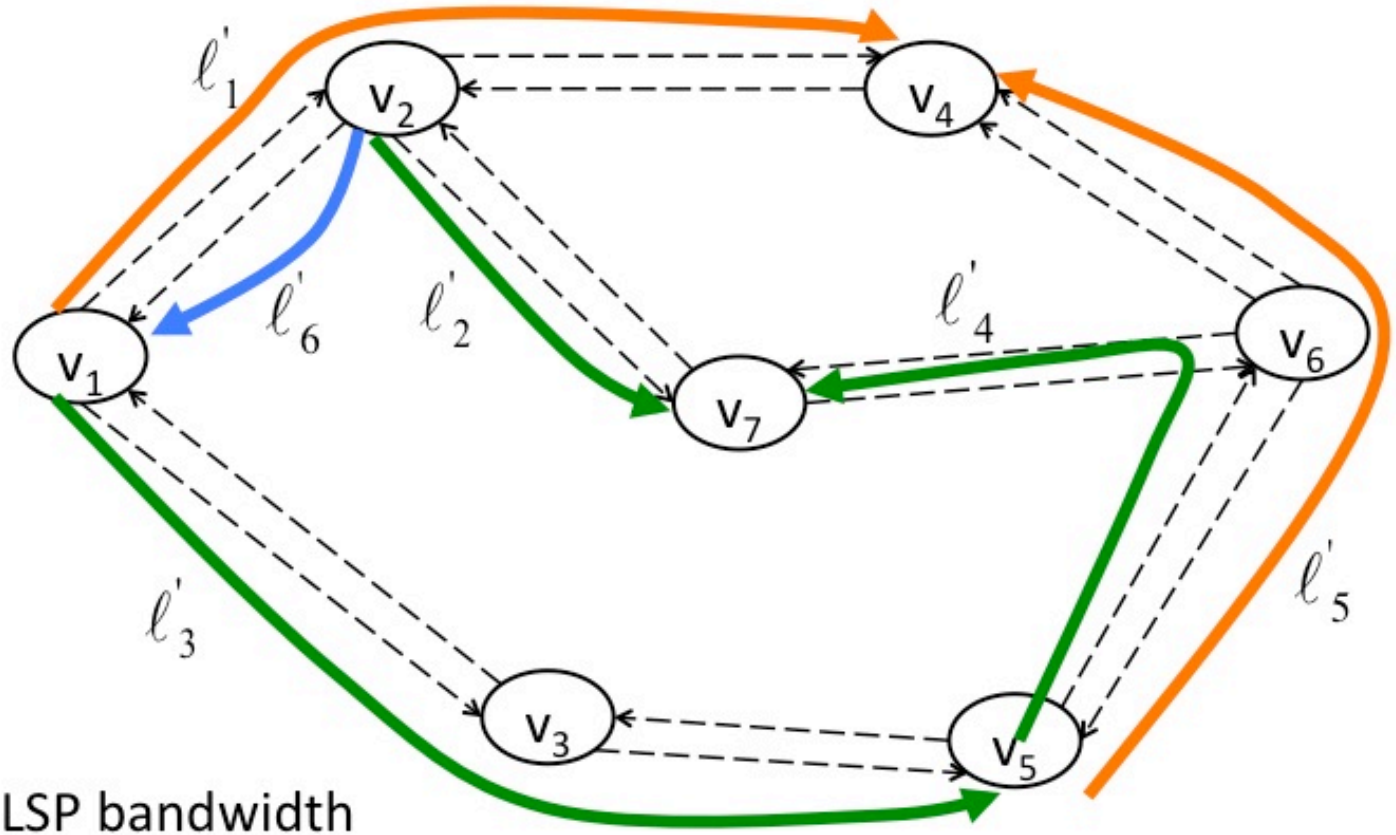


Physical vs. logical topology: Dual Failure

A non survivable topology



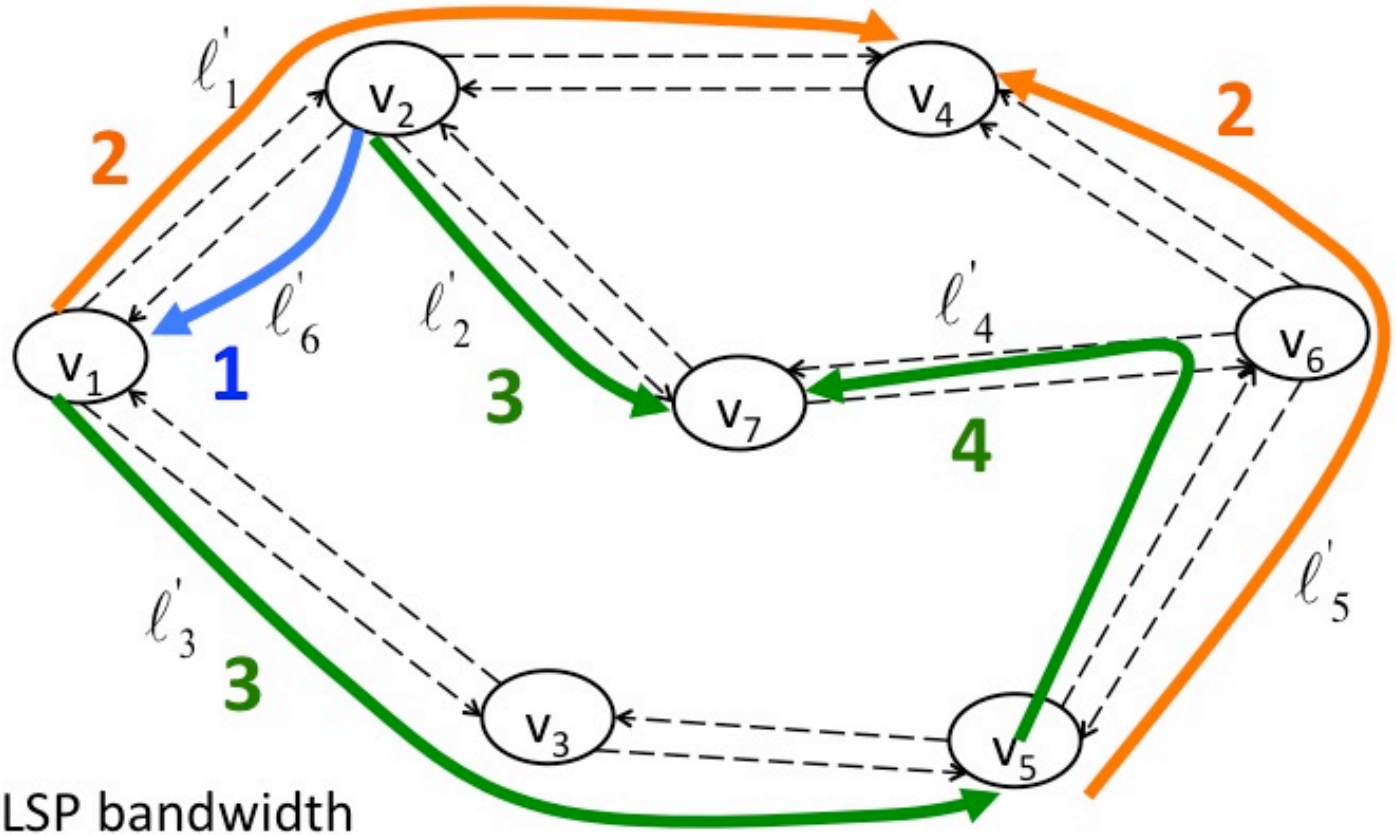
Capacity concerns: Feasible logical ← physical mapping



LSP bandwidth demands:

$$\ell'_1 \mapsto 2; \ell'_2 \mapsto 3; \ell'_3 \mapsto 1; \ell'_4 \mapsto 4; \ell'_5 \mapsto 2; \ell'_6 \mapsto 1$$

Capacity concerns: Feasible logical ← physical mapping



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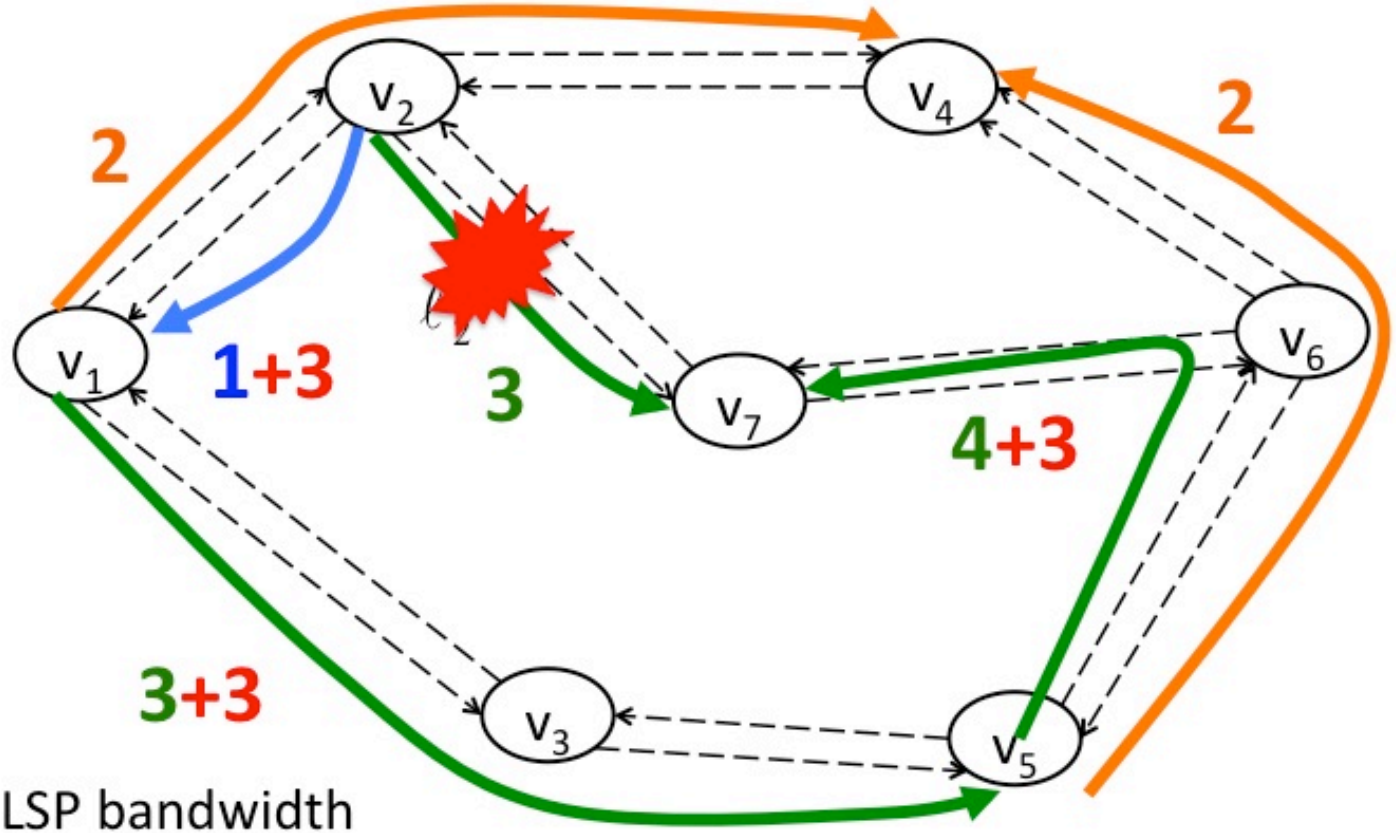
$$\ell'_1 \mapsto 2; \ell'_2 \mapsto 3; \ell'_3 \mapsto 1; \ell'_4 \mapsto 4; \ell'_5 \mapsto 2; \ell'_6 \mapsto 1$$

Capacity concerns: Feasible IP restoration



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$$\ell'_1 \mapsto 2; \ell'_2 \mapsto 3; \ell'_3 \mapsto 1; \ell'_4 \mapsto 4; \ell'_5 \mapsto 2; \ell'_6 \mapsto 1$$

Logical Survivable Topology Design Problem

For a given backbone network:

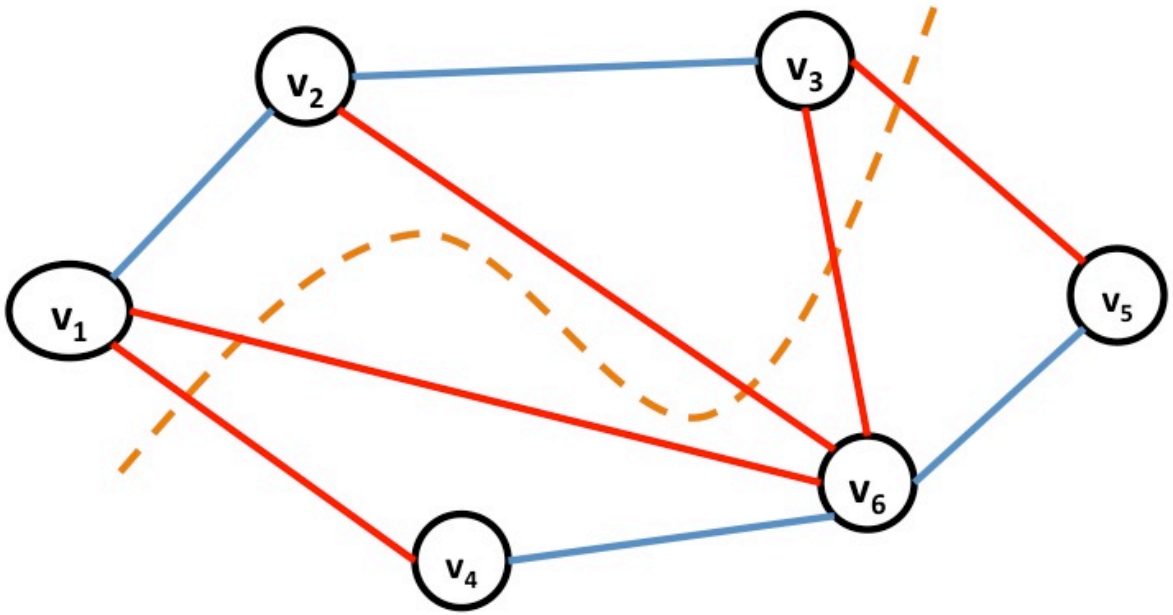
- Physical Topology $G_{\mathsf{P}} = (V_p, E_p)$
- Logical Topoploy $G_{\mathsf{L}} = (V_{\mathsf{L}}, E_{\mathsf{L}})$
- A set of single/multiple link (node) failures $\mathcal{F}$
  - $\mathcal{F}\,\{\{\ell_1\};\{\ell_2\};\{\ell_3\};\{\ell_1,\ell_3\}\}$

Finding a routing (mapping) of each logical link on the physical topology such that:

- Minimize the mapping cost
- In order that the logical topology is survivable

Cutset

Graph $G = (E, V)$, Cut: $S \subset V, T = V \setminus S$
Example: $S = \{v_1, v_2, v_3\}; T = \{v_4, v_5, v_6\}$
Cutset $C = \langle S, T \rangle = \{(u, v) \in E | u \in S, v \in T\}$



Cutset (Cont'd)

- A logical topology is survivable if for *any* cutset, the number of failed logical links going through the cutset is smaller than the cardinality of that cutset.
  - That is, there is always at least one survivable logical link connecting two subsets S and T, for any subset $S \subset V$
- Each cutset corresponds to a constraint.
  - Exponential number of cutset constraints.
  - Difficult to solve even for small network instances.

Literature review

- Most references $\longmapsto$ ILP (Integer Linear Program) model, only scalable on particular topologies. Use heuristics to deal with meaningful sizes data instances.
- Modiano and Narula-Tam (2001) : particular topologies (e.g., rings) + relaxation for mesh topologies.
- Todimala and Ramamurthy (2007): ILP model, only scalable on particular topologies. Reason: an exponential number of cutsets in the graph underlying the logical topology.
- Kurant and Thiran (2007) : heuristic, mapping from a logical topology to a simplified one which preserves the survivability.

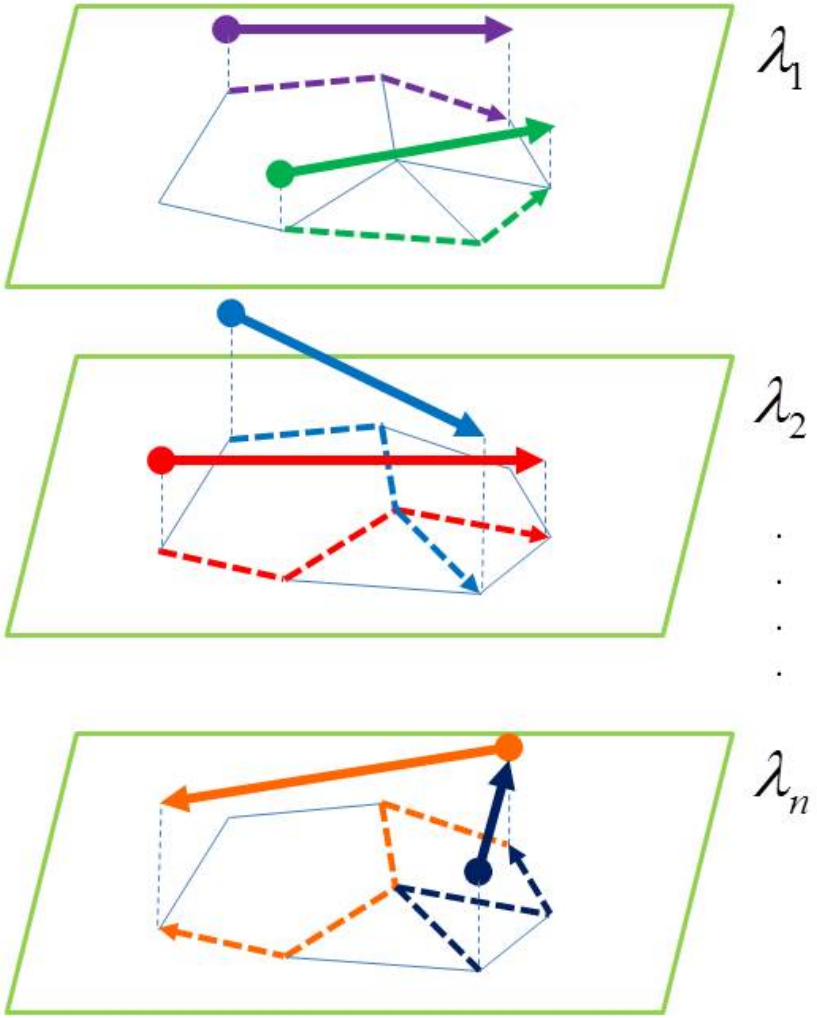
Literature review (Cont'd)

- Liu and Ruan (2007): a more flexible context - several logical links can be added if no survivable logical topology exists. Still lacks scalability due to the exponential # of cutset constraints.
- Kan *et al.* (2009) : jointly the capacity assignment and logical survivability, derived some cutset constraints to guarantee the survivability of a logical topology.
- Lin *et al.* (2011) : weakly vs. strongly survivable routing where strongly is related to limitations imposed on the routings by physical capacity limits.
- Most proposed ILP models based on the cutset theorem: a huge number of cutset constraints.
  - Usable only with data instance of (very) small size.
  - A great effort made to reduce the number of generated cutset constraints by exploiting some special graph structures.
  - Little effort put to deal efficiently with the general case.

Generalities

- Lightpath: a connection from source to destination over the same wavelength.
- A configuration is a list of mappings of logical links $\hookrightarrow$ physical paths, on a single wavelength:
  - Each logical link is associated with a lightpath
  - All lightpaths belonging to the same configuration: same wavelength
- A solution is a collection of configurations, one per wavelength such that:
  - Requests are satisfied
  - Logical topology is survivable

Configuration Examples



Configurations

A configuration c:

- $f^c_{\ell\ell'}$ = 1 if virtual link ℓ' is routed over physical link ℓ in configuration c, 0 otherwise.

Provides information on how many logical links cannot be protected using additional variables and penalty coefficients:

- $\text{PENAL}^{\text{NP}} = 10^4$
- $a^c_{\ell'} = 1$ if there exists one lightpath to route logical link ℓ', 0 otherwise, easily deduced from $f^c_{\ell\ell'}$.

Cutset Optimization Model

Decision variables:

- $(z_c)_{c \in C}$: $z_c = 1$ if configuration c is selected, 0 otherwise.
- $(x^F_{\ell'})_{F \in \mathcal{F}, \ell' \in E_{\mathsf{L}}}$: $x^F_{\ell'} = 1$ if logical link ℓ' is routed but cannot be protected in case links of failure set F fail, 0 otherwise.

Objective:

$$\min \sum_{c \in C} \sum_{(\ell,\ell') \in E_{\mathsf{P}} \times E_{\mathsf{L}}} f^c_{\ell\ell'} z_c + \sum_{(\ell',F) \in E_{\mathsf{L}} \times \mathcal{F}} x^F_{\ell'} \mathsf{PENAL}^{\mathsf{NP}}. \quad (1)$$

Cutset Optimization Model (Cont'd)

Constraints:

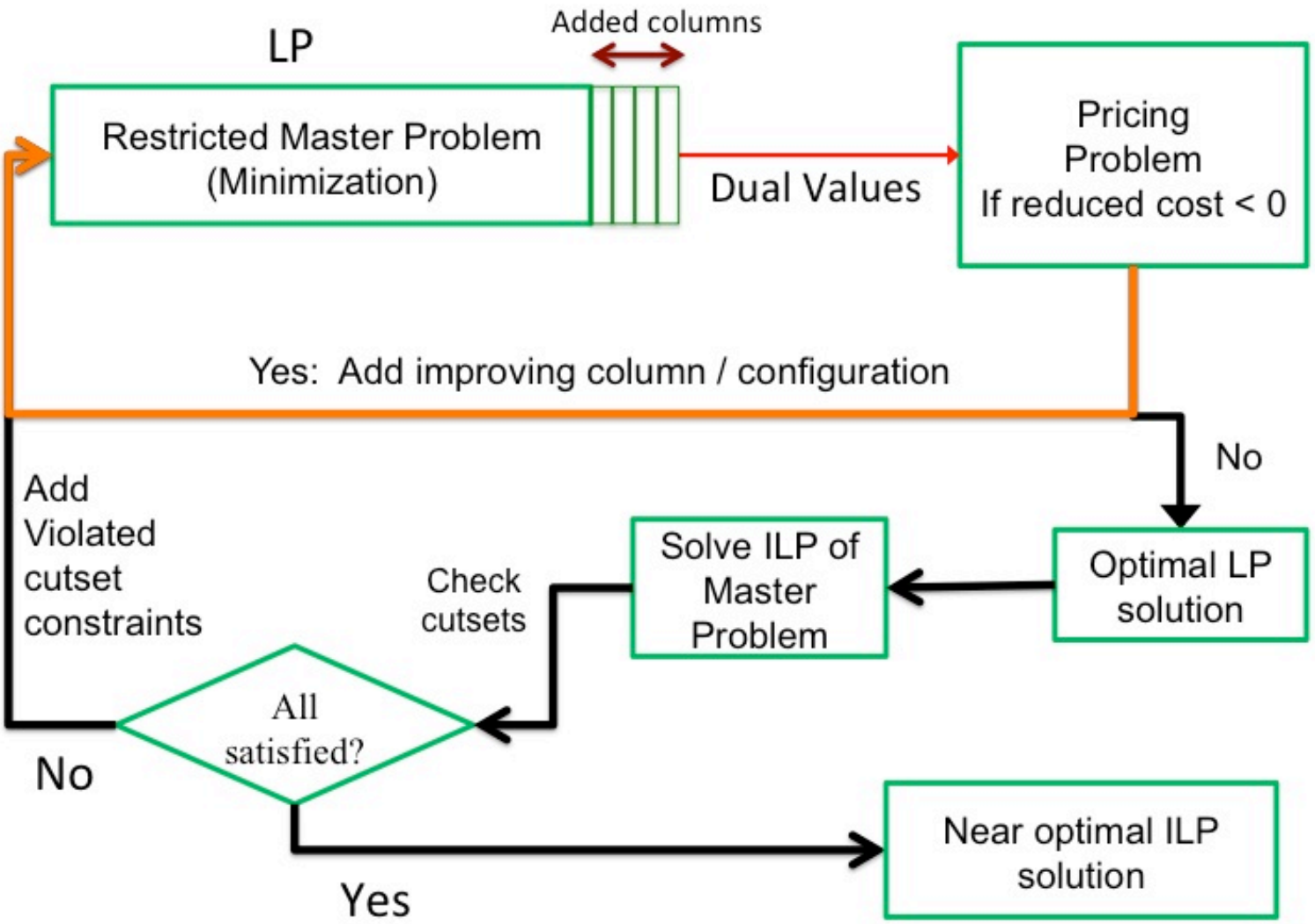
$$\sum_{c \in C} a^c_{\ell'} z_c \geq 1 \qquad \ell' \in E_{\mathsf{L}} \tag{2}$$

$$\sum_{c \in C} \sum_{\ell \in F} \sum_{\ell'' \in CS(S, V_{\mathsf{L}} \setminus S)} f^c_{\ell\ell''} z_c \leq \quad |\langle S, V_{\mathsf{L}} \setminus S \rangle| - 1 + x^F_{\ell'}$$

$$S \subset V_{\mathsf{L}}, F \in \mathcal{F}, \ell' \in E_{\mathsf{L}} \tag{3}$$

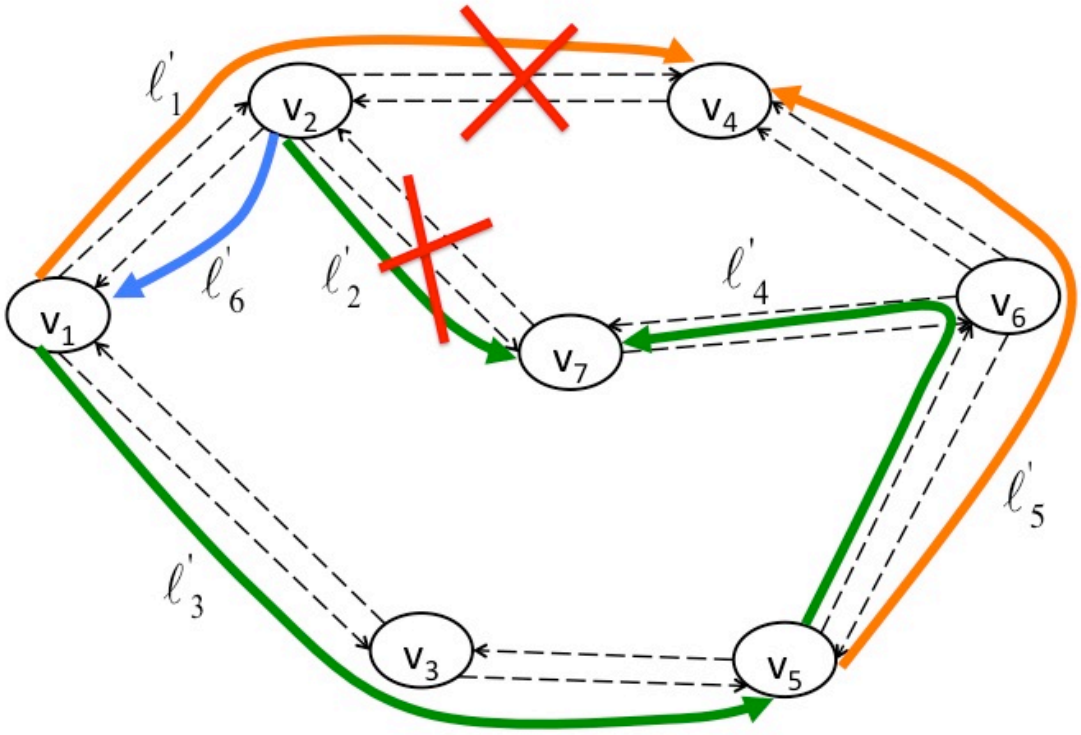
$$z_c \in \{0, 1\} \qquad c \in C. \tag{4}$$

Transform Cutset Constraints into Lazy Constraints



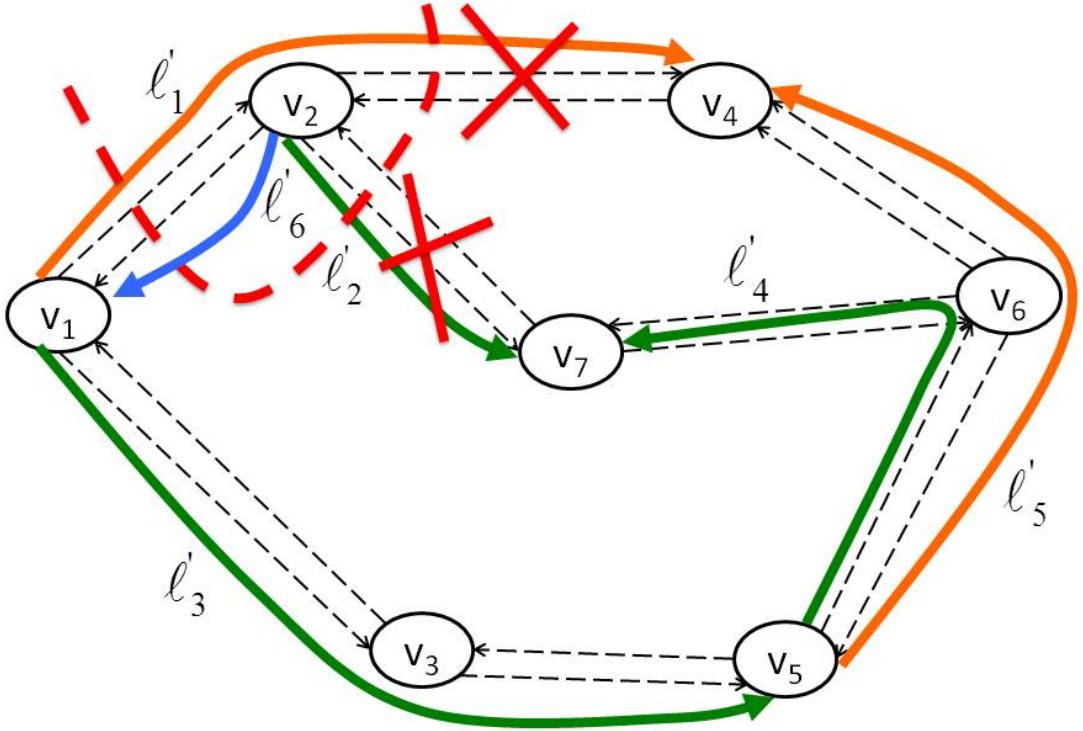
Example

- Failed physical links: $(v_2, v_4), (v_2, v_7)$.
- Source node of ℓ'_2 (resp. ℓ'_1): v_2. Try to reach the destination node v_7 (resp. v_4): Use, e.g., Dijkstra to compute minimum shortest tree
- Logical paths: $\ell'_6 \to \ell'_3 \to \ell'_4$ and $\ell'_6 \to \ell'_3 \to \ell'_5$



Example

- Failed physical links: $(v_2, v_4), (v_2, v_7)$.
- Source node of ℓ'_2 (resp. ℓ'_1): v_2. Try to reach the destination node v_7 (resp. v_4): Use, e.g., Dijkstra to compute minimum shortest tree
- Deduce a cutset to be added to the set of constraints



Pricing Problem

Objective: Reduced Cost

$$\overline{\mathsf{COST}} = \sum_{(\ell,\ell')\in E_{\mathsf{P}}\times E_{\mathsf{L}}} f^{c}_{\ell\ell'} - \sum_{\ell'\in E_{\mathsf{L}}} u^{\mathsf{D}}_{\ell'} a_{\ell'}$$
$$+ \sum_{S\subset V_{\mathsf{L}}} \sum_{F\in\mathcal{F}} \sum_{\ell'\in E_{\mathsf{L}}} \sum_{\ell\in F} \sum_{\ell''\in CS(S,V_L\setminus S)} u^{\mathsf{F}}_{S,\ell'} f_{\ell\ell''}$$

Constraints, route one unit of flow on the physical network from $\mathsf{SRC}(\ell)$ to $\mathsf{DST}(\ell)$, for all $v \in V_{\mathsf{L}}$:

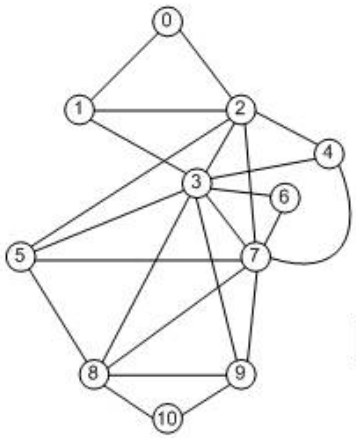
$$\sum_{\ell\in\omega^{+}(v)} f_{\ell\ell'} - \sum_{\ell\in\omega^{-}(v)} f_{\ell\ell'} = \begin{cases} a_{\ell'} & \text{if } v_s(\ell') = v \\ -a_{\ell'} & \text{if } v_d(\ell') = v \\ 0 & \text{otherwise} \end{cases}$$

Data sets

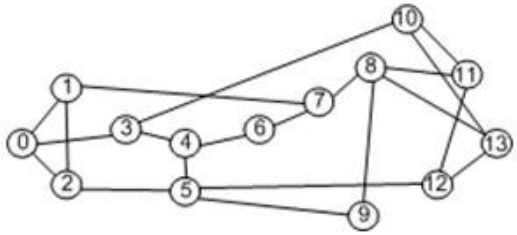
Four different physical topologies:

Topologies	# nodes	# spans = (# links)/2	Average nodal degree
NJLATA	11	23	4.2
NSF	14	21	3.0
EURO	19	37	3.9
24-NET	24	43	3.4

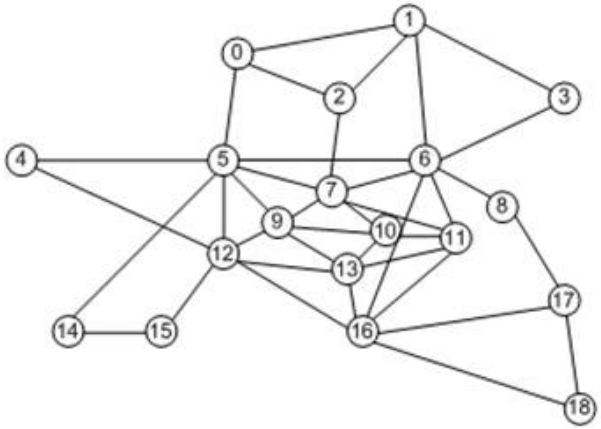
Data sets



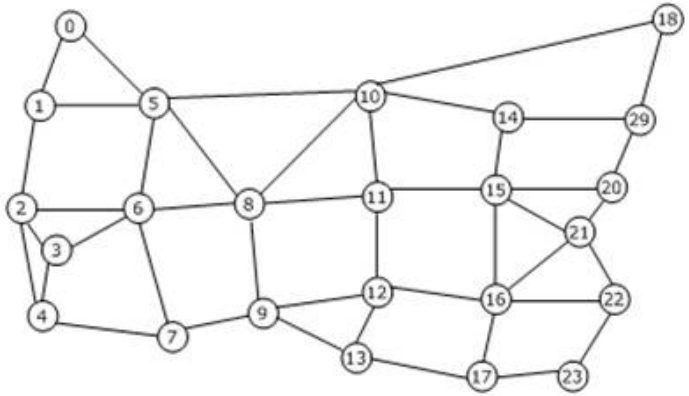
NJLATA



NSF



EURO



24-NET

http://www.optical-network.com/topology.php

Existence of a survivable logical topology

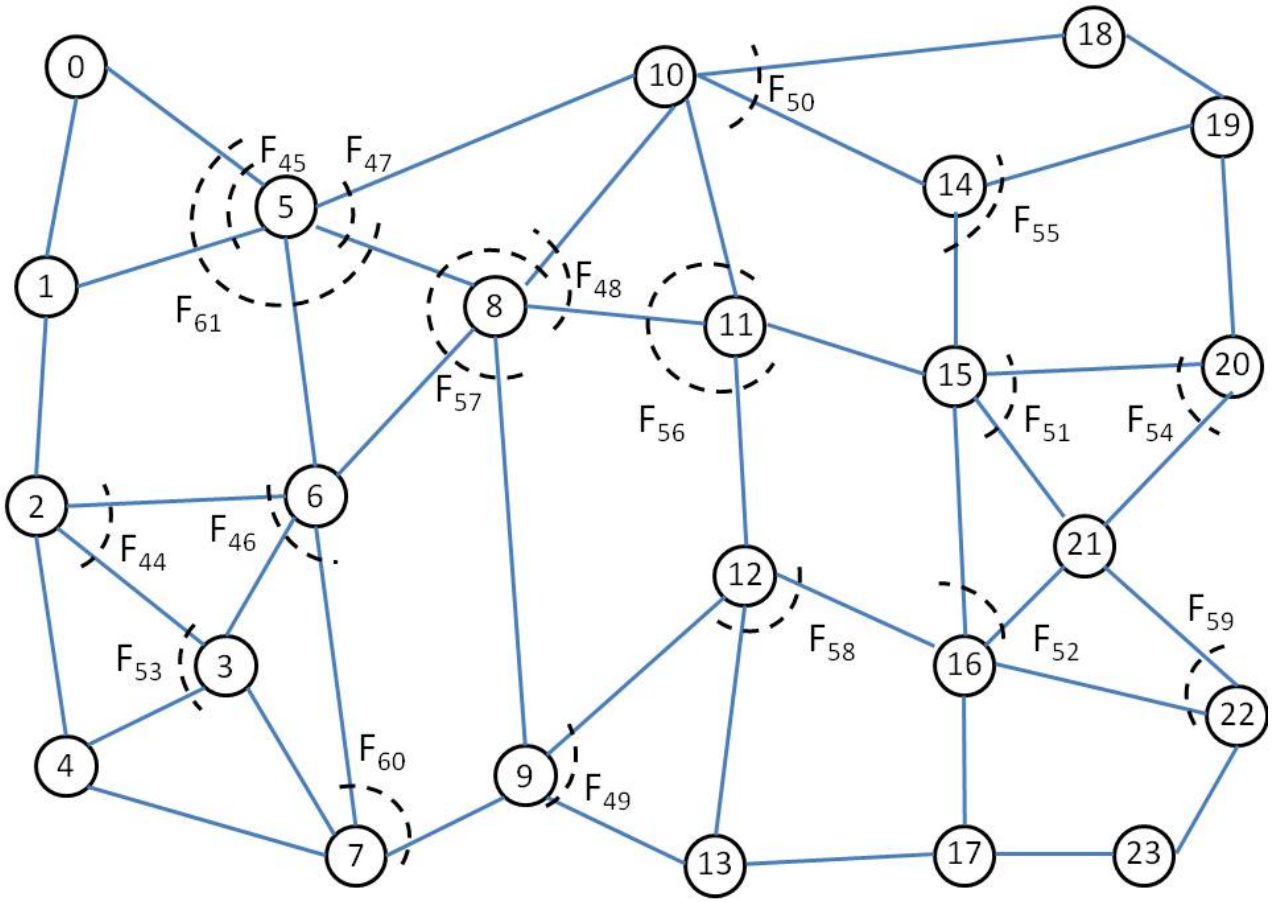
Instances	Topo	#survivable topologies		# unprotected log. links
		Cutset	Todimala *et al.* (2007)	Cutset
NJLATA	degree 3	100		0
	20-edge	100		0
NSF	21-edge	99	76	1
	25-edge	100	100	0
EURO	degree-3	99	87	2
	30-edge	98	83	4
	35-edge	100	100	2
24-NET	40-edge	97	93	1
	45-edge	100	87	2

Performance of the enhanced column generation cutset model

Instances	Topo.	Single Link Failures					
		# Configurations		gap	#λ	CPU	# cutset constraints
		gener.	selec.				
NJLATA	degree 3	45.8	34.1	0.01	3.3 ± 0.6	8.2	4.5
	20-edge	48.1	40.0	$< 10^{-3}$	4.4 ± 0.8	8.7	3.1
NSF	21-edge	53.6	42.0	0.03	5.0 ± 1.2	11.5	5.7
	25-edge	57.8	50.0	0.01	5.8 ± 1.1	11.3	3.4
EURO	degree-3	83.7	58.0	0.03	4.9 ± 0.9	43.8	10.2
	30-edge	84.1	60.0	0.03	5.6 ± 1.4	44.5	10.5
	35-edge	93.8	70.0	0.02	6.4 ± 1.2	54.2	9.0
24-NET	40-edge	106.2	80.0	0.02	7.9 ± 2.2	103.1	11.9
	45-edge	113.4	90.0	0.01	8.6 ± 1.8	116.1	9.4

Instances	Topo.	Single Node Failures					
		# Configurations		gap	#λ	CPU	# cutset constraints
		gener.	selec.				
NJLATA	degree 3	34.6	34.1	$< 10^{-3}$	3.6 ± 0.6	4.6	0.3
	20-edge	40.3	40.0	$< 10^{-3}$	4.4 ± 0.8	5.7	0.2
NSF	21-edge	42.0	42.0	$< 10^{-3}$	5.4 ± 0.9	6.3	0
	25-edge	50.0	50.0	$< 10^{-3}$	5.9 ± 1.1	7.9	0
EURO	degree-3	59.5	58.0	$< 10^{-3}$	5.3 ± 1.0	17.8	0.8
	30-edge	61.9	60.0	$< 10^{-3}$	5.9 ± 1.2	19.3	0.9
	35-edge	71.1	70.0	$< 10^{-3}$	6.7 ± 1.2	25.3	0.5
24-NET	40-edge	88.6	80.0	$< 10^{-3}$	8.2 ± 1.5	51.5	5.0
	45-edge	97.2	90.0	$< 10^{-3}$	8.9 ± 1.4	65.0	3.6

24-NET Topology



Failure sets

Sets	Set elements	
F_e	$= \{e\}, e \in E$	
F_{44}	$= \{\{2,6\},\{2,3\}\}$	$F_{45} = \{\{0,5\},\{1,5\}\}$
F_{46}	$= \{\{2,6\},\{3,6\},\{6,7\}\}$	$F_{47} = \{\{5,10\},\{5,8\}\}$
F_{48}	$= \{\{8,10\},\{8,11\}\}$	$F_{49} = \{\{9,12\},\{9,13\}\}$
F_{50}	$= \{\{10,18\},\{10,14\}\}$	$F_{51} = \{\{15,20\},\{15,21\}\}$
F_{52}	$= \{\{15,16\},\{16,21\}\}$	$F_{53} = \{\{2,3\},\{3,4\}\}$
F_{54}	$= \{\{15,20\},\{21,20\}\}$	$F_{55} = \{\{14,15\},\{14,19\}\}$
F_{56}	$= \{\{10,11\},\{8,11\},\{12,11\}\}$	
F_{57}	$= \{\{8,10\},\{8,5\},\{8,6\},\{8,9\}\}$	
F_{58}	$= \{\{12,13\},\{12,16\}\}$	$F_{59} = \{\{21,22\},\{16,22\}\}$
F_{60}	$= \{\{7,6\},\{7,9\}\}$	
F_{61}	$= \{\{0,5\},\{1,5\},\{6,5\},\{5,8\}\}$	
F^2	$F_1^2 = \{F_{44}, F_{45}, F_{47}, F_{48}, F_{49}, F_{50}, F_{51}, F_{52}\}$ $F_2^2 = F_1^2 \cup \{F_{53}, F_{54}, F_{55}\}$ $F_3^2 = F_2^2 \cup \{F_{58}, F_{59}, F_{60}\}$	
F^3	$F_1^3 = \{F_{46}\}$	$F_2^3 = F_1^3 \cup \{F_{56}\}$
F^4	$F_1^4 = \{F_{57}\}$	$F_2^4 = \{F_{61}\}$

Conclusion

- With the recourse of Column Generation, an enhanced scalable cutset model has been designed
- Future work
  - Embed capacity constraints
  - Future work: how to improve the solution of the models in order to solve even larger data instances

Any question?