A Branch-and-Price Method for a Ship Routing and Scheduling Problem with Cargo Coupling and Synchronization Constraints

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Outline

- Background and motivation
- Path-flow models
- Solution approach
- Computational study
- Conclusions

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Tramp shipping

- Contracts of affreightment
  - Pickup and delivery ports
  - Specified quantity
  - Time windows
- Spot market for optional cargoes
- Heterogeneous fleet
  - Capacity
  - Initial position
  - Cost structure
  - Speed
  - Cargo compatibility
- Maximize profit

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Project shipping

- A special segment of tramp shipping
- Unique cargoes transported on a one-time basis
  - Parts of a process facility, yachts, train sets
- Special stowage challenges
  - Shape, stability, sea fastening, weight and lifting
  - Engineering unit in order to calculate the possibility of transporting the cargoes

- Cargo coupling and synchronization

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Cargo coupling and synchronization

- Cargo coupling
  - The shipping company cannot transport a cargo unless other parts of the same order are transported as well, even though these parts may have different origins
- Synchronization
  - The different parts of an order require synchronized delivery within some time window
  - Expensive equipment, storage problems

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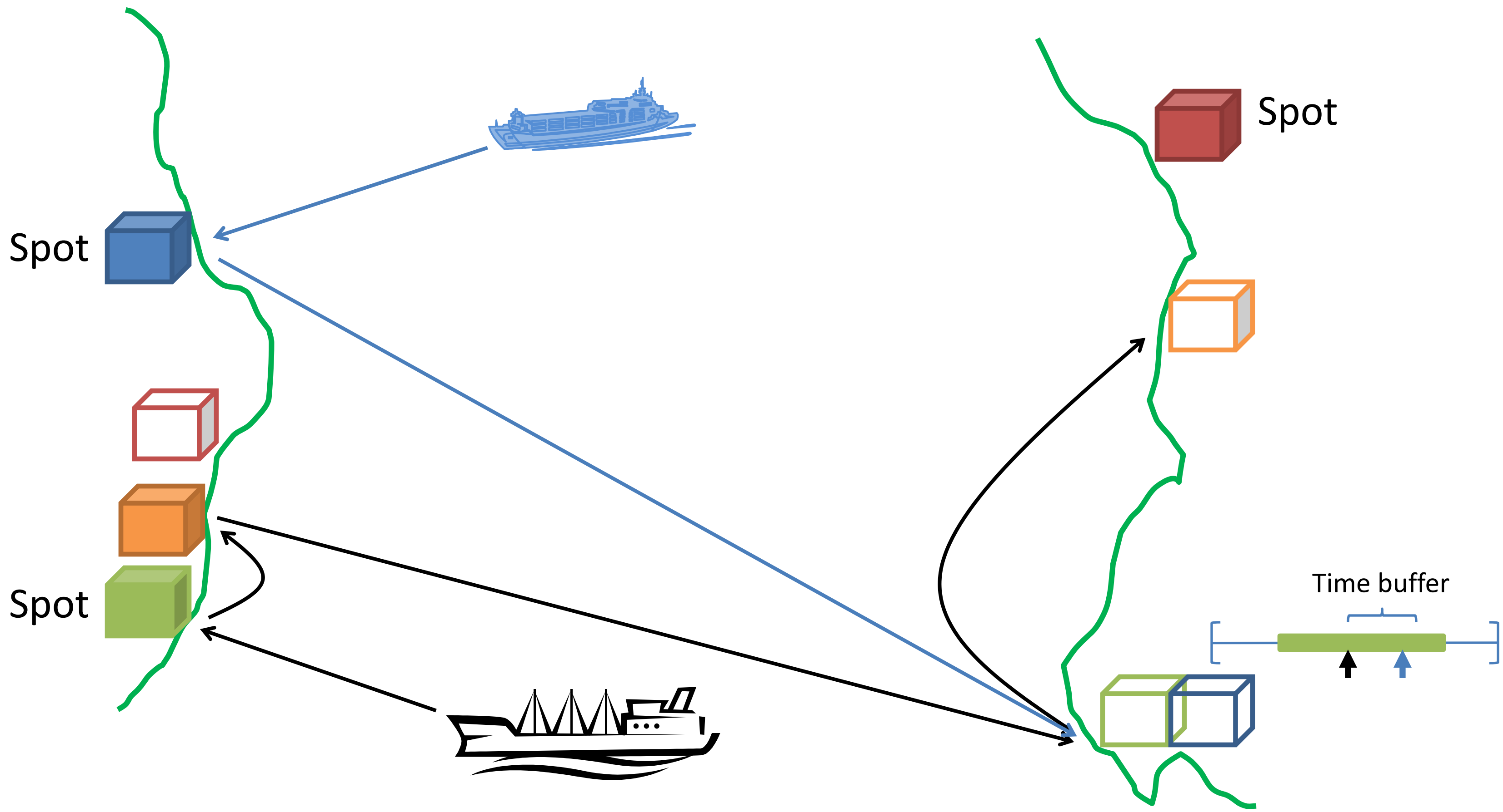
Project shipping - example



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Project shipping

Some pictures


WWW. JUMBOSHIP.NL
JUMBO JAVELIN


QC-14


DOCKWISE YACHT TRANSPORT

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DOCKWISE
SUPER SERVANT 3

Project shipping - summary

- Heterogeneous fleet

- Cargoes
  - Mandatory and optional
  - Time windows
  - Coupled
  - Synchronized deliveries

Path flow models

Paths

- A path is a sequence of pickups and deliveries
- Capacity never violated
- Pickup visited before the corresponding delivery
- At least one feasible schedule exists (with respect to time windows)

Path flow model 1

$$\max z = \sum_{v \in V} \sum_{r \in R_v} P_{vr} y_{vr}$$

$\sum_{v \in V} \sum_{r \in R_v} A_{ivr} y_{vr} = 1$	$i \in N_C$	Mandatory cargoes
$\sum_{v \in V} \sum_{r \in R_v} A_{ivr} y_{vr} \leq 1$	$i \in N_O$	Optional cargoes
$\sum_{r \in R_v} y_{vr} = 1$	$v \in V$	Convexity
$\sum_{v \in V} \sum_{r \in R_v} A_{ivr} y_{vr} = w_{N_K}$	$i \in N_K, N_K \in K$	Coupled cargoes
$t_{jv} \geq t_{iv} + \sum_{r \in R_{ijv}} \left(T_{ijv} + T_{iv} + \bar{T}_i \right) y_{vr} - \bar{T}$	$i, j \in N, v \in V,$	Time
$\underline{T}_{ivr} A_{ivr} y_{vr} \leq t_{iv} \leq \bar{T}_{ivr} A_{ivr} y_{vr}$	$i \in N, v \in V,$	Time windows
$T_{N_S}^{S} \leq \sum_{v \in V} \left(t_{iv} - t_{jv} \right) \leq T_{N_S}^{S}$	$N_S \in S, i, j \in N_S$	Synchronization
$y_{vr} \in \{0,1\}$	$v \in V, r \in R_v$	
$w_{N_K} \in \{0,1\}$	$N_K \in K$	

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Path flow models

Schedule

- A schedule for a given path gives the exact time for start of service at each node on the path

Path flow model 2

$$\max z = \sum_{v \in V} \sum_{r \in R_v} P_{vr} y_{vrw}$$

$\sum_{v \in V} \sum_{r \in R_v} \sum_{w \in W_r} A_{ivr} y_{vrw} = 1$	$i \in N_C$	Mandatory cargoes
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$\sum_{w \in W_r} y_{vrw} \in \{0,1\}$	$v \in V, r \in R_v$	
$y_{vrw} \geq 0$	$v \in V, r \in R_v, w \in W_r$	
$w_{N_K} \in \{0,1\}$	$N_K \in K$	

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$y_{vrw} \geq 0$	$v \in V, r \in R_v, w \in W_r$	
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Model comparison

Path flow formulation 1

- One column per path
- Weaker LP-bound
- Duals related to nodes and arcs in the subproblems

Path flow formulation 2

- Many columns per path
- Stronger LP-Bound
- Duals related to nodes and visiting times in the subproblems

Solution Approach

Solution Approach

- A priori column generation (PF1)
  – Andersson et. al, 2011 *Ship routing and scheduling with cargo coupling and synchronization constraints.* Computers & Industrial Engineering 61(4) p. 1107 – 1116.
- Branch-and-price (PF1 and PF2)
  – Dynamic generation of columns
    - Elementary shortest path problems with resource constraints
    - Solved by Dynamic Programming

Subproblem

- Defined on a graph $G_v = (N_v, A_v)$
  - N_v consists of all pickup and delivery nodes that ship v may visit
  - A_v consists of all arcs that ship v can traverse
- Assumptions
  - Triangle inequality holds for both costs and travel times

Pricing problem PF1

$$max_{r \in R_v} = \sum_{(i,j) \in r} d_{ijv}$$

$$d_{ijv} = -C_{ijv} + (T_{ijv} + \overline{T}_i)\eta_{ijv} + \begin{cases} R_i - \alpha_i - \sum_{k=1}^{|\mathcal{K}|} \gamma_{ik} - \underline{T}_i \underline{\sigma}_{iv} + \overline{T}_i \overline{\sigma}_{iv} & \\ -\underline{T}_{(i+n)} \underline{\sigma}_{(i+n)v} + \overline{T}_{(i+n)} \overline{\sigma}_{(i+n)v} & \text{if } i \in \mathcal{N}^P \\ -\beta_v & \text{if } i = o(v), \end{cases}$$

Pricing problem PF2

$$max_{r \in R_v} = \sum_{(i,j) \in r} d_{ijv} + \tau(r)$$

$$d_{ijv} = -C_{ijv} + \begin{cases} R_i - \alpha_i - \sum_{k=1}^{|\mathcal{K}|} \gamma_{ik} & \text{if } i \in \mathcal{N}^P, \\ -\beta_v & \text{if } i = o(v). \end{cases}$$

Calculating optimal schedule

$$\tau(r) = \max \sum_{(i,j)\in r} \delta_i t_{iv}$$

subject to

$$t_{iv} + T_{ijv} - t_{jv} \leq 0, \qquad \forall (i,j) \in r$$

$$\underline{T_i} \leq t_{iv} \leq \overline{T_i} \qquad \forall (i,j) \in r$$

Cannot be calculated exactly until path is completed

Dominance for PDPTW

- Røpke and Cordeau (2009)
- Label L_1 dominates L_2 if:

$\eta(L_1) = \eta(L_2)$	Same node
$t(L_1) \leq t(L_2)$	Less time
$c(L_1) \leq c(L_2)$	Less cost
$V(L_1) \subseteq V(L_2)$	Subset of cargoes picked up
$O(L_1) \subseteq O(L_2)$	Subset of cargoes onboard

Dominance for PF1

- Label L_1 dominates L_2 if:

$$\eta(L_1) = \eta(L_2)$$
$$t(L_1) \leq t(L_2)$$
$$c(L_1) \geq c(L_2)$$
$$V(L_1) \subseteq V(L_2)$$
$$O(L_1) = O(L_2) \text{ if } \exists\, \eta_{ijv} \neq 0$$
$$O(L_1) \subseteq O(L_2) \text{ otherwise}$$

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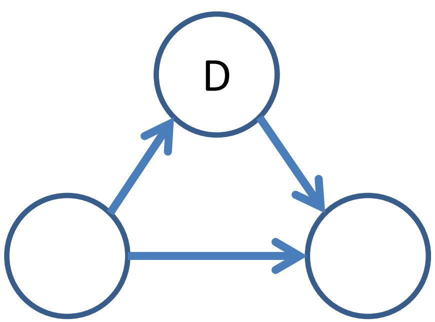

Dominance for PF2

- Label L_1 dominates L_2 if:

$$\eta(L_1) = \eta(L_2)$$
$$t(L_1) \leq t(L_2)$$
$$c(L_1) + \underline{\tau}(L_1) \geq c(L_2) + \overline{\tau}(L_2)$$
$$V(L_1) \subseteq V(L_2)$$
$$O(L_1) \subseteq O(L_2)$$

Dominance for PF2

- Label L_1 dominates L_2 if:

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$$t(L_1) \leq t(L_2)$$
$$c(L_1) + \underline{\tau}(L_1) \geq c(L_2) + \overline{\tau}(L_2)$$
$$V(L_1) \subseteq V(L_2)$$
$$O(L_1) \subseteq O(L_2)$$

Branching

- Hierarchical branching strategies
  1. Whether a cargo is picked up or not
  2. A given cargo is transported by a given ship
  3. Branching on arc flow

Computational Study

Test instances

- Instances extracted from real life data
  - 20 – 32 cargoes
  - 4 ships
  - 4 – 8 pairs of coupled and synchronized cargoes
- Three test cases
  - A: Original case from real shipping company
  - B: More coupled and synchronized cargoes
  - C: Time buffer = 0

Computational results 1:3

Instance	PF1 - PreGen gen. time	master time	total time	PF1 time	difference	PF2 time	difference
20.A	5696	343	6039	464	-92 %	11	-100 %
20.B	6520	395	6915	275	-96 %	10	-100 %
20.C	112	35	147	66	-55 %	10	-93 %
22.A	19261	607	19868	1856	-91 %	128	-99 %
22.B	23091	720	23811	1417	-94 %	323	-99 %
22.C	345	79	424	96	-77 %	10	-98 %
24.A	38338	1348	39686	142	-100 %	16	-100 %
24.B	49107	1719	50826	134	-100 %	19	-100 %
24.C	1467	247	1714	238	-86 %	19	-99 %
26.A	81942	1199	83141	200	-100 %	26	-100 %
26.B	95600	1961	97561	236	-100 %	51	-100 %
26.C	9049	540	9589	569	-94 %	29	-100 %
Average	27544	766	28310	474	-98 %	54	-100 %

Computational results 2:3

	PF1	PF2	
instance	time	time	difference
28.A	670	88	-87 %
28.B	1013	87	-91 %
28.C	344	75	-78 %
30.A	7327	8370	14 %
30.B	8494	1057	-88 %
30.C	34304	405	-99 %
32.A	7581	927	-88 %
32.B	8302	1407	-83 %
32.C	>36000	433	-INF
Avg	11559	1428	

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Computational results 3:3

	PF1			PF2		
instance	% in sub	nodes	columns	% in sub	nodes	columns
28.A	33 %	27	2843	100 %	7	721
28.B	32 %	37	3572	99 %	7	806
28.C	32 %	31	2068	99 %	21	942
30.A	68 %	1441	8281	100 %	35	1335
30.B	69 %	1481	8693	100 %	43	1430
30.C	22 %	35919	21129	99 %	103	1755
32.A	74 %	463	6091	100 %	23	1168
32.B	73 %	483	6632	100 %	55	1266
32.C	N/A	N/A	N/A	97 %	267	2357
Average	51 %	4985	7414	99 %	62	1309

Summary

- A priori generation of paths is time consuming and not possible for larger instances
- Calculating service times in the subproblems works better than calculating them in the master problem