TAKING ADVANTAGE OF DEGENERACY IN MATHEMATICAL PROGRAMMING

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and many students and post-docs

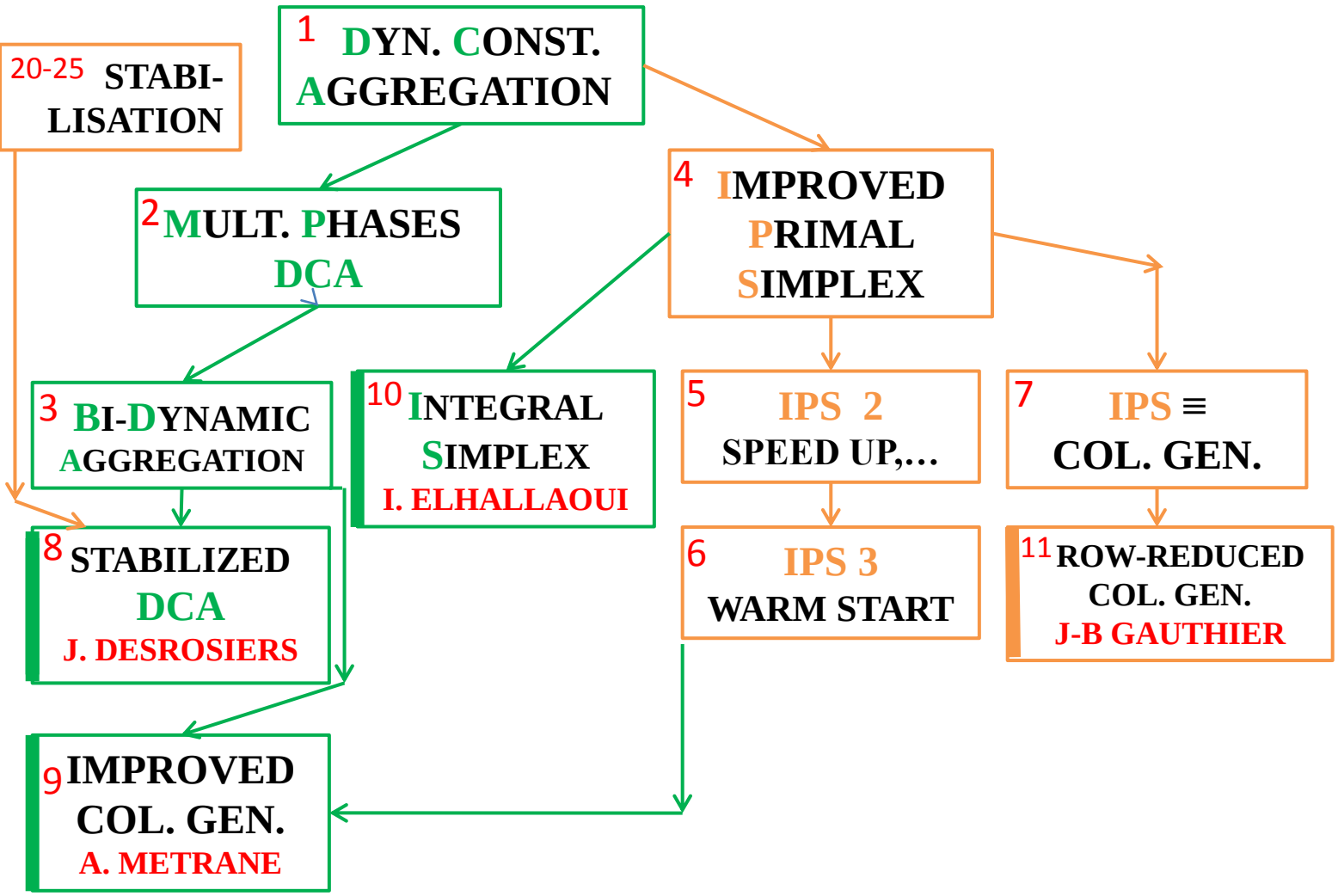
OVERVIEW

- THE TEAM PRESENS 4 OTHERS TALKS IN THIS RESEARCH FIELD
  - ROW REDUCED COLUMN GENERATION J.-B. GAUTHIER, …
  - INTEGRAL SIMPLEX USING DECOMPOSITION I. ELHALLAOUI….
  - IMPROVED COLUMN GENERATION … A. METRANE,…
  - STABILIZED DYNAMIC CONSTRAINT AGGREGATION… J. DESROSIERS,…
- I START WITH A MOTIVATION OF THIS RESEARCH FIELD
- I PRESENT A SYNTHESIS OF THE PREVIOUS WORKS OF THE TEAM IN THIS FIELD
- I GIVE THE RELATIONS OF THE 4 TALKS WITH THE PREVIOUS WORKS
- I PRESENT A VIEW OF ON GOING WORKS
- I GIVE THE LIST OF THE PAPERS

MOTIVATION OF THIS RESEARCH FIELD

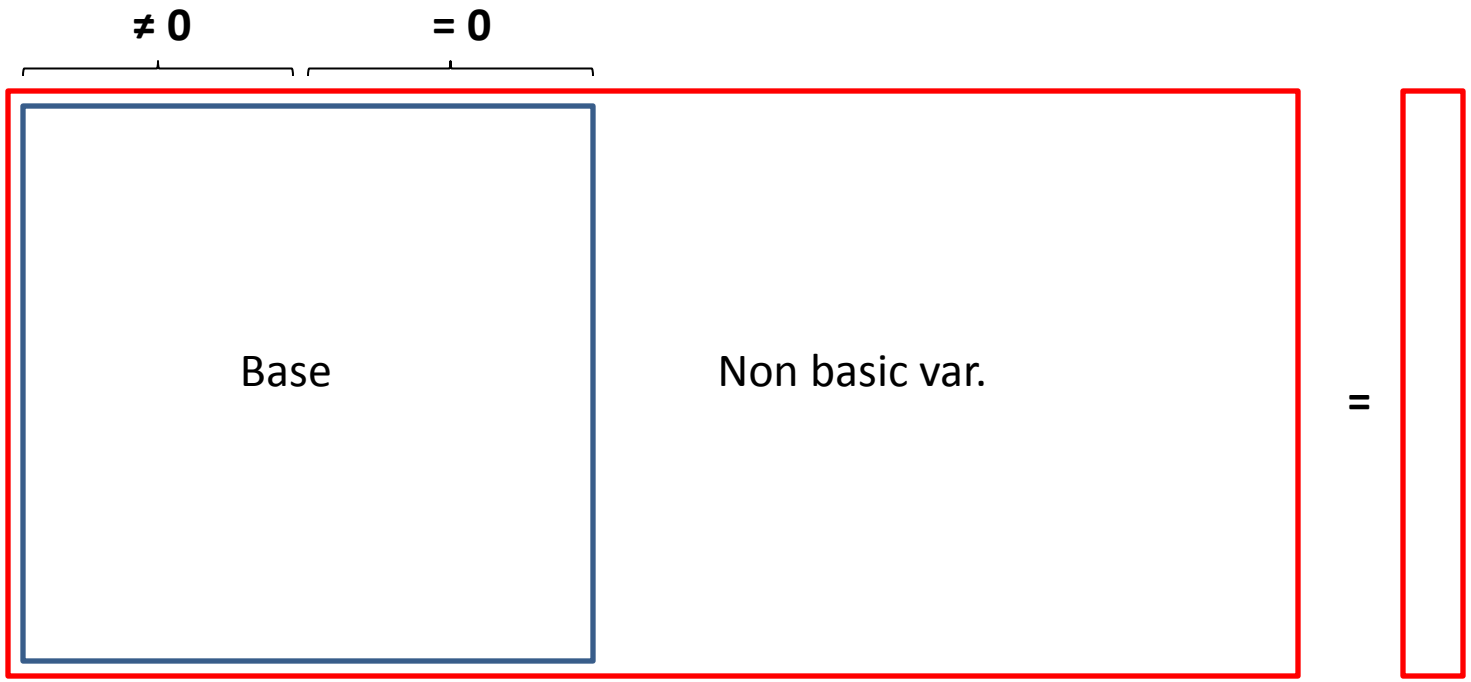
- POINTS TO IMPROVE IN THE PRIMAL SIMPLEX
  - AVOID THE EFFECT OF DEGENERACY
    - PERTURBATION AVOID CYCLING BUT PRODUCES SMALL STEP SIZE
    - INITIAL INTEGER SOLUTION CREATES DEGENERACY AND SLOW DOWN THE PROCESS
  - WORK WITH A SUBSET OF CONSTRAINTS
    - COLUMN GENERATION WORK WITH A SUBSET OF VARIABLES
    - THE NUMBER OF CONTRAINTS HAS MORE EFFECT ON CPU TIME THAN NUMBER OF VARIABLES
  - START WITH INITIAL SOLUTION AND TAKE ADVANTAGE OF DEGENERACY TO REDUCE THE NUMBER OF CONSTRAINTS

PREVIOUS WORKS



SET PARTITIONING LINEAR PROGRAMING MATH PROG.

IPS ON A DEGENERATED L P



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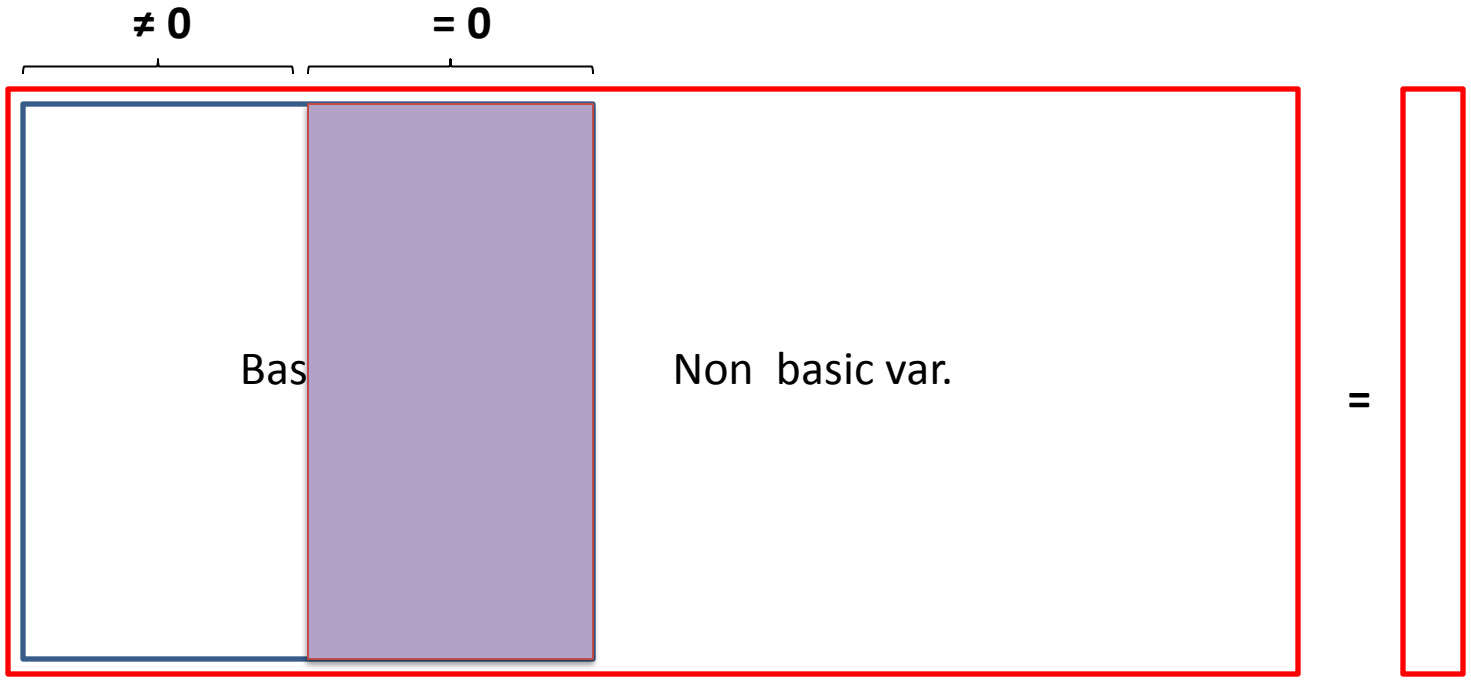
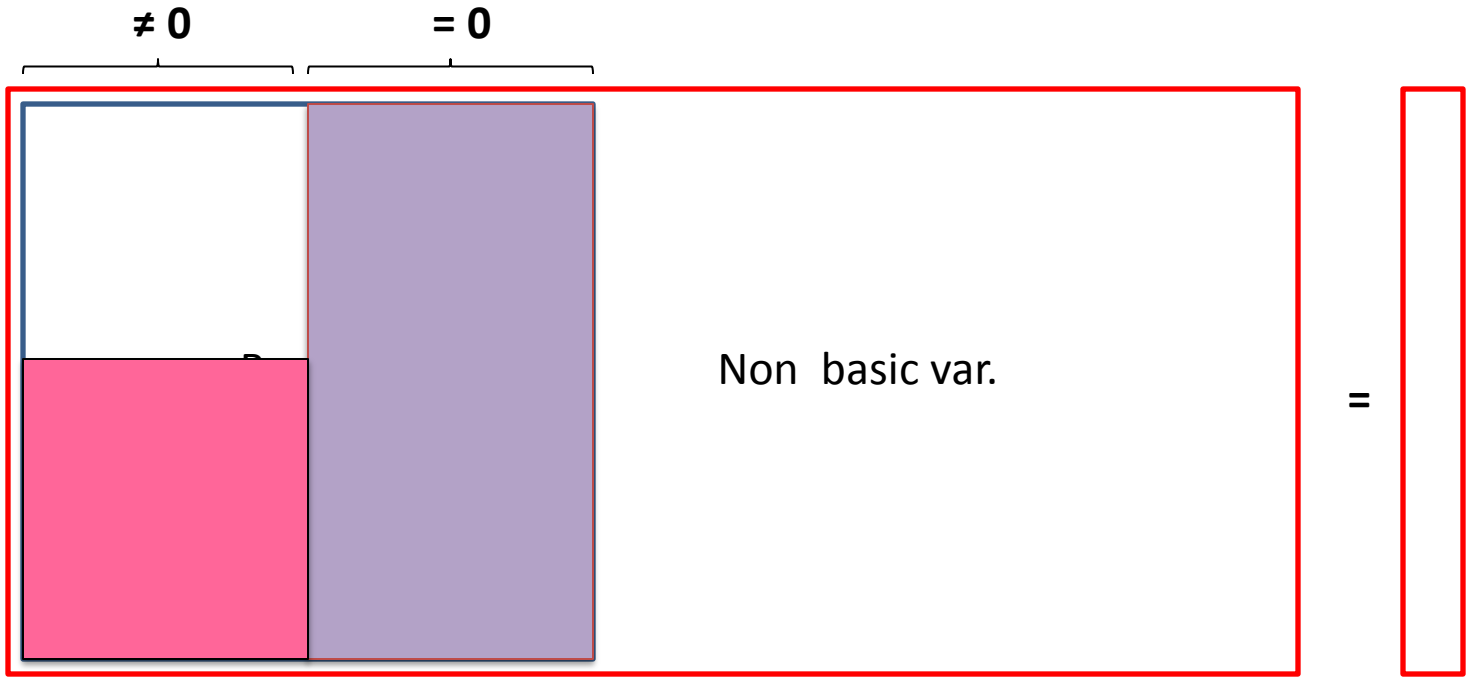


- Remove the degenerate variables from the base

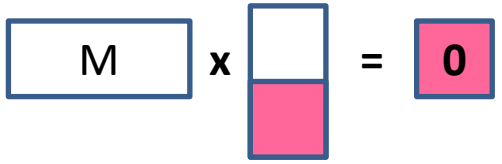
IPS ON A DEGENERATED L P

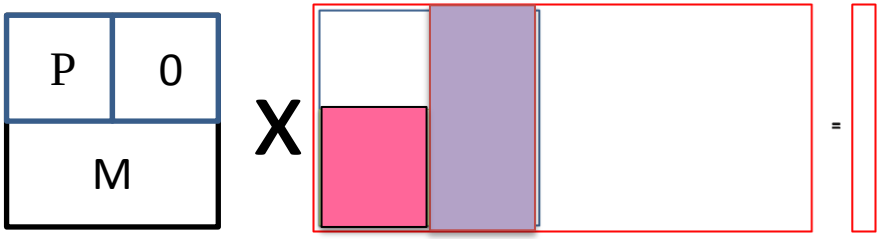


- Remove the degenerate variables from the base
- Remove the dependant rows

REDUCED AND COMPLEMENTARY PROBLEMS

M: a compatibility matrix





Compatible var.

Incompatible var.

Reduced problem: RP

0

Redondant rows

Complementary problem: CP

1111. . . 111

= $\bar{b}$

= 0

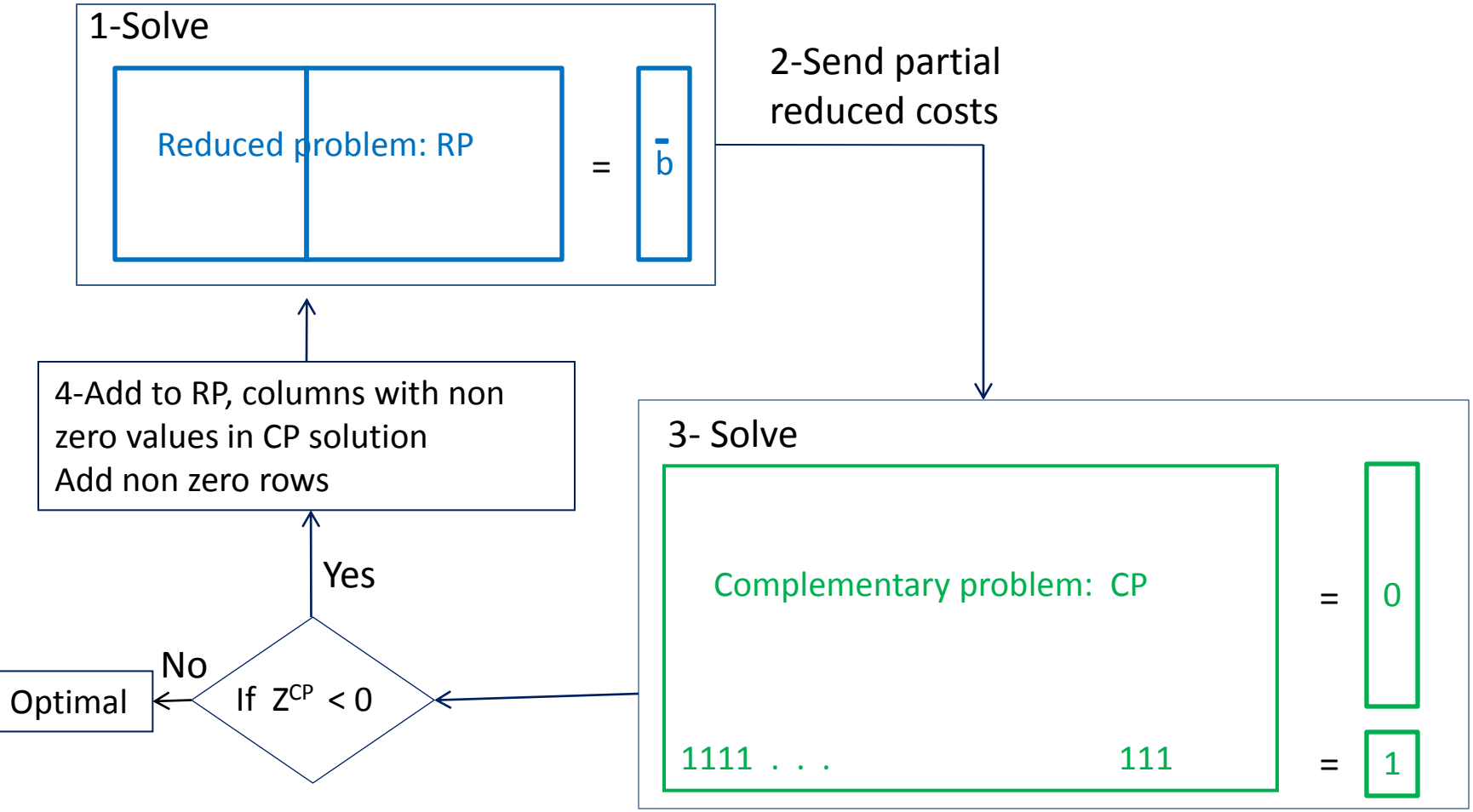
= 1

P=set of non degenerated rows

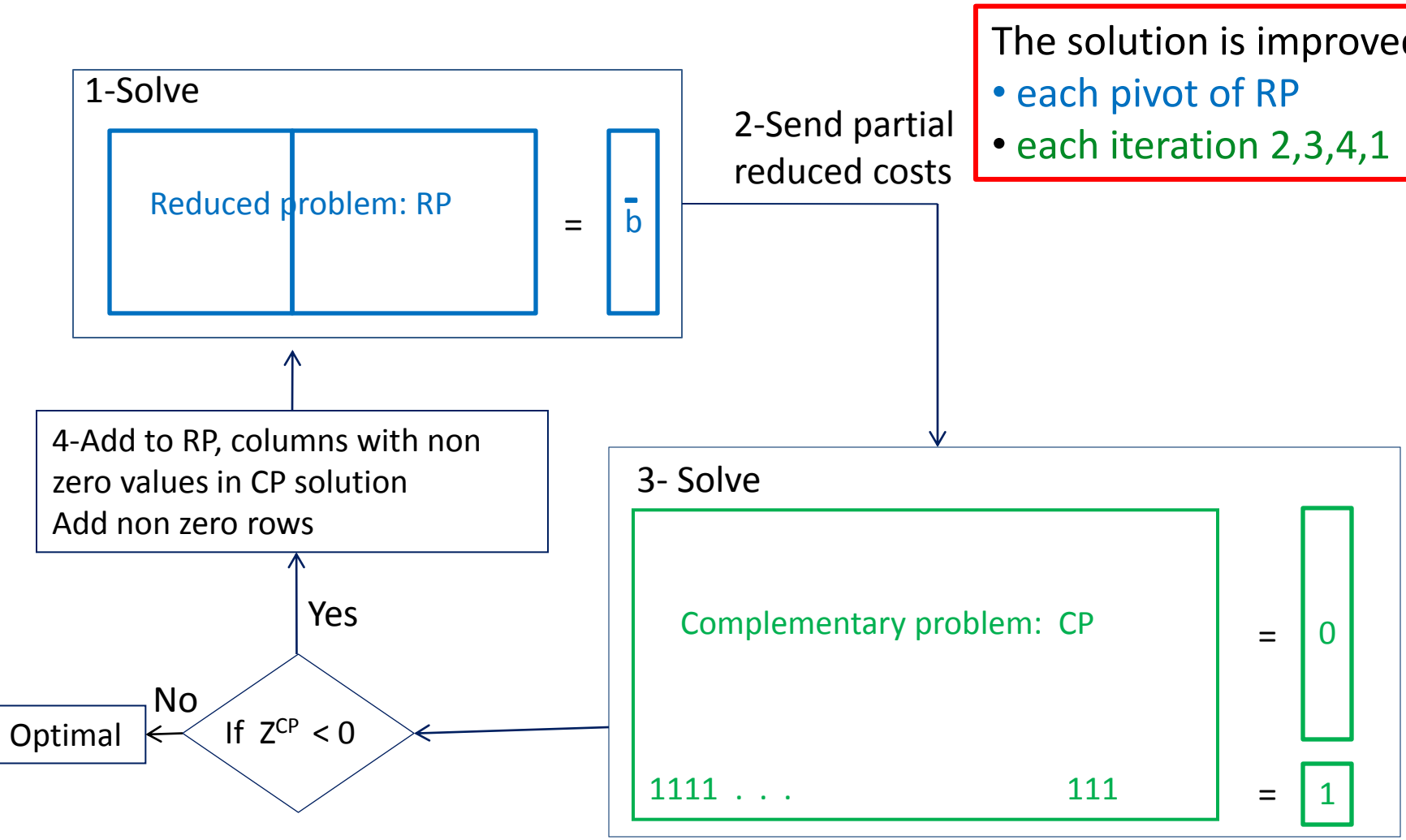
Z=set of degenerated rows

Compatible var.: $(M\ A_j) = \bar{A}_j^z = 0$

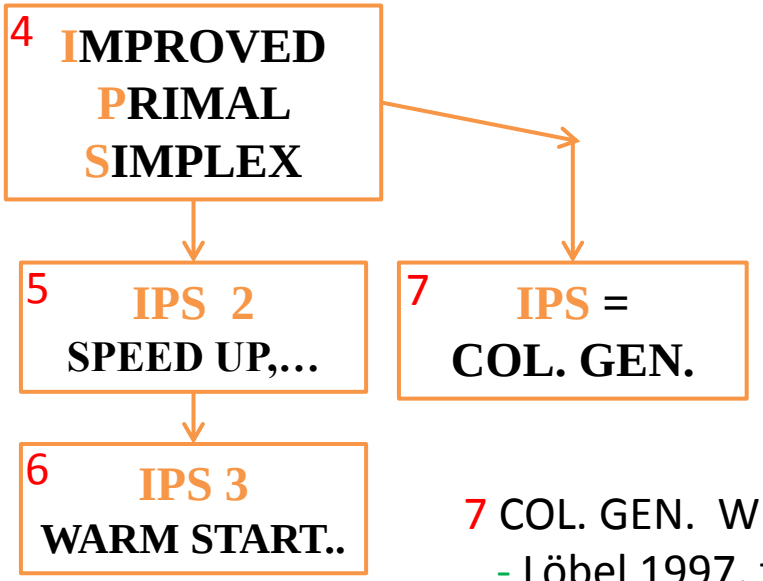
IMPROVED PRIMAL SIMPLEX



IMPROVED PRIMAL SIMPLEX



SUPLEMENTARY RESULTS



5 -USE PRESOLVE AND DUAL SIMPLEX,

-SOLVE CP MANY TIME AT EACH ITERATION, …

6 -WARM START, USE M = [| I] ,

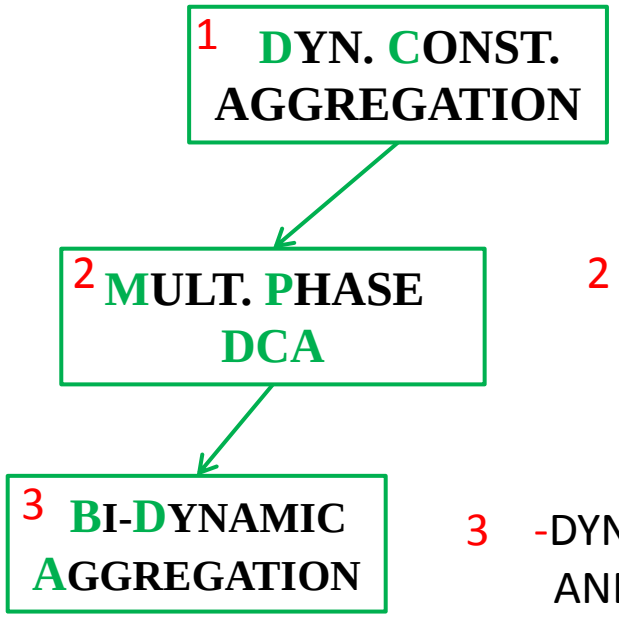
- REMOVE DEGENERATED CONST. AT UPPER BOUND

7 COL. GEN. WITH ORIGINAL VARIABLES IN THE MASTER PROBLEM

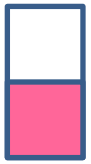
- Löbel 1997, for Vehicle Routing Problem
  - Subproblem = shortest path
  - Add arcs of the shortest path in the master problem
- Degenerated const. are suppressed in the M.P., they are redundant
- Some of them are added to M.P. when incompatible variables are added. They are removed in S.P.

IPS is a Column Generation on the original formulation modified dynamically

DCA and IMPROVEMENTS



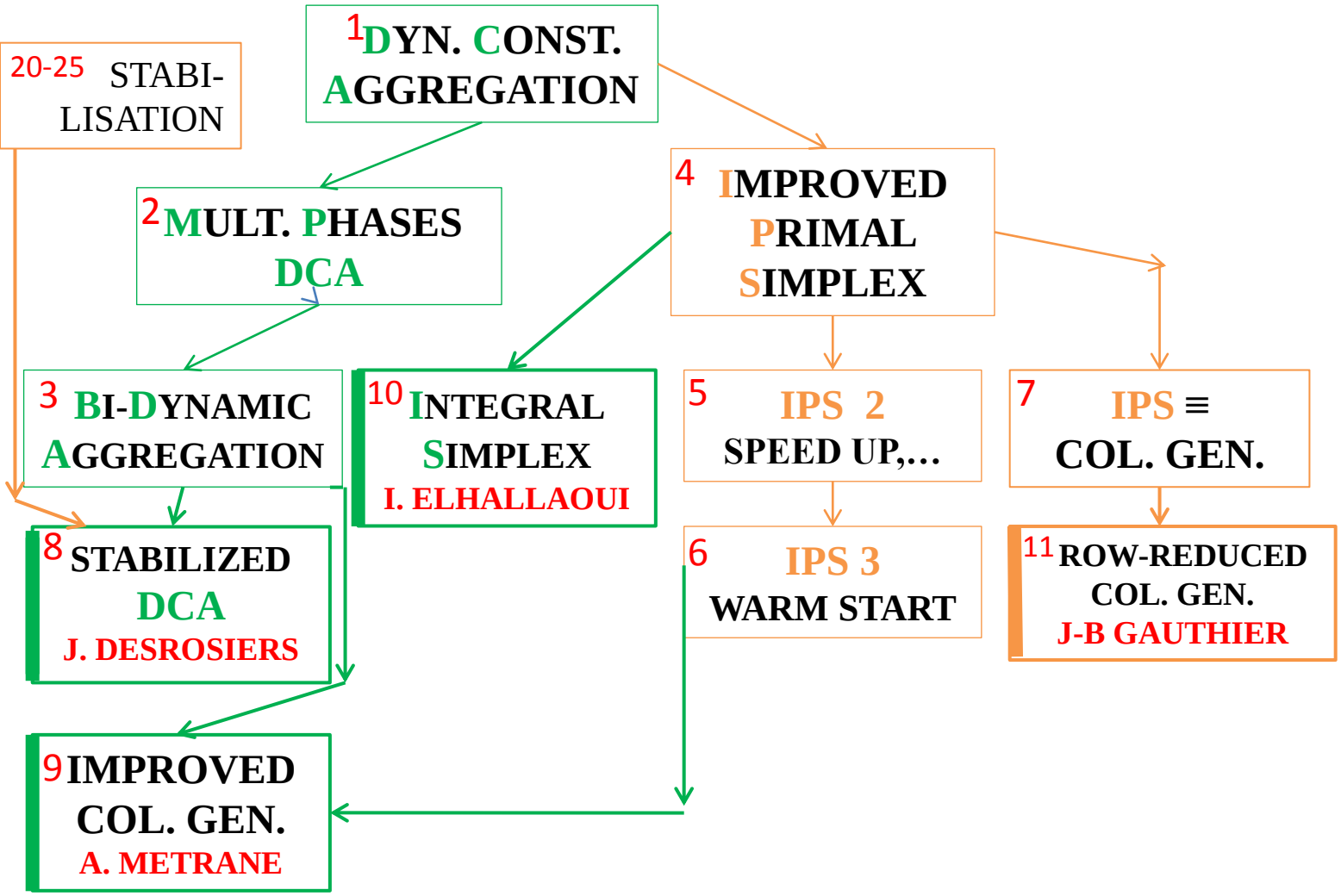
1 -REMOVE THE DEGENERATE VARIABLES
-REMOVE IDENTICAL ROWS $\subset$ DEPENDENT ROWS
-SPECIALIZED PROCEDURES REPLACE CP
-INTEGRATED IN GENCOL



2 -PARTIAL PRICING IN THE SUBPROBLEM
-SELECT IN PHASE k COLUMNS WITH $|\bar{A}_j^z|^1 \leq k$
-REDUCES CPU TIME BY 4 to 50

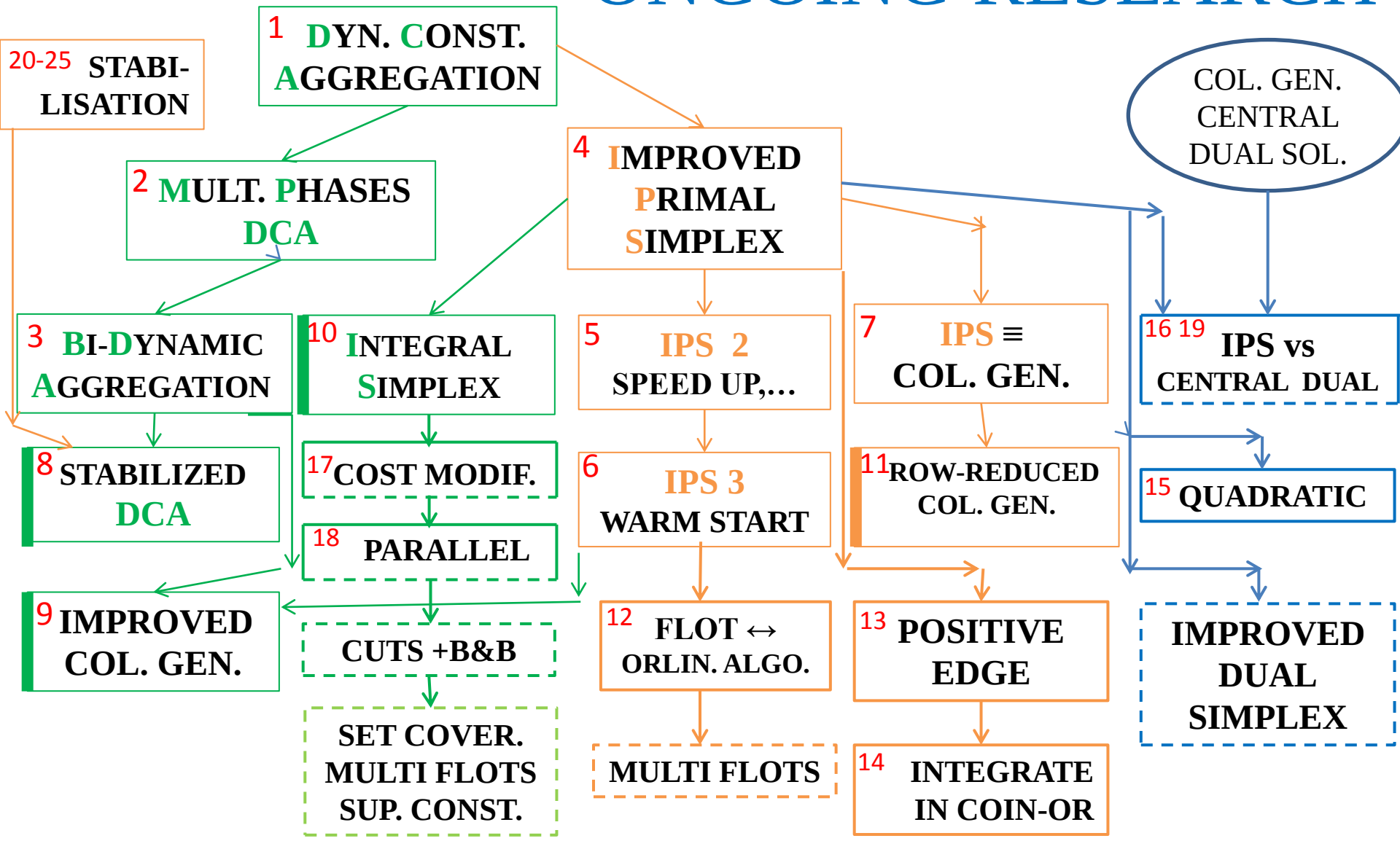
3 -DYNAMIC AGGREGATION IN THE MASTER PROBLEM AND IN THE SUBPROBLEM
-REDUCES CPU TIME UP TO 100

IN THIS CONFERENCE

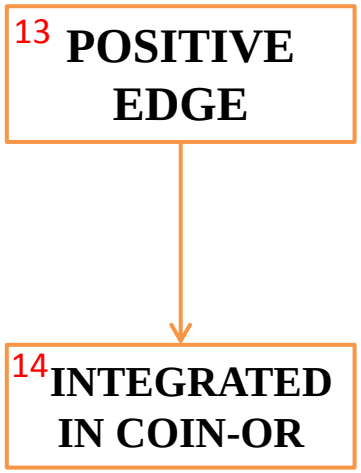


SET PARTITIONING LINEAR PROGRAMING MATH. PROG.

ONGOING RESEARCH



NEW PRICING RULE
SELECTING NON DEGENERATE PIVOT



13 – SELECT FIRST THE COMPATIBLE VARIABLES

- M x A USES TOO MUCH TIME FOR LARGE PROBLEMS
  (10^{15} OPERATIONS WITH A (100 000 x 400 000))
- POSITIVE EDGE RULE WORKS DIRECTLY ON A_J
  STOCHASTIC TEST WITH **P** (error) < 2^{20}
  (SAME COMPLEXITY THAN COMPUTING A REDUCE COST)

14 – REDUCES CPU TIME BY 3 to 5

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APPLICATIONS

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