Column generation for the robust rescheduling of resources

Dennis Huisman

Co-authors: Luuk Veelenturf, Daniel Potthoff, Leo Kroon, Gábor Maróti & Albert Wagelmans

Dennis Huisman

Econometric Institute & ECOPT, Erasmus Univ. Rotterdam
Process quality & Innovation, Netherlands Railways

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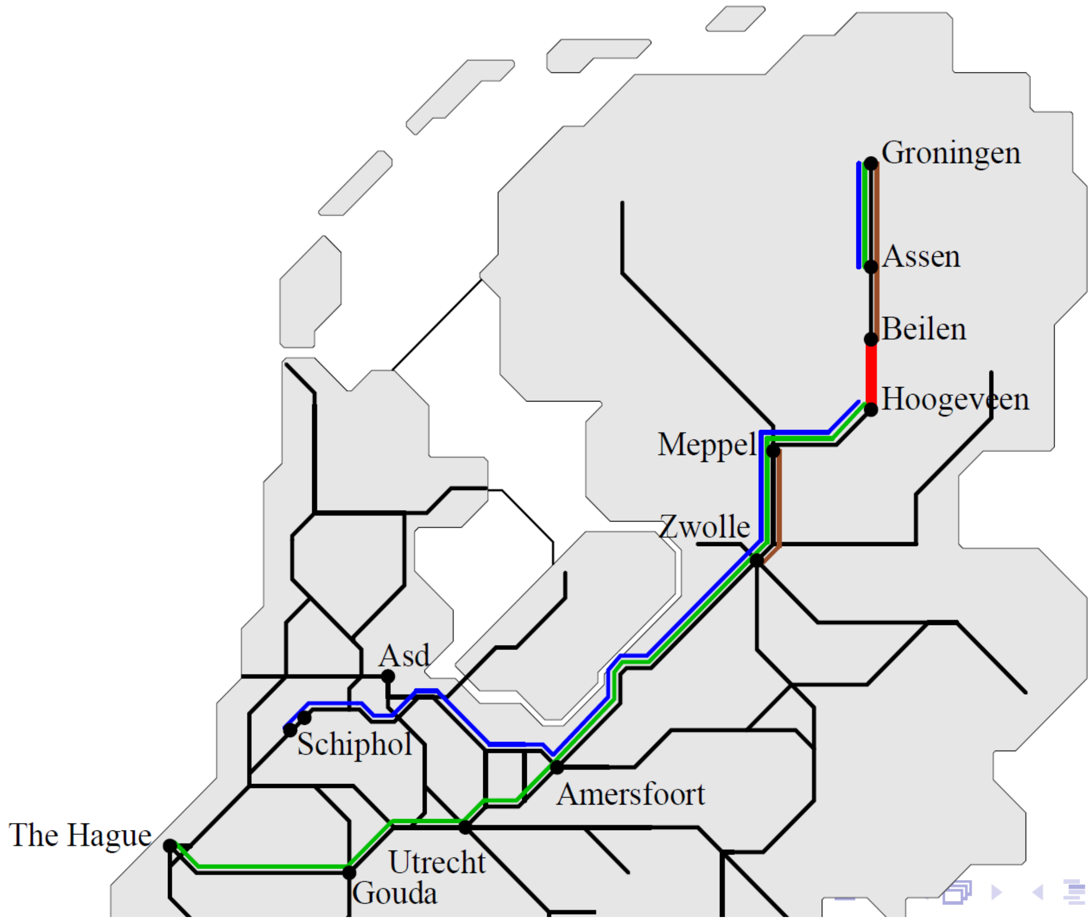
Disruptions happen every day …

Beilen – Hoogeveen

Monday 10 September 07:10

Reason	Due to broken overhead lines no train traffic is possible between Beilen and Hoogeveen.
Expectation	Passengers should take into account an extra travel time of approximately 30 minutes. The disruption is expected to last until 10:00.
Travel advice	Shuttle buses are available at stations Beilen and Hoogeveen.

… and a lot when it snows …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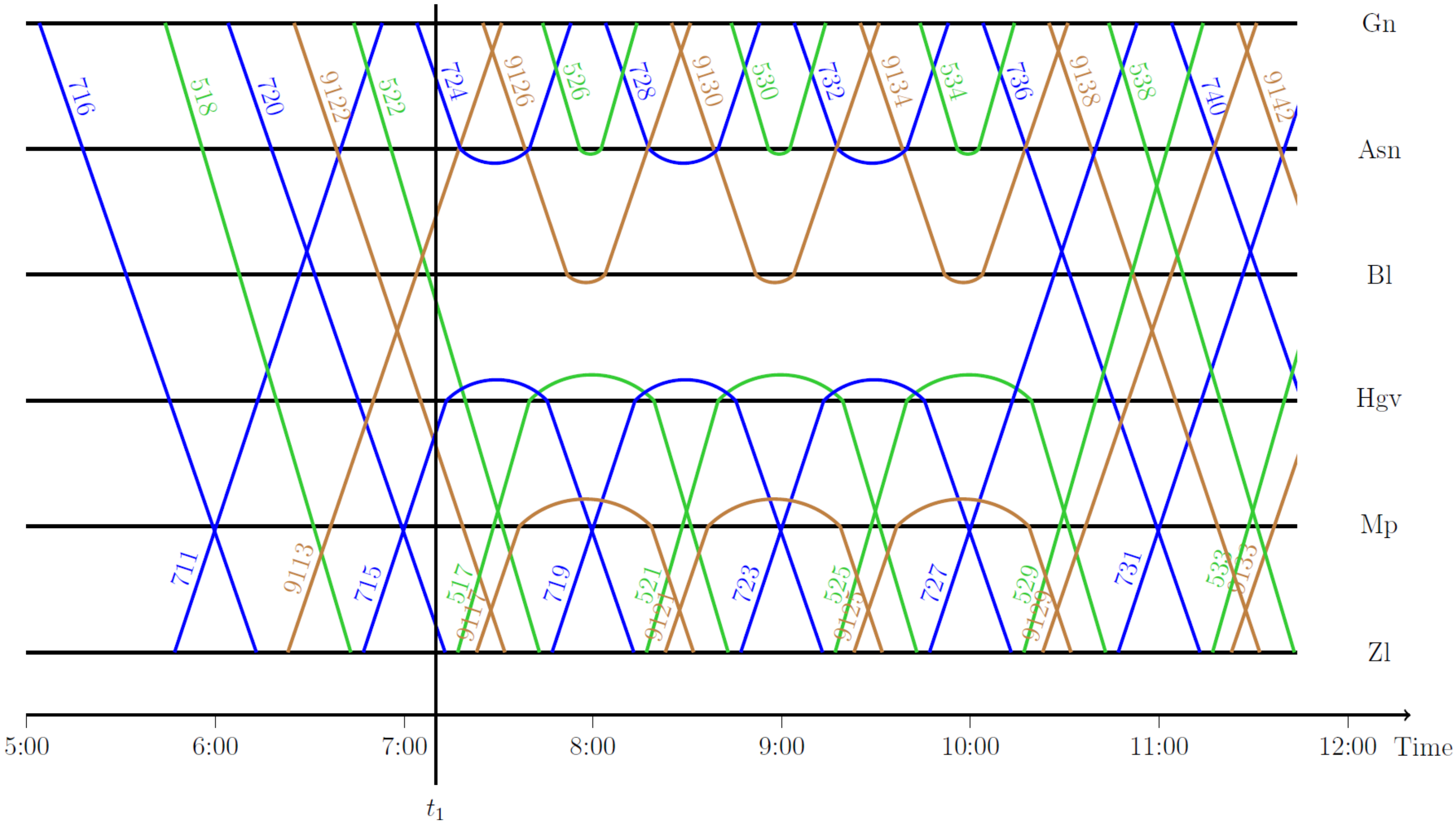
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6:00
7:00
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9:00
10:00
11:00
12:00
Time
t_1

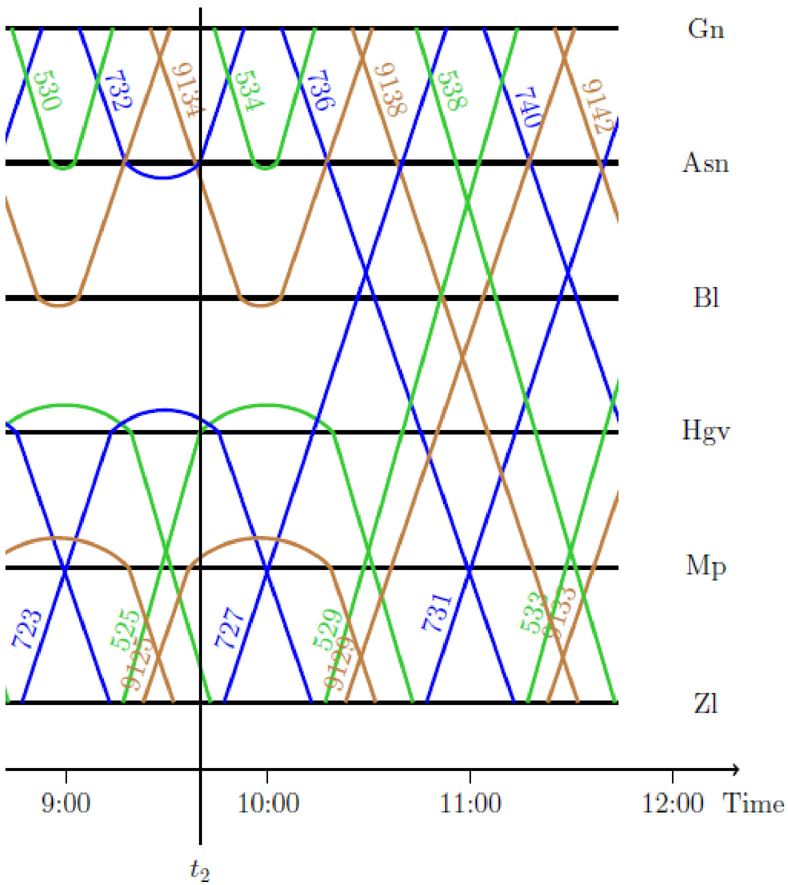
Uncertainty in duration

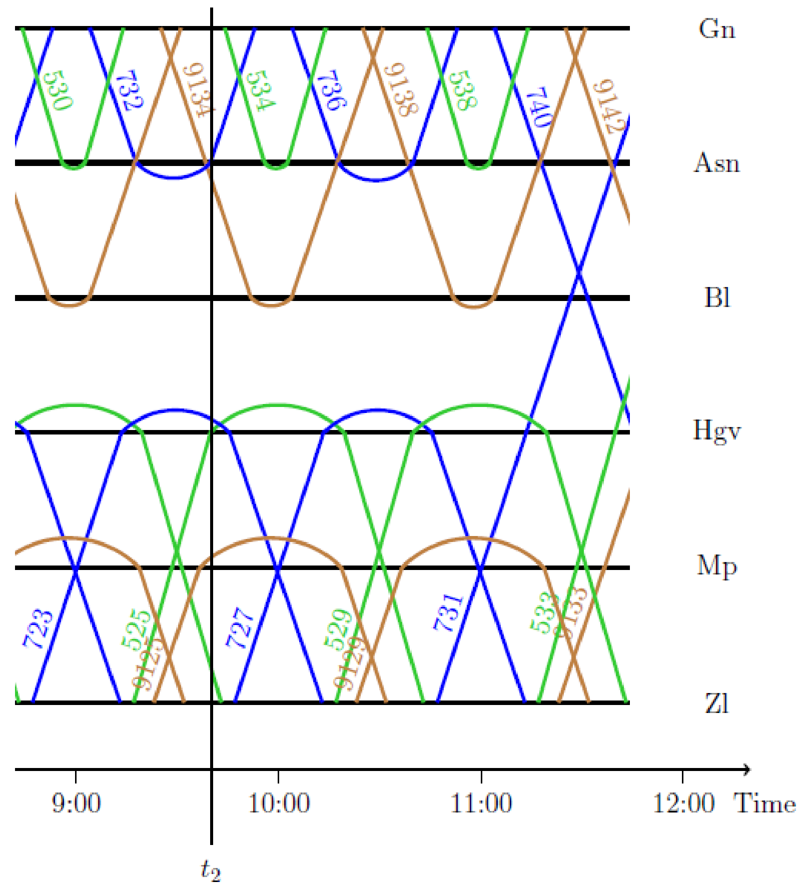
Beilen – Hoogeveen

Monday 10 September 07:10, **update at 9:40**

Reason	Due to broken overhead lines no train traffic is possible between Beilen and Hoogeveen.
Expectation	Passengers should take into account an extra travel time of approximately 30 minutes. The disruption is expected to last until **11:00**.
Travel advice	Shuttle buses are available at stations Beilen and Hoogeveen.

Possible scenarios





Rescheduling problems in general

- Schedule a set of timetabled *tasks* on a certain number of servers.
- A *duty* is a sequence of tasks on the same server.

- Rescheduling problem (RSP):

  Modify the duties due to a certain disruption such that:
  - As many as tasks as possible are covered by a server.
  - The modifications in the schedule are minimal.

- *Completion* of a duty is the new feasible sequence of tasks from the start of the disruption to the end of the duty.

Notation

Δ: set of unfinished original duties

N: set of tasks

K^{δ}: set of all feasible completions for original duty δ

a^{δ}_{ik} = 1, if completion k for original duty δ contains task i
 0, otherwise

c^{δ}_{k}: cost of duty k for driver δ

f_i: cost for not covering task *i*

Decision variables:

x^{δ}_{k} = 1, if completion k for original duty δ is selected
 0, otherwise

z_i = 1, if task i is uncovered
 0, otherwise

Mathematical Formulation Rescheduling Problem

$$\min \quad \sum_{\delta \in \Delta} \sum_{k \in K^\delta} c_k^\delta x_k^\delta + \sum_{i \in N} f_i z_i \tag{1}$$

$$\text{s.t.} \quad \sum_{\delta \in \Delta} \sum_{k \in K^\delta} a_{ik}^\delta x_k^\delta + z_i \geq 1 \qquad \forall i \in N \tag{2}$$

$$\sum_{k \in K^\delta} x_k^\delta = 1 \qquad \forall \delta \in \Delta \tag{3}$$

$$x_k^\delta, z_i \in \{0,1\} \quad \forall \delta \in \Delta, \forall k \in K^\delta, \forall i \in N \tag{4}$$

Rescheduling under uncertainty (1)

- Time t_1: disruption starts, and estimates about the duration are available (limited set of scenarios)
- h_1 : optimistic estimate of the duration known at t_1
- Time t_2: new information about the duration of the disruption available, disruption ends at t_3 ($t_3 \geqq t_1 + h_1$)

- Naïve approach:
  - Stage 1: reschedule duties at time t_1 based on the optimistic scenario
  - Stage 2: reschedule duties again at time t_2 if duration takes longer

Rescheduling under uncertainty (2)

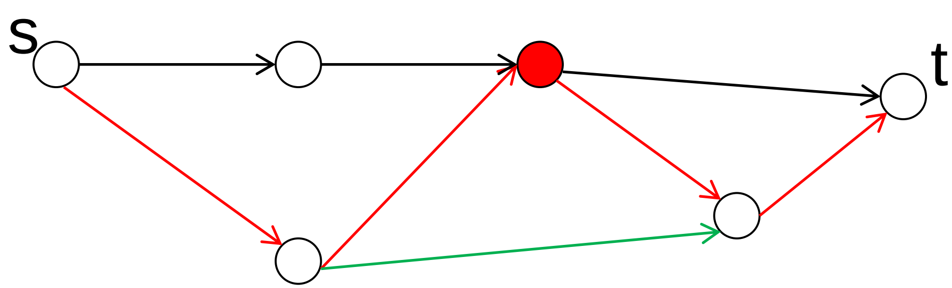
- Disadvantage naïve approach:
  - High risks that duties become infeasible when optimistic scenario is not realized -> additional tasks will be uncovered in stage 2

- To overcome this disadvantage, we suggest a *quasi robust solution approach*

- Main idea is that in the first stage, we prefer to choose a duty that is still feasible when all other scenarios (including the pessimistic) one occurs.

Definitions

Definition *A feasible completion k of a duty δ is called* ***recoverable robust*** *if there exists a recovery alternative γ_s of duty δ for all scenarios $s \in S$.*



Definition *A schedule is called* ***quasi robust*** *if $q > 0$ duties are recoverable robust.*

Mathematical Formulation
Quasi-robust Rescheduling Problem

- Add constraint:

$$\sum_{\delta \in \Delta} \sum_{k \in R^{\delta}} x_k^{\delta} \geq q$$

  where R^{δ} is the subset of all feasible (recoverable) robust completions for original duty δ

- Note that if $q = 0$, we have the naïve approach and if $q = |\Delta|$, then all duties must have a robust feasible completion.

Solution Approach

- We extend the approach from Potthoff et al. (2010) that was developed to solve the Operational Crew Rescheduling Problem
- Potthoff et al. (2010) uses a column generation algorithm combined with a Lagrangian heuristic

- Modifications in:
  - Restricted master problem (trivial)
  - Pricing problem (modify the graph in a preprocessing step)

D. Potthoff, D. Huisman and G. Desaulniers, "Column Generation with Dynamic Duty Selection for Railway Crew Rescheduling", *Transportation Science* (2010).

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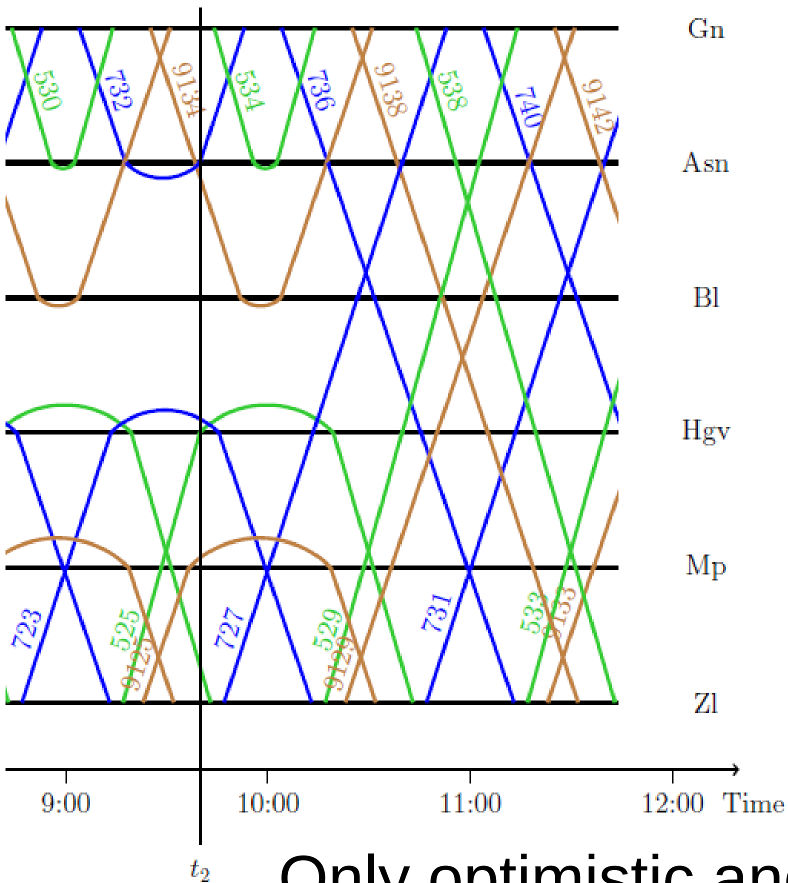
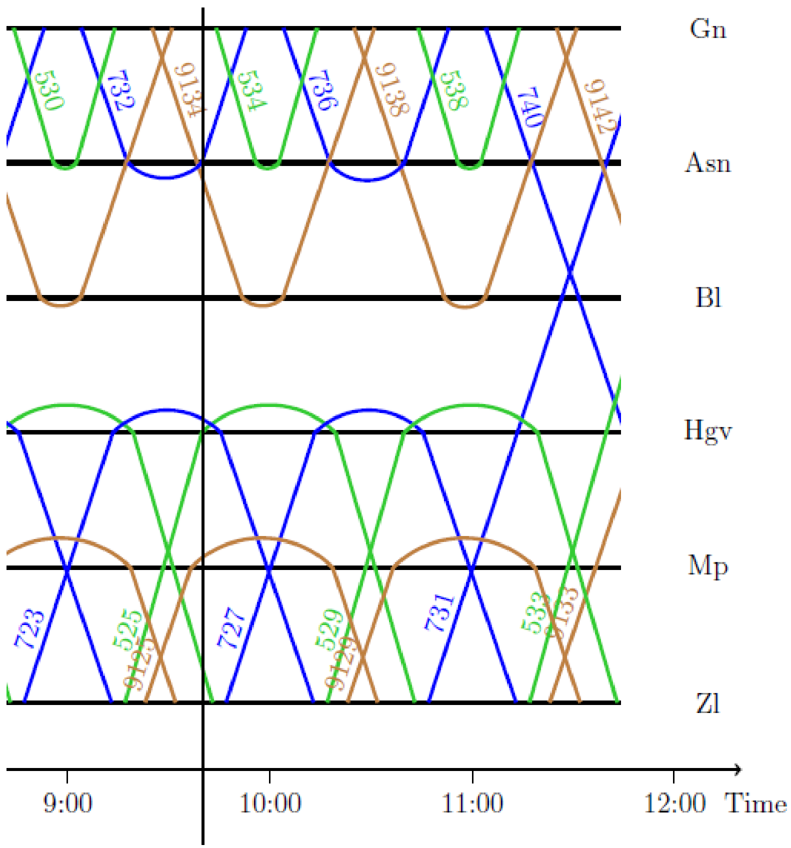
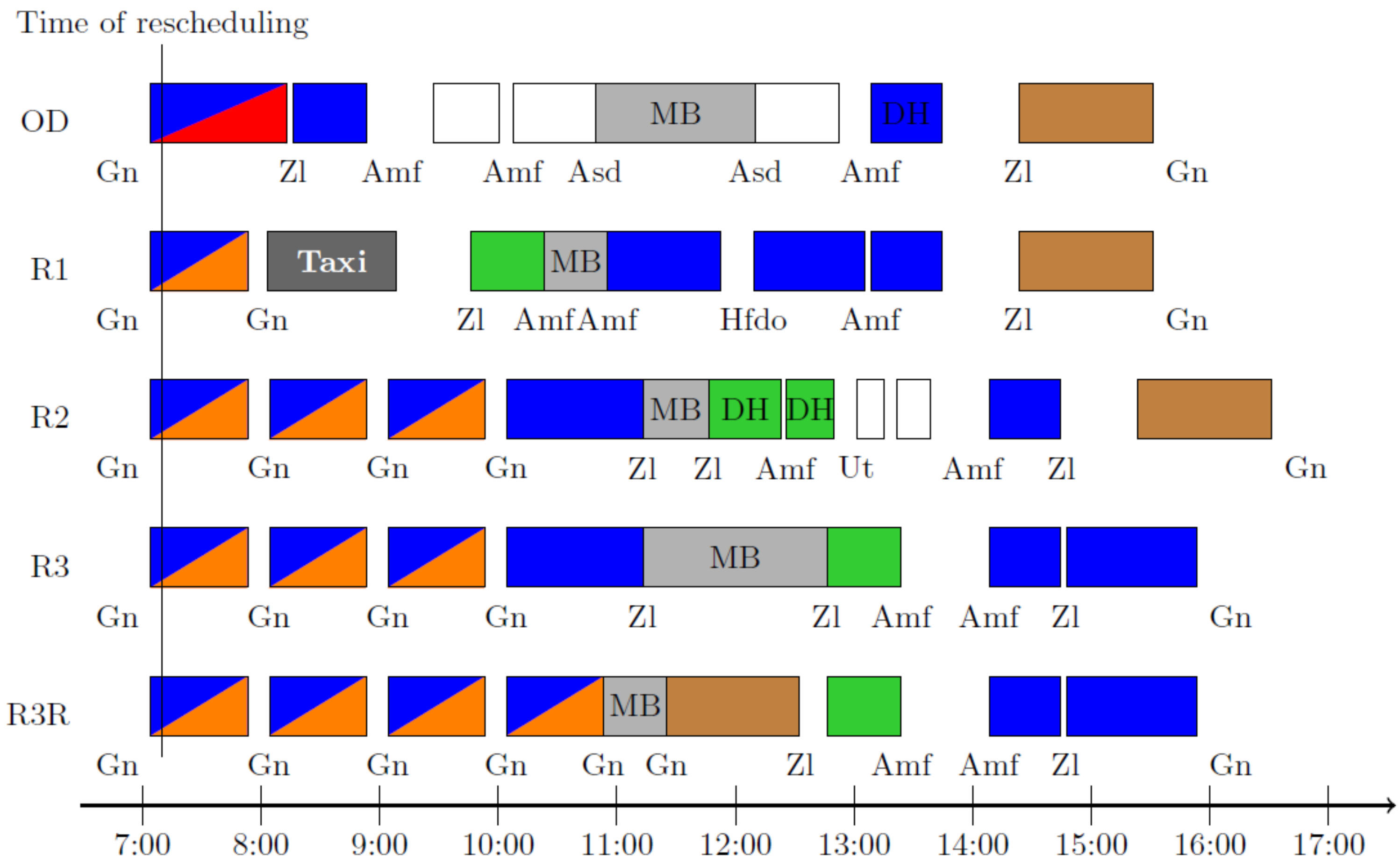
Return to the application …





Only optimistic and pessimistic scenario

Feasible completions of a duty



Computational results (1)

- Test instances are based on disruptions in the past on the Dutch railway network
- For most of these practical instances, the naïve approach works fine (no cancellations). However, sometimes tasks need to be cancelled in stage 2.
- However, when rules are tightened (no standby duties, no overtime allowed) the naïve approach performs worse.
- We illustrate the benefits on two instances with tightened rules.

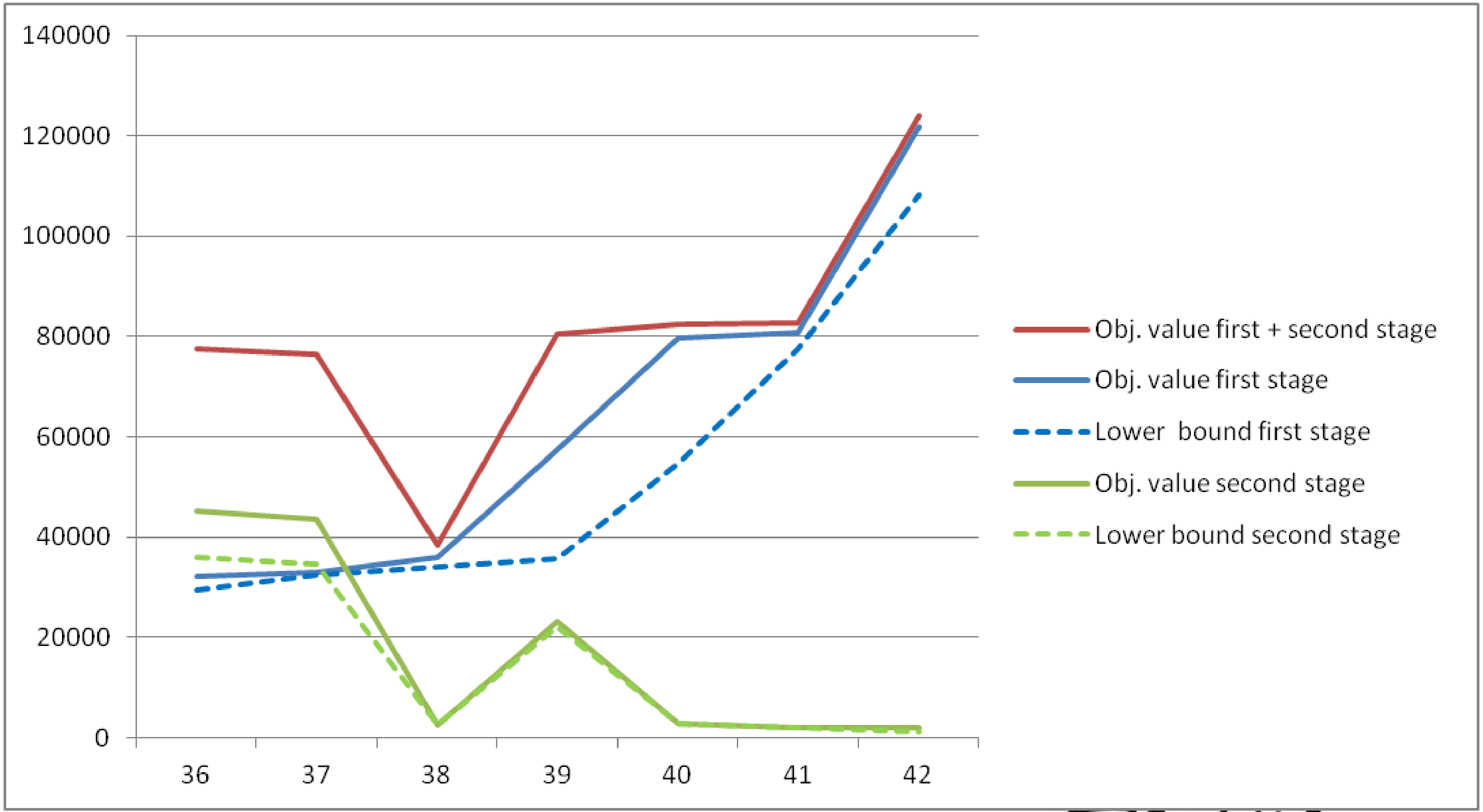
Computational results (2)

Disruption Beilen-Hoogeveen (42 original duties)

q	Uncovered tasks stage 1	Uncovered tasks stage 2	Uncovered tasks total	Cpu (sec.)
0-36	1	2	3	17
37	1	2	3	16
38	1	0	1	18
39	2	1	3	18
40	3	0	3	19
41	3	0	3	16
42	5	0	5	17

Computational results (3)

Disruption Beilen-Hoogeveen (42 original duties)



Computational results (4)

Disruption round ‘s Hertogenbosch (98 original duties)

q	Uncovered tasks stage 1	Uncovered tasks stage 2	Uncovered tasks total	Cpu (sec.)
0-85	5	2	7	142
86-87	5	3	8	149
88-89	5	1	6	153
90-91	5	3	8	154
92	6	3	9	190
93-94	5	4	9	162
95	5	3	8	185
96	5	5	10	160
97	6	0	6	192
98	7	0	7	191

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Computational results (5)

Disruption round 's Hertogenbosch (98 original duties)

Conclusions

- Results demonstrate that quasi robust solution approach performs better than naïve approach.
- When a conservative choice of *q* is made, many additional tasks are uncovered in the first stage.

- From a practical point of view (railway problem): currently no need for a quasi robust solution approach since naïve approach works good enough given current rules and standby duties.

Implementation at NS

- Re-scheduling duties for a disruption tomorrow:
  - Algorithm implemented in planning system CREWS in 2010
  - Has been used a few times since then, e.g. February 5, 6 and 7, 2012 (when an adjusted timetable was operated)

- Real-time re-scheduling duties
  - Algorithm of Potthoff et al. (2010) implemented in new dispatching system CREWS-RTD, which NS purchased in 2010.
  - Experiments in Operations Control Center in Spring 2011.
  - New staff was hired and trained in August/September 2011
  - Since end of Oct 2011: shadow experiments 7*16
    with up to now 3 or 4 solutions implemented in practice

Important dates

Abstract submission:
- Opens: January 15, 2013
- Closes: April 30, 2013

Notification of acceptance
May 31, 2013

Deadline registration:
Early: June 30, 2013
Late: September 3, 2013

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