Annual holiday planning for the crew of a public transport company

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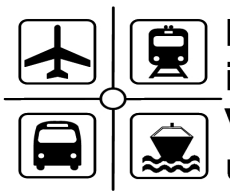


Professur für BWL, insbesondere Verkehrsbetriebslehre und Logistik

Background of the Project

Problem

- Description
- Social fairness
- Holiday-Point-System (Application for leave)

Solution Approach

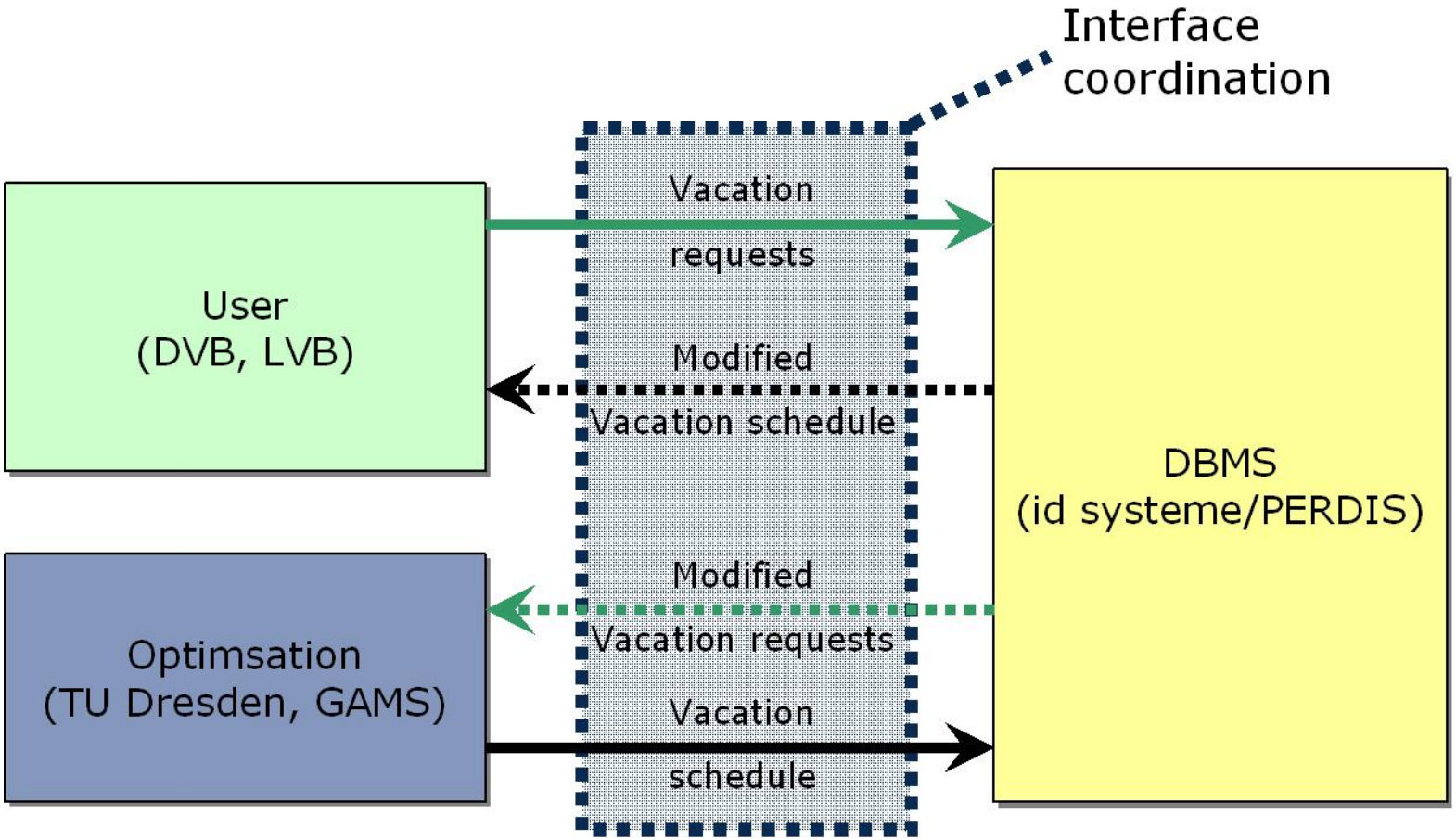
- Master problem
- Subproblem
- Preliminary Numerical Results

Future Work

Background of the Project

- Innovation competition in 2007 in East-Germany by the ministry of traffic, construction and urban development (BMVBS): 'Economy-meets-Sciences'
- From 157 submitted projects 11 have been selected (award winners)
- Transferring methods from the transportation science to transportation companies taking „*Socially acceptable holiday planning*“ and „*Customer oriented line planning*“ as examples
- Supported by the BMVBS (Ref.-No.: 03WWSN037)

Scheduling system (under progress)



Problem description

- City of Dresden: 836 drivers (tram, bus)
- Drivers are qualified for trams or busses or for both
- Some drivers are additionally qualified for operational management tasks
- Connection between pairs of drivers (e.g. married couples)
  - Holiday together
  - Holiday not together
- Holiday entitlement (number of leave days in planning horizon)
- Duty roster (given for the planning horizon) $\Rightarrow$ Staff supply
- For each day the number of needed drivers is known $\Rightarrow$ Staff demand

  $\Rightarrow$ For each day maximum number of drivers allowed to be on holiday

Problem

Social fairness

- Family situation (children required to attend school)
- Holiday in last year (leave day at christmas)
- Bonus points for splitted duties

Holiday-Point-System

External pricing system

aim of the system: conflict avoidance

100 HPs

individual distribution

...	April 2009					...	October 2009					...
...	Fr 10	Sa 11	So 12	Mo 13	Tu 14	...	Mo 26	Tu 27	We 28	Th 29	Fr 30	...
...	20	20	30	40	30	...	0	0	2	2	3	...

(Easter)

Holiday-Point-System

External pricing system

aim of the system: conflict avoidance

~~100 Hps~~
30 HPs

individual distribution

	April 2009						October 2009				
	Fr 10	Sa 11	So 12	Mo 13	Tu 14		Mo 26	Tu 27	We 28	Th 29	Fr 30
	20	20	30	40	30		0	0	2	2	3

(Easter)

1st application: 2 days from April 13th to April 14th → 70 HPs

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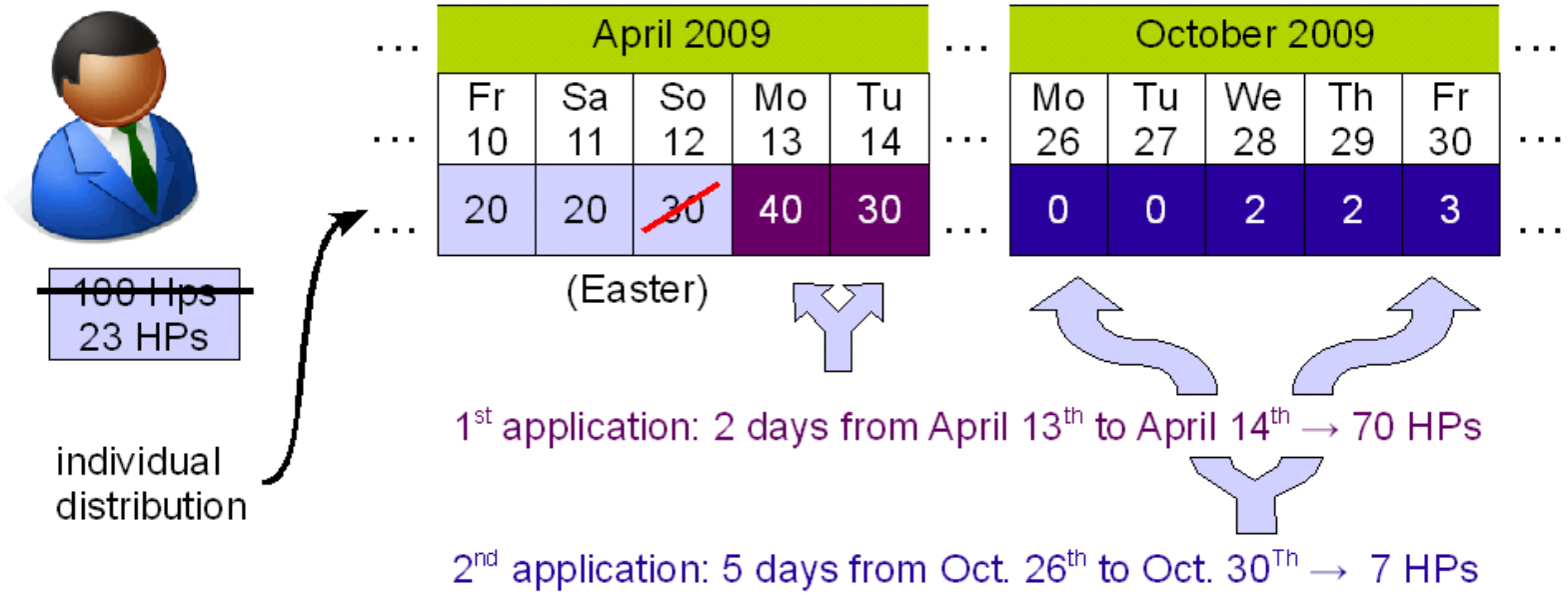

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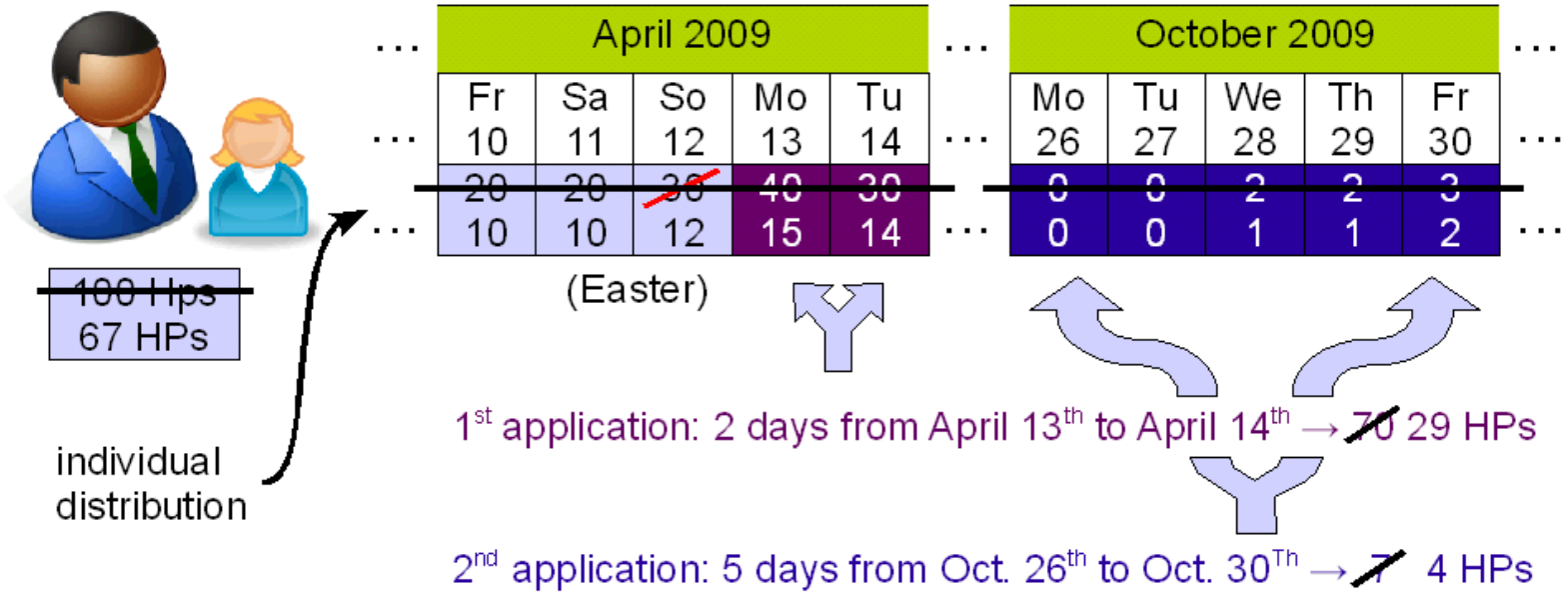

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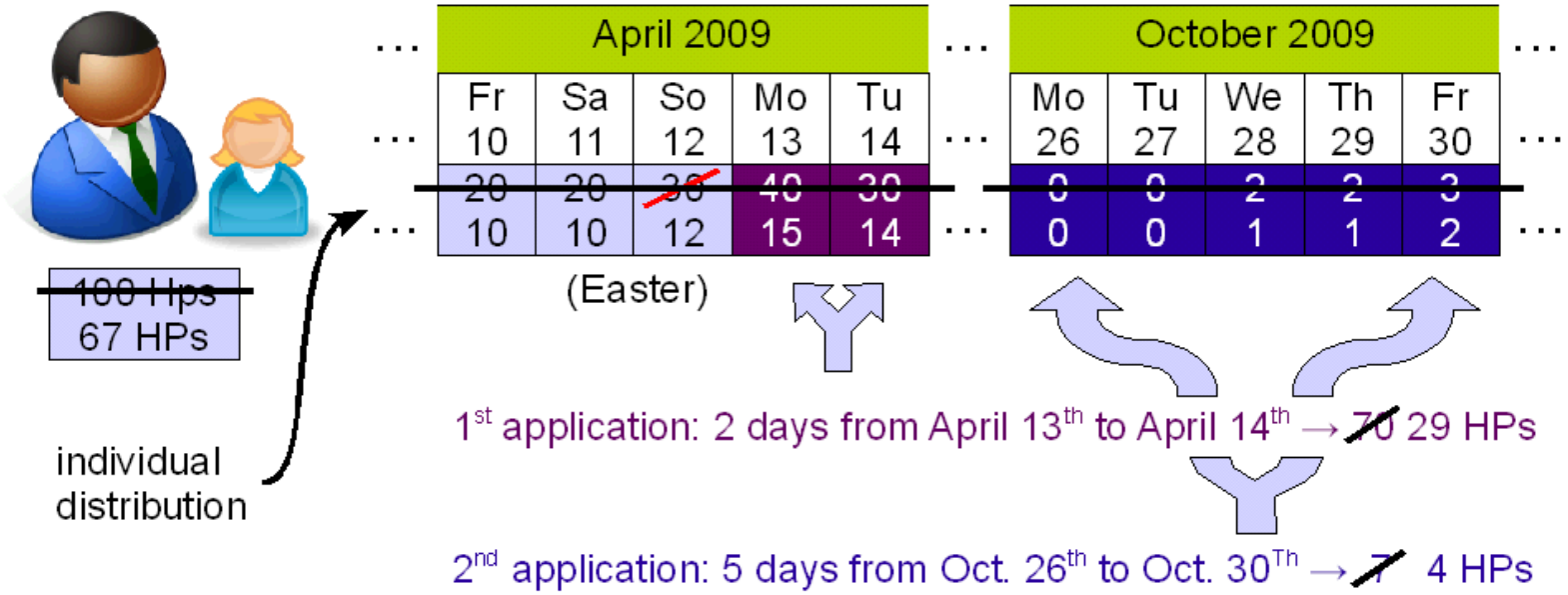

- discount for flexibility in duration or range

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External pricing system

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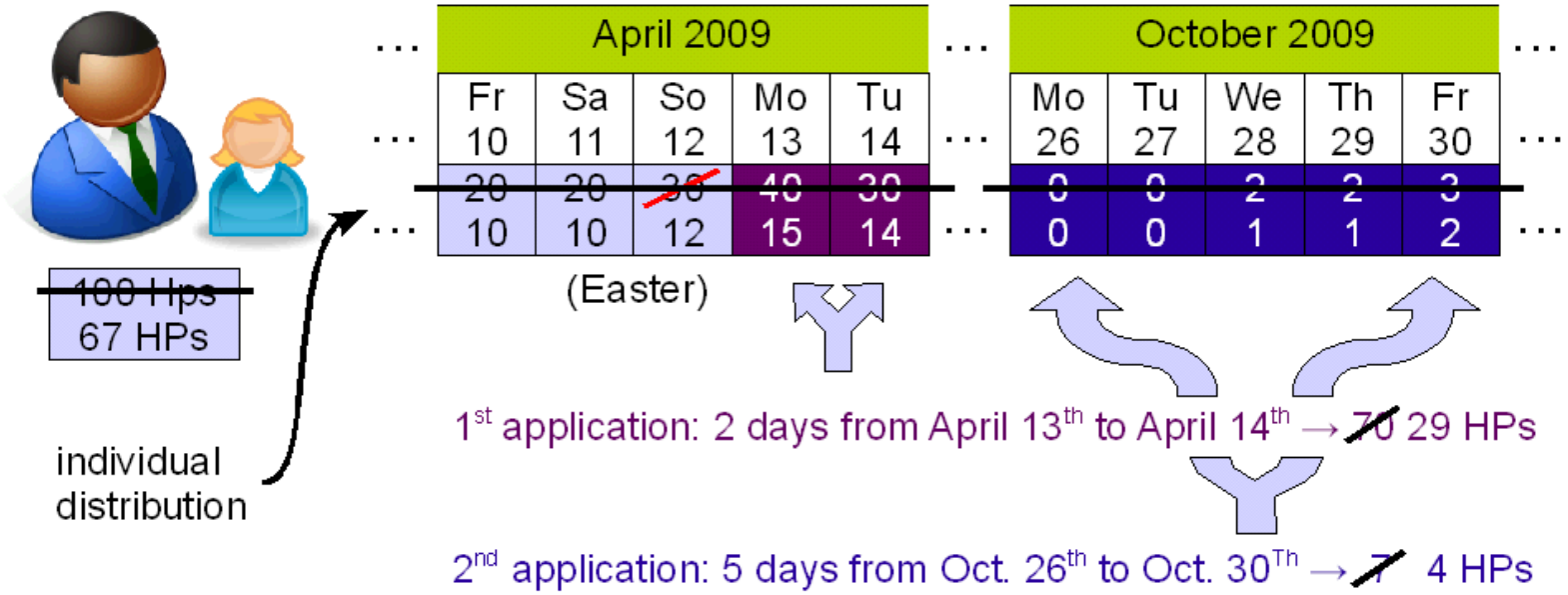


- discount for flexibility in duration or range
- updated every year depending on utilization, granted holidays and so on

Solution Approach

Implementation

Algebraic Modelling Language: GAMS/Cplex

Upper Bound

Column Generation

Lower Bound

- Integer solution based on the generated holiday schedules (Cplex).

Master problem

Sets

G	groups of drivers; index: g
T	days; index: t
H	annual holiday schedules; index: h

Parameters

c_{gh}	nonnegative, normalized and logarithmic weighted utility of annual holiday schedule h of group g
v_{qtgh}	number of drivers with qualification q on leave on day t according to annual holiday schedule h
b_{qt}	maximum allowable number of drivers with qualification q on leave on day t

Variables

y_{gh}	$=1$, if holiday schedule h of group g is selected (0, otherwise)

Model

Maximizing the logarithmic weighted utility

$$\max F = \sum_{g,h} c_{gh} y_{gh}$$

Selecting for each group one or none annual holiday schedule

$$\sum_{h} y_{gh} \leq 1 \qquad g \qquad (\sigma_g)$$

Maximum number of drivers on leave

$$\sum_{g,h} v_{qtgh} y_{gh} \leq b_{qt} \qquad q, t \qquad (\pi_{qt})$$

Domains of variables

$$y_{gh} \in \{0, 1\} \qquad g, h$$

Subproblem

Sets

D_g	drivers of holiday group g
A_d	application for leaves of driver d; index: a
J	pairwise application for leaves of one driver or of two drivers which have to be approved together (or not); quadrupels: $(a, d, a', d') \in J$
N	pairwise application for leaves of one driver or of two drivers which cannot be approved together; quadrupels: $(a, d, a', d') \in N$
S	stairs of piecewise linear approximation of a logarithmic function; index: s

Example

Drivers d and $\hat{d}$ are a married couple which want to have holiday together. Let a and a' the application for leaves of driver d where a' is an alternative to a. Respectively $\hat{a}$ and $\hat{a}'$ are the application for leaves of driver $\hat{d}$.

$$(a, d, a', d),\ (\hat{a}, \hat{d}, \hat{a}', \hat{d}) \in N$$

$$(a, d, \hat{a}, \hat{d}),\ (a', d, \hat{a}', \hat{d}) \in J$$

Subproblem

Parameters

u_{ad}	utility of driver d if application for leave a is approved
$\tilde{\pi}_{ad}$	opportunity cost of application for leave a of driver d derived from the dual variables π_{qt}
σ_g	dual variable related to the constraint 'selecting at most one holiday schedule for each group'

Example

Driver d has qualification q. According to his application for leave a he has applied for holiday from period $t = 4$ to period $t = 10$.

$$\tilde{\pi}_{ad} = \sum_{t=4}^{10} \pi_{qt}$$

Subproblem

Variables

X_{ad} = 1, if application for leave a of driver d is approved (0, otherwise)

U_{ds} part worth utility of driver d in the stair s; $U_{ds} \in [0, 1/ \mid S \mid]$

Remark

The maximum total utility a driver can achieved is 1.

Subproblem

Maximizing reduced cost of holiday group

$$\max \bar{c}_g = -\sigma_g + \sum_{d \in D_g} \sum_{s=1}^{|S|} (\ln(1+s) - \ln(s)) U_{ds} - \sum_{a,d} \pi_{ad} X_{ad}$$

Utility of driver

$$\sum_{a} u_{ad} X_{ad} \geq \sum_{s=1}^{|S|} U_{ds} \quad d \in D_g$$

Maximum number of days of holiday of driver

$$\sum_{a,d} h_{ad} X_{ad} \leq E_d \qquad d$$

Jointly on holiday

$$X_{ad} - X_{a'd'} = 0 \qquad (a, d, a', d') \in J$$

Not together on holiday

$$X_{ad} + X_{a'd'} \leq 1 \qquad (a, d, a', d') \in N$$

Domains of variables

$$X_{ad} \in \{0, 1\} \qquad a, d$$

$$U_{ds} \in [0, \frac{1}{\mid S \mid}] \qquad d, s$$

Preliminary Numerical Results

Randomly generated instance

- planning horizon 400 days (overlapping with next years)
- 800 drivers
- 400 single driver groups
- 200 groups with 2 drivers
- 2 types of qualifications
- Applications for leaves of drivers
  - one application with a duration $\in \{9, 10, \ldots, 21\}$ days (relative high probability that holiday will be applied for the middle of the year)
  - duration of the other applications: $\in \{3, 4, \ldots, 8\}$ days
- holiday entitlement: 40 days (including off days according to unknown duty rosters)
- $\mid S \mid = 10$

Preliminary Numerical Results

Upper bound: 177.13 (best possible of first iteration (2% or 0.1 absolute))
Lower bound: 170.26 (best possible: 173.33)
Computation time: 7 min

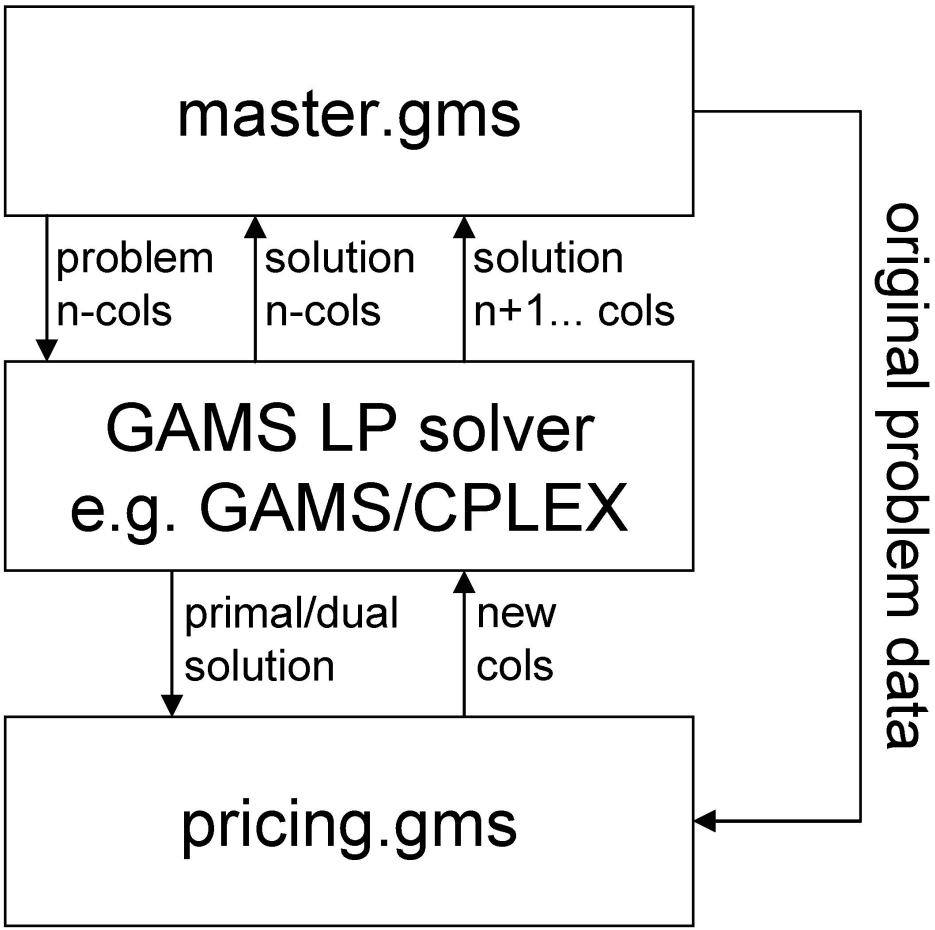
Iteration	Ofv Master	Reduced cost
1	0.00	175.06
2	145.90	148.71
3	163.65	75.76
4	169.56	11.71
:	:	:
8	173.02	1.08
9	173.23	0.63

driver \ day	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	..
1	0	0	0	0	0	0	0	0	0	0	1	1	1	1	1	..
2	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	..
								:								
7	0	0	0	0	0	0	1	1	1	1	1	1	1	1	0	..
								:								
11	1	1	1	1	1	0	0	0	0	0	0	0	0	0	0	..
12	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	..
13	1	1	1	1	1	1	1	1	1	0	0	0	0	0	0	..
								:								

Future Work

- Using GAMS enhancement regarding column generation

GAMS enhancement



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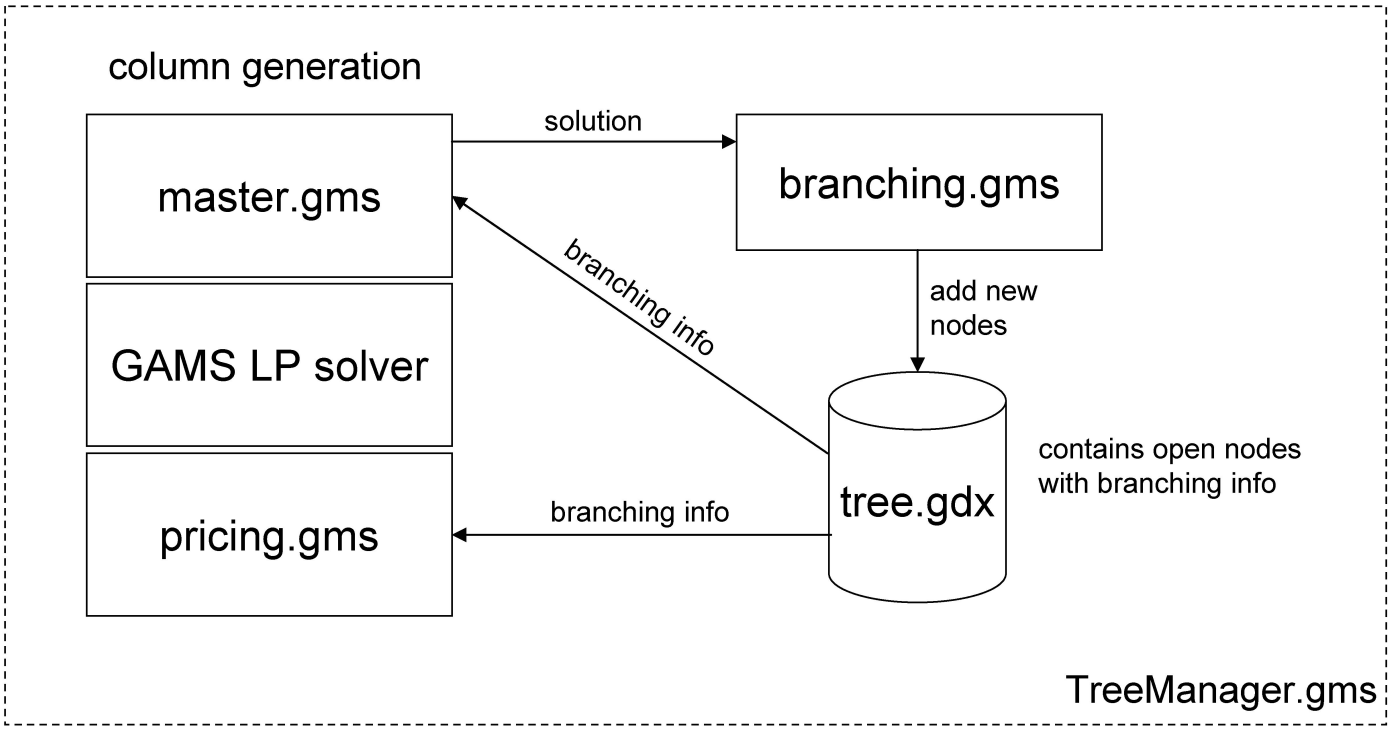

B&B: manages tree
(node selection, pruning, …)

calls master & branching.gms

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Thank you very much for your attention!