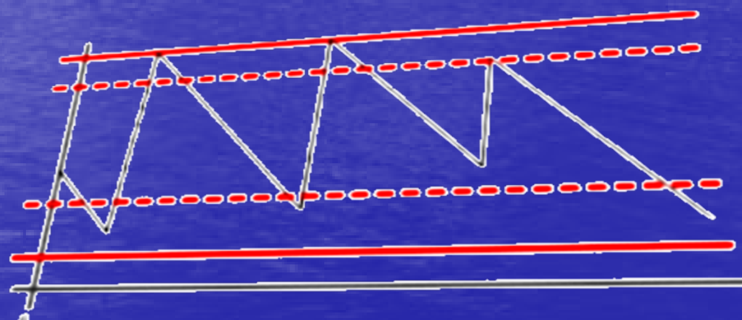
A Branch-and-Price Approach for a Ship Routing Problem with Multiple Products and Inventory Constraints



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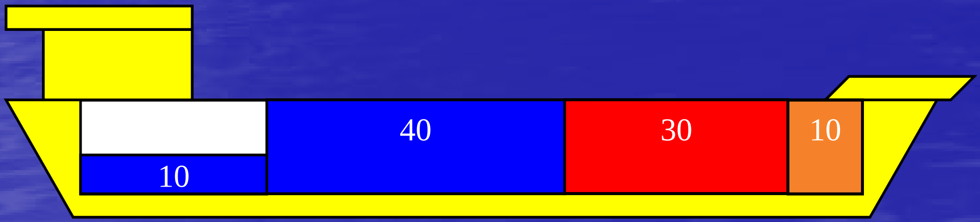
Outline

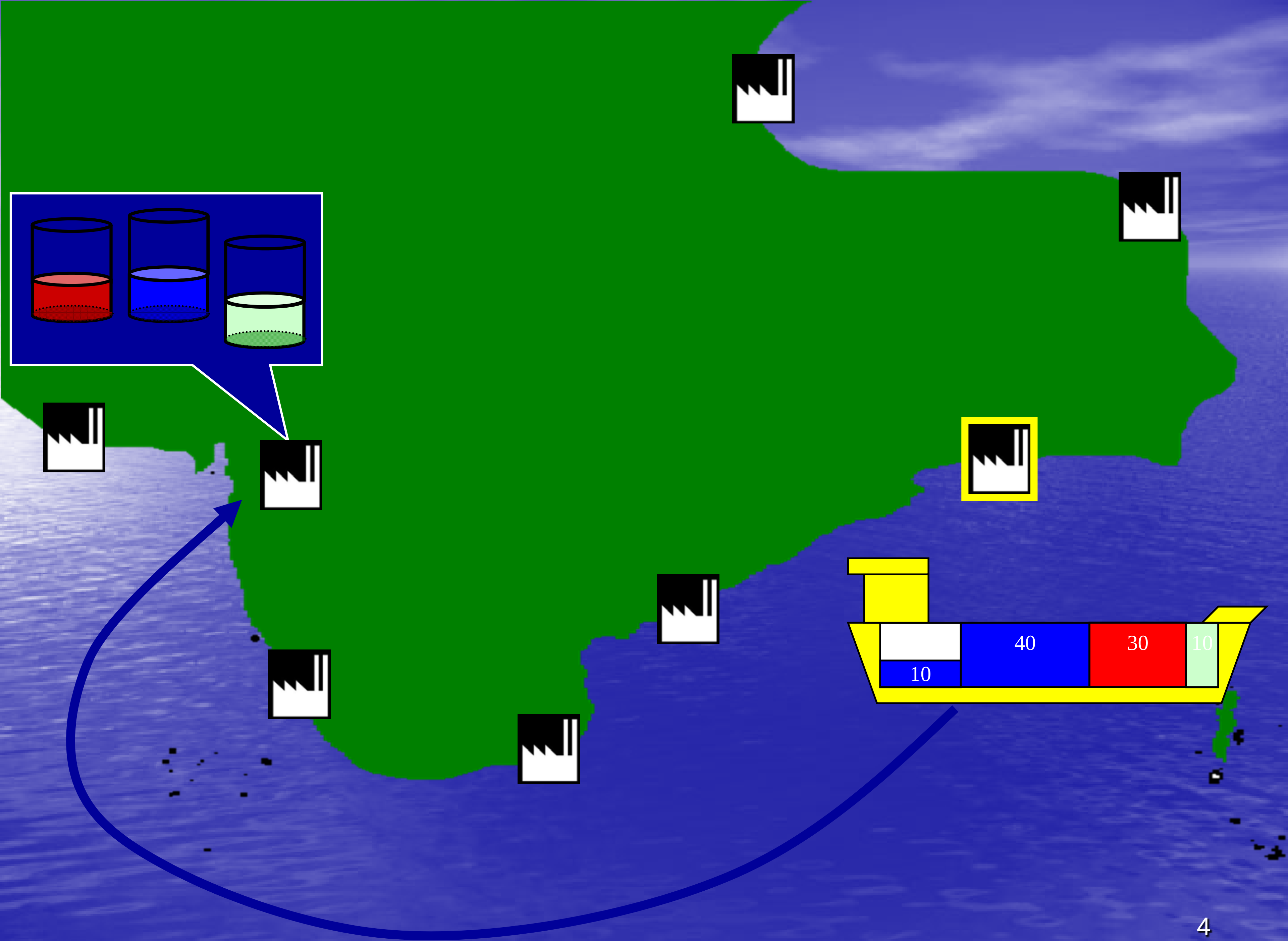
- Practical Application
- Problem description
- Mathematical Formulation
- Column generation algorithm

Practical Application

Oil Company

- 1 refinery (no inventory constraints)
- ±12 depots (one per harbor)
- ± 8 grades
- ± 7 ships
- compartments (differ in size, 3-6)




10
40
30
10

Practical Application

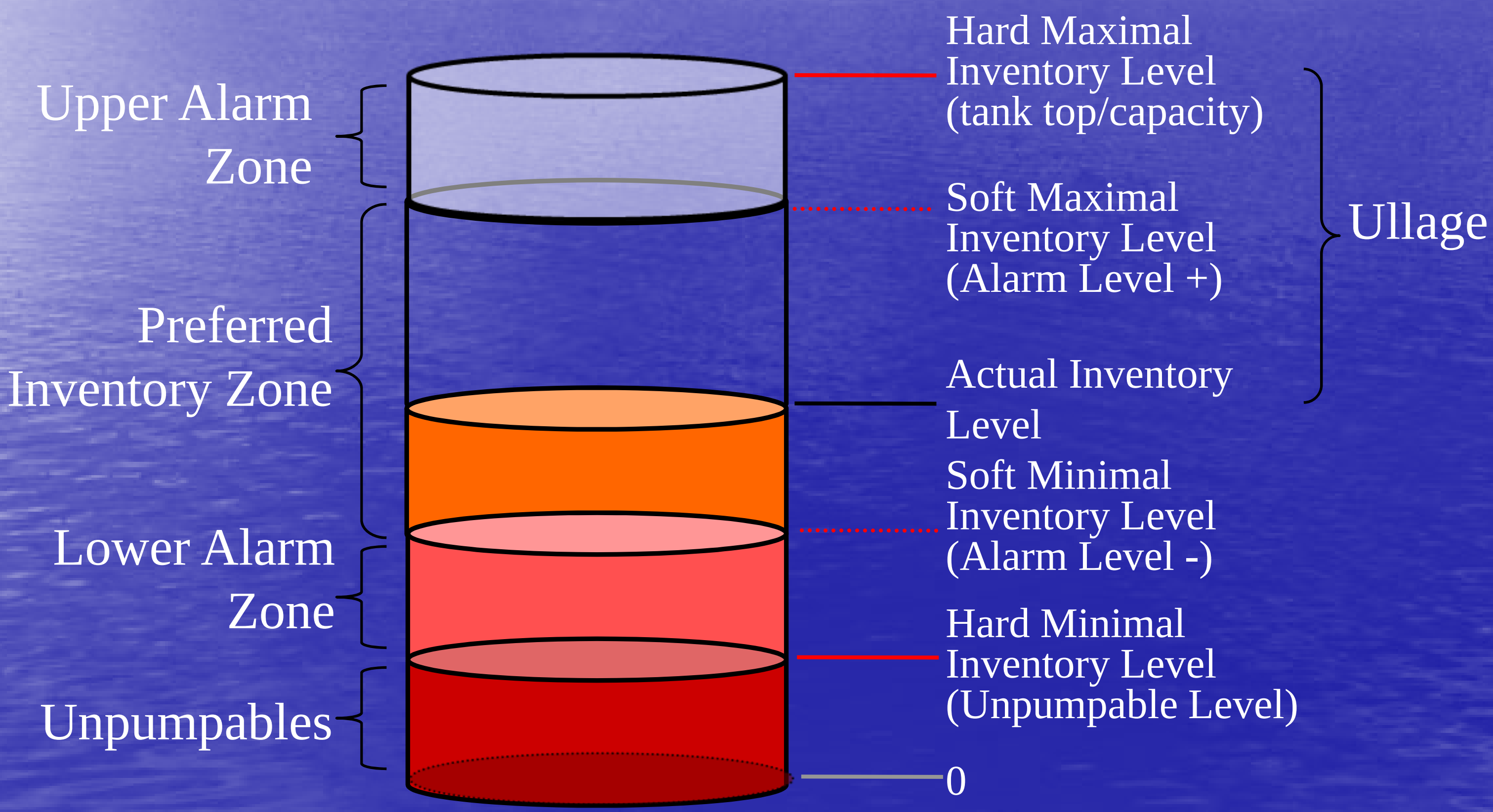
- There is a schedule
- Question: Can we adjust the schedule due to changes in the data?

- We construct an “optimal” schedule from scratch

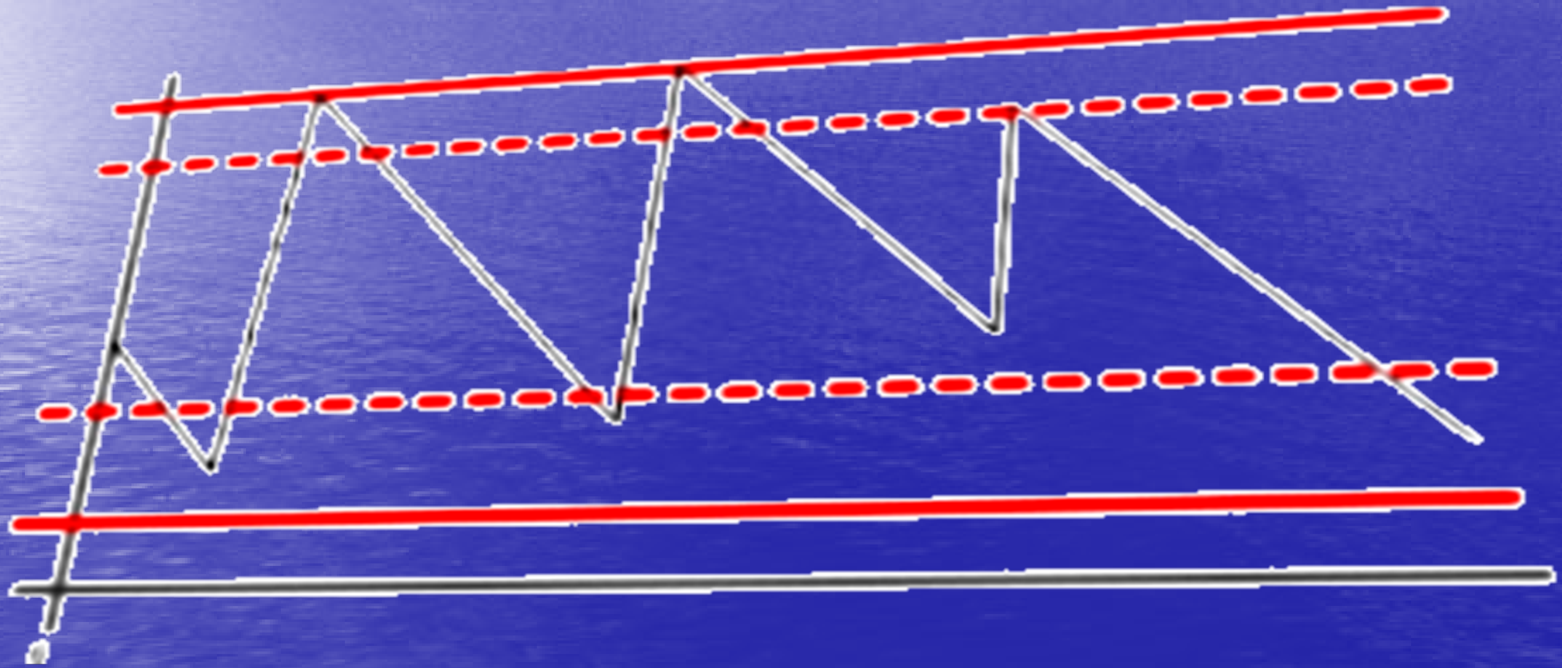
Problem Description

- Ships
  - Compartments with capacities
  - Travel times for each possible trip
  - Fixed (un)loading times
  - Wait at harbor is possible
- Harbor
  - Inventory constraints

Inventory levels



Inventory levels



Notation (Christiansen, 1998)

- (i,m): harbor i, number of arrival m
- (i,m,j,n): traveling from (i,m) to (j,n)

- V: set of ships
- H: set of harbors
- G: set of grades
- C: set of compartments

Decision Variables

- λ_{cr}: compartment *c* travels schedule *r*
- θ_{igs}: harbor *i* follows for grade *g* sequence *s*

Mathematical Formulation (1)

$$\min \sum_{c \in C} \sum_{r \in R_c} C_{cr}^{C} \lambda_{cr} + \sum_{i \in H_S} \sum_{g \in G} \sum_{s \in S_{ig}} C_{igs}^{HG} \theta_{igs}$$

Sailing cost route r (only for c=1)

Penalties for violating alarm levels

Mathematical Formulation (2)

$$\sum_{v\in V}\sum_{r\in R_c} B_{imc_v^* r}\lambda_{c_v^* r} + \sum_{s\in S_{ig}} Y_{imgs}\theta_{igs} = 1 \qquad \forall i,m,g$$

1 if (i,m) is in schedule r to serve g

0 if (i,m) is in sequence s to (un)load g

$$\sum_{c\in C}\sum_{r\in R_c} Q^{C}_{imcgr}\lambda_{cr} - \sum_{s\in S} Q^{HG}_{imgs}\theta_{igs} = 0 \qquad \forall i,m,g$$

Quantity of g (un)loaded at (i.m) in schedule r

Quantity of g (un)loaded at (i,m) in sequence s

Mathematical Formulation (3)

$$\sum_{v\in V}\sum_{r\in R_{c_v^*}} T^{C}_{imc_v^*r}\lambda_{c_v^*r} - \sum_{s\in S_{ig}} T^{HG}_{imgs}\theta_{igs} = 0 \qquad \forall i,m,g$$

Time (un)loading starts at (i,m) in schedule r, first compartment

Time (un)loading starts at (i,m) in sequence s

$$\sum_{r\in R_{c_v^*}} T^{C}_{imc_v^*r}\lambda_{c_v^*r} - \sum_{r\in R_c} T^{C}_{imcr}\lambda_{cr} = 0 \qquad \forall v,i,m,c$$

Time (un)loading starts at (i,m) in schedule r, first compartment

Time (un)loading starts at (i,m) in schedule r, compartment c

Mathematical Formulation (3)

$$\sum_{r \in R_c} \lambda_{cr} = 1 \qquad \forall c$$

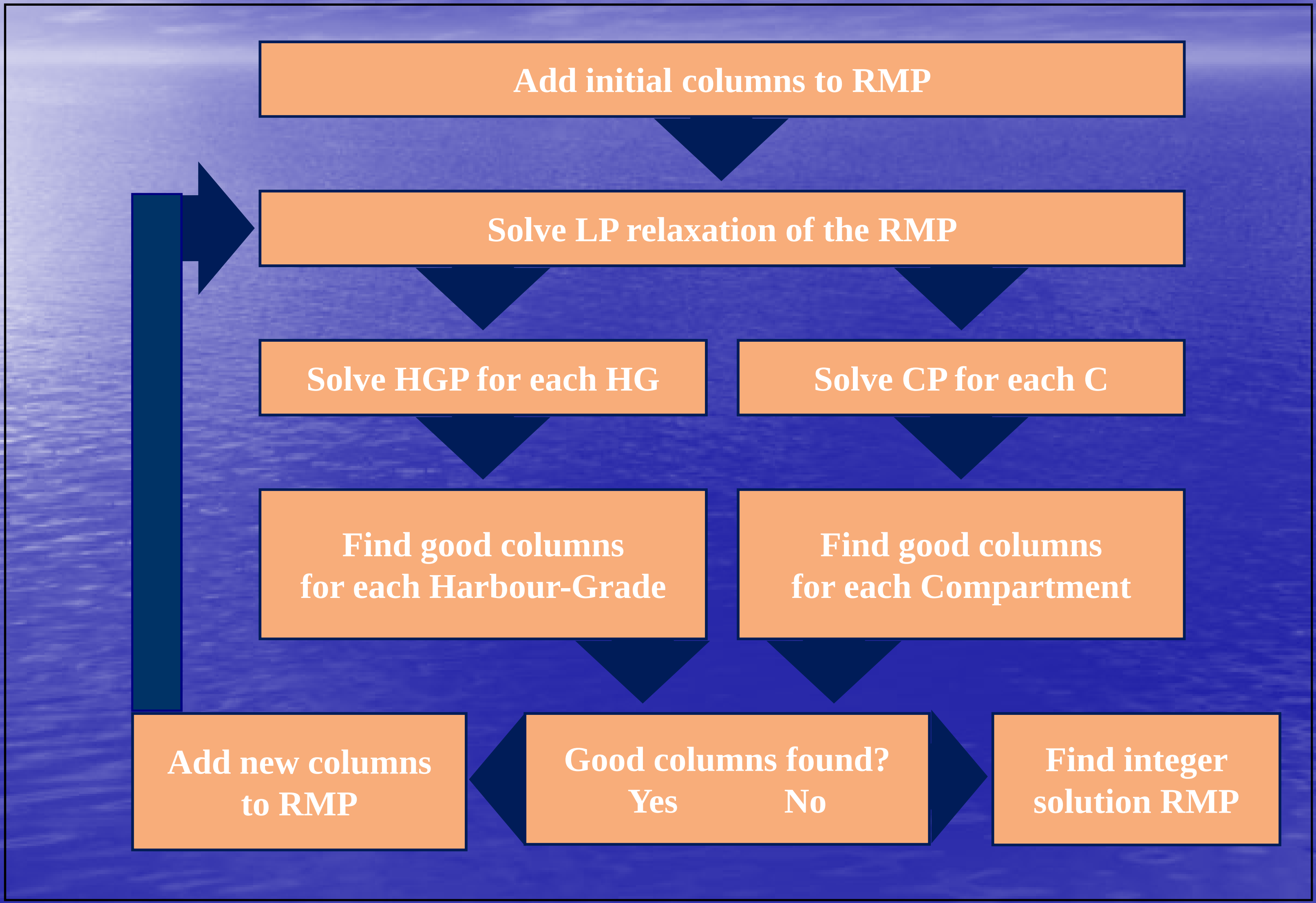
$$\sum_{s \in S_{ig}} \theta_{igs} = 1 \qquad \forall i, g$$

$$\lambda_{cr} \geq 0 \qquad \forall c, r$$

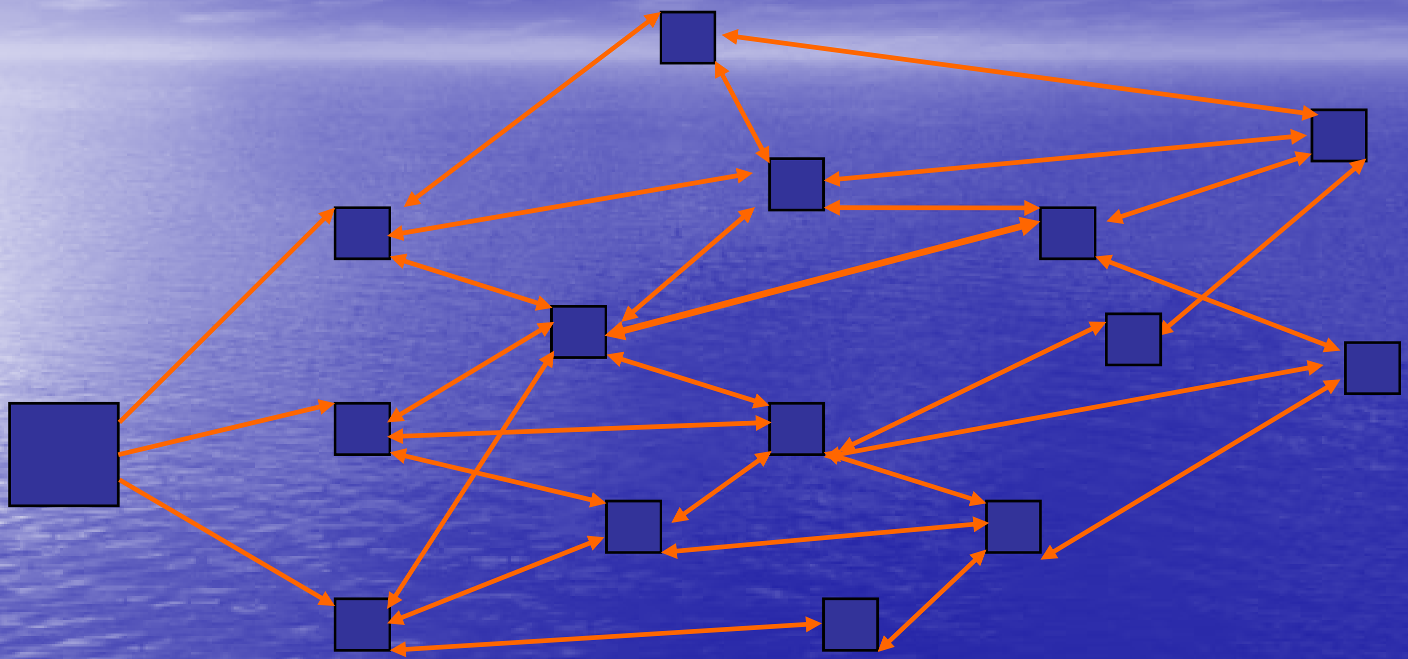
$$\theta_{igs} \geq 0 \qquad \forall i, g, s$$

$$\sum_{r \in R_c} X_{imjncr} \lambda_{cr} \in \{0,1\} \qquad \forall c, (i, m, j, n)$$

Branch-and-Price Framework

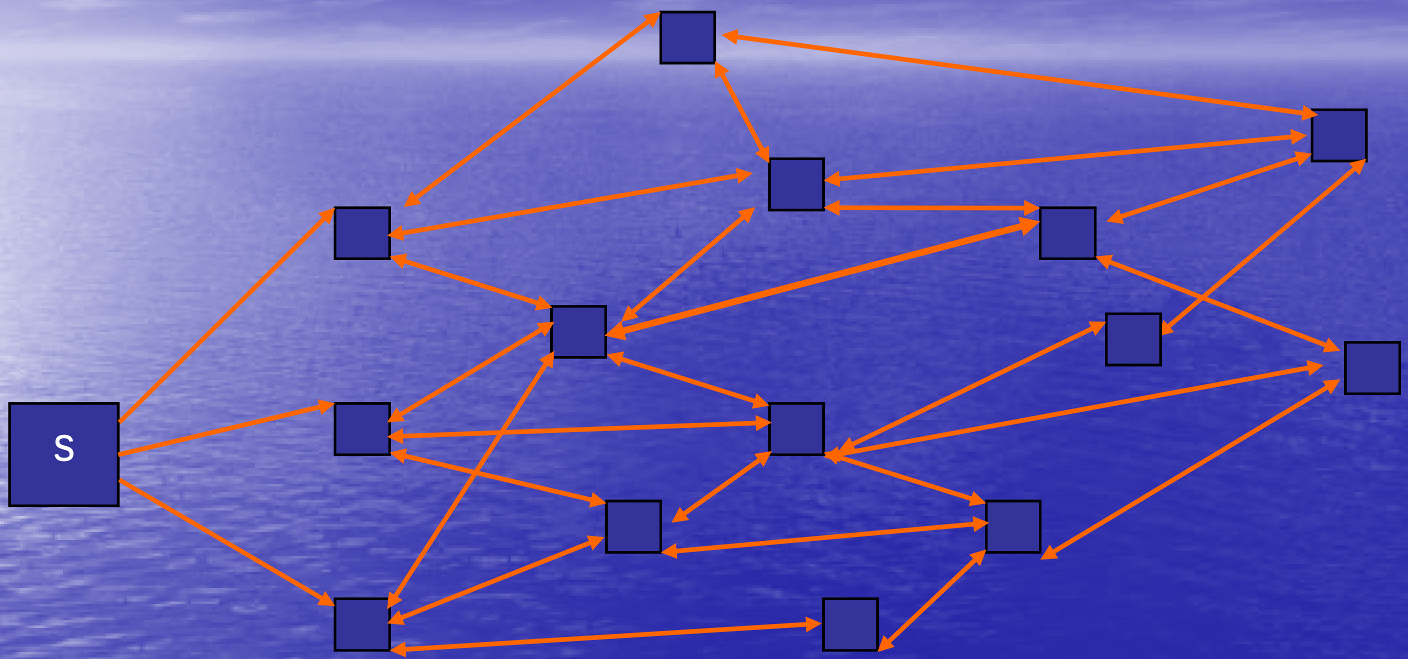


Compartment Sub problem



- Node: harbor arrival plus quantity per grade
- Reduced cost per arc:
  - Fixed
  - Time dependent

Compartment Sub problem



- Shortest path from s to any other node (taking into account time aspect)
- Can be solved with Dynamic Programming (Christiansen, 1998/9)

Branching

- Arcs in the network
- Aggressive 1-branch

Implementation Issues

- Pricing problem: exact or heuristically?
- Difficult to get a feasible solution of the model: relax and penalize constraints?
- Cycles in the compartment sub problem (generates redundant columns)

Results

- Will follow

Questions?