~~ALGORITHM ENGINEERING~~

ALGORITHMS BY ENGINEERS

(AN EGOCENTRIC VIEW ON RAILWAY OPTIMIZATION)

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PROJECTS:

FASTER (1994) PARTNER (2003-6)

FARO (1995) ARRIVAL (2006-9)

TRIS (1996-9) ...

TRIO (1996-9)

COLUMN GENERATION
AUSSOIS 2008, JUNE 7-11

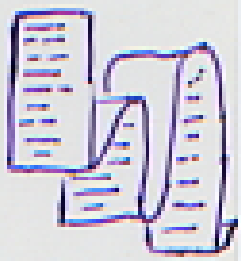
RAILWAY OPTIMIZATION PROBLEMS

(IN ORDER OF APPEARENCE IN BOLOGNA)

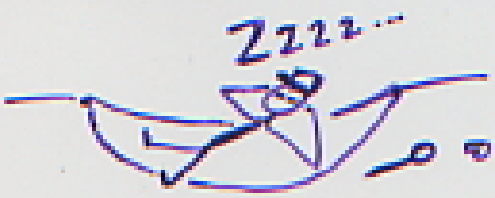
- CREW ASSIGNMENT
- TRAIN TIMETABLING
- TRAIN-UNIT ASSIGNMENT
- TRAIN PLATFORMING

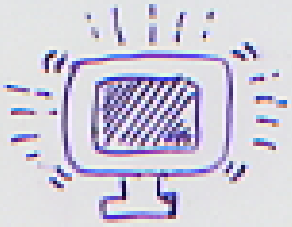
COMMON FEATURES:

→ LARGE-SIZE REAL-WORLD INSTANCES

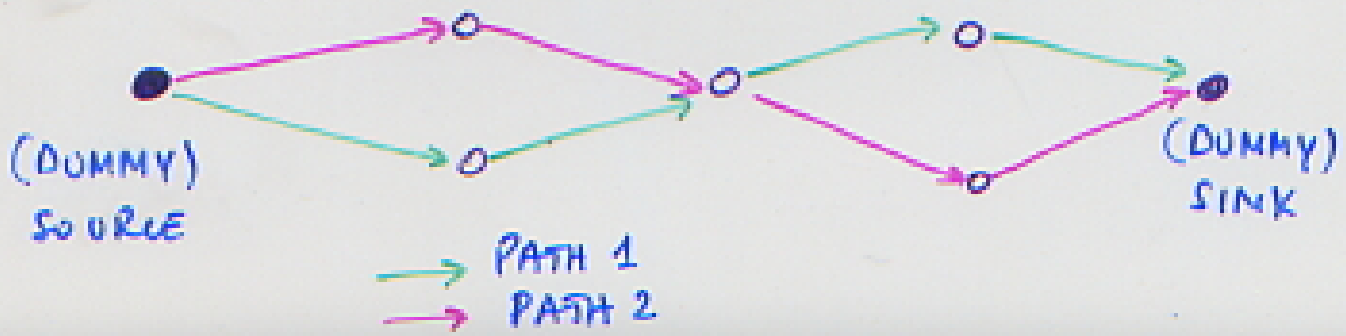


→ PLANNING LEVEL (PLENTY OF TIME TO BE SOLVED)





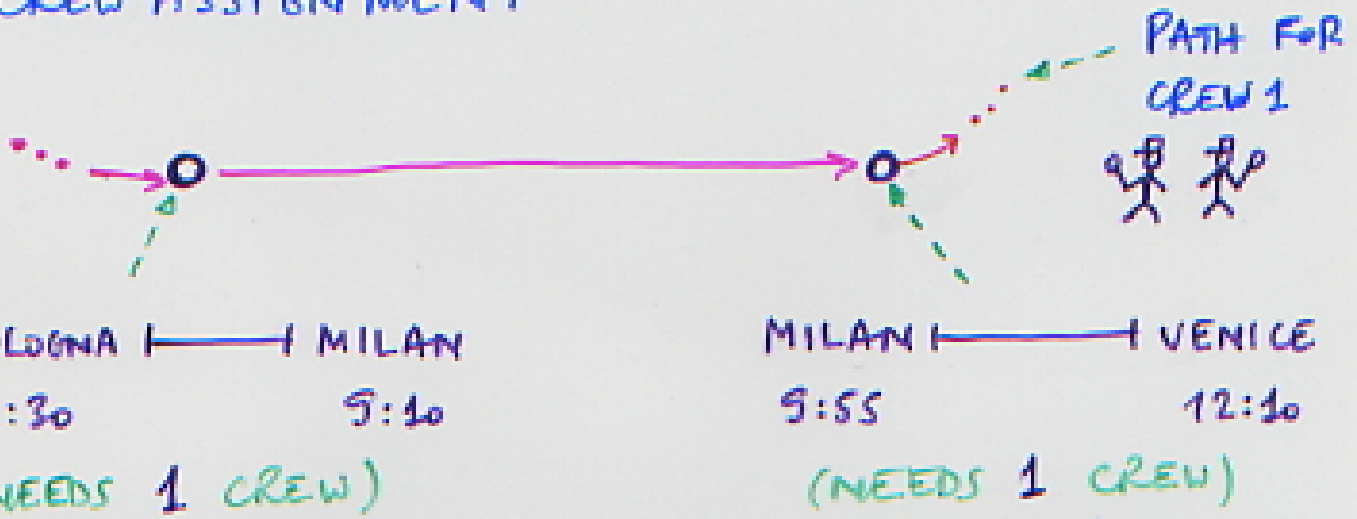
→ CAN BE MODELED AS INTEGER MULTICOMMODITY FLOW PROBLEMS: FOR EACH "COMMODITY" (CREW, TRAIN, ...) FIND "PATH" FROM SOURCE TO SINK IN ACYCLIC GRAPH



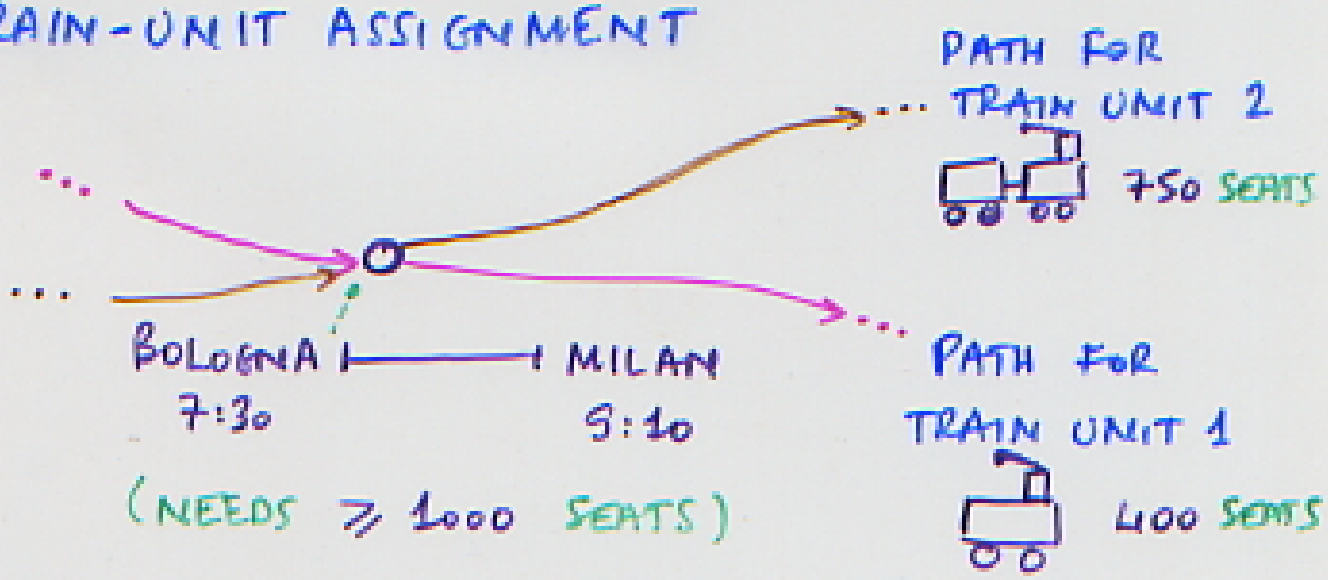
"ASSIGNMENT" PROBLEMS

NODES IN ACYCLIC GRAPH REPRESENT SERVICES TO BE COVERED

EX. CREW ASSIGNMENT



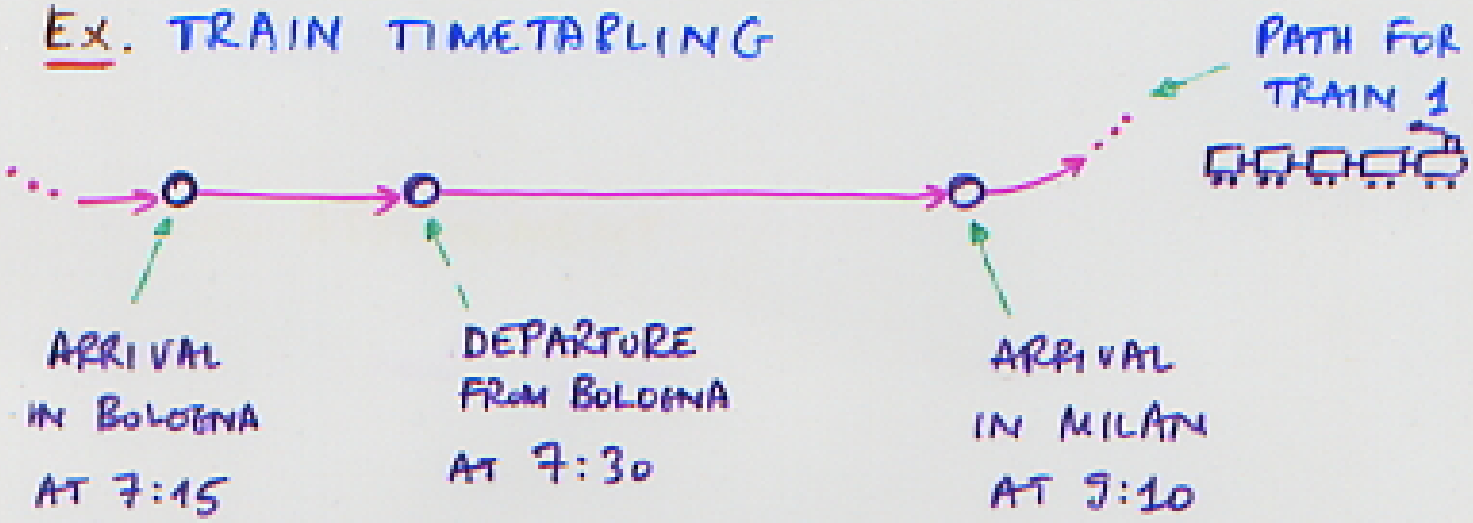
EX. TRAIN-UNIT ASSIGNMENT



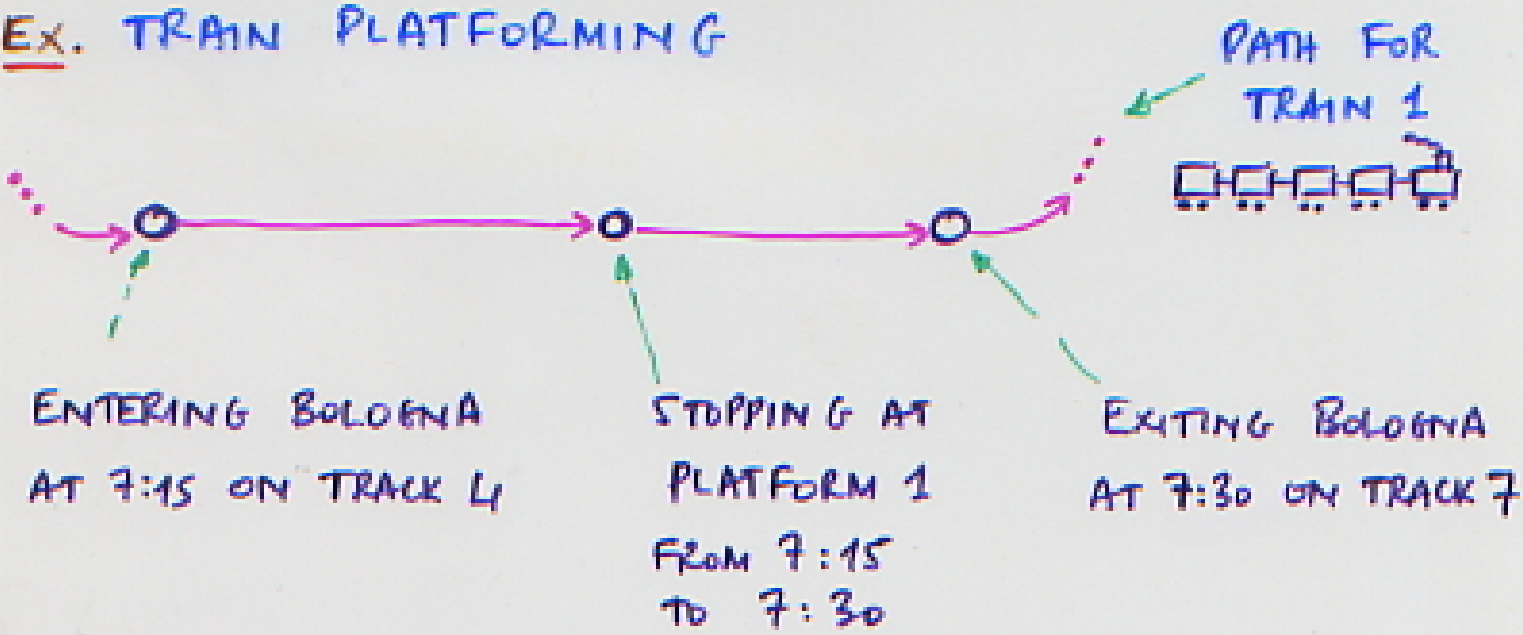
"TIMETABLING" PROBLEMS

NODES IN ACYCLIC GRAPH REPRESENT RESOURCES WITH LIMITED CAPACITY

Ex. TRAIN TIMETABLING



Ex. TRAIN PLATFORMING



→ FOR "ASSIGNMENT" PROBLEMS THE MAIN OBJECTIVE IS WELL DEFINED

(MIN # OF CREWS, MIN # OF TRAIN UNITS)

AND EACH UNIT OF OBJECTIVE MAY COST A LOT

(CREW ASSIGNMENT AT TRENITALIA:

1 UNIT ≡ 6 CREWS ≡ 12 CONDUCTORS)

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OPTIMALITY VS. ROBUSTENESS

→ FOR "TIMETABLING" PROBLEMS THE MAIN OBJECTIVE IS LESS CLEAR

(MIN "DEVIATION" W.R.T. "IDEAL" TIMETABLE OR "IDEAL" PLATFORM)

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OPTIMALITY VS. ROBUSTNESS

STATE-OF-MY-ART APPROACHES

1990's | 2000's

- OPTIMAL SOLUTION HOPELESS
- HEURISTICS BASED ON ILP MODELS

1990's	2000's
• LP RELAXATION IS TOO LARGE ⇒ LAGRANGIAN RELAX.	• LP RELAXATION CAN BE HANDLED BY GOOD LP SOLVERS
• MCF FORMULATION WITH ARC VAR.S	• MCF FORMULATION WITH PATH VAR.S + COLUMN GENERATION

- DIVING HEURISTICS (SUITED BOTH FOR LAGRANGIAN RELAX. & COLUMN GENERATION)

2000's STATE-OF-MY-ART APPROACHES

COLUMN GENERATION $\Rightarrow$

NEED TO HAVE CONTROL ON THE CONSTRAINTS

$\Rightarrow$ START WITH "GOOD" FORMULATION

(MAIN) VARIABLES:

$$x_{i,p} = \begin{cases} 1 & \text{IF COMMODITY } i \text{ IS ASSIGNED PATH } p \\ 0 & \text{OTHERWISE} \end{cases}$$

$c_{i,p}$ = COST PAID IF $x_{i,p} = 1$

I = SET OF COMMODITIES (CREWS, TRAINS, ...)

P_i = COLLECTION OF PATHS FOR COMMODITY i

CREW ASSIGNMENT (1 CREW/SERVICE)

$$\min \sum_{i \in I} \sum_{p \in P_i} c_{i,p}\, x_{i,p}$$

$$\sum_{i \in I} \sum_{\substack{p \in P_i: \\ p \text{ COVERS } j}} x_{i,p} \geq 1, \qquad j \in J$$

$$\sum_{p \in P_i} x_{i,p} \leq 1, \qquad i \in I$$

J = SET OF SERVICES TO BE COVERED

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SET COVERING CONSTRAINTS

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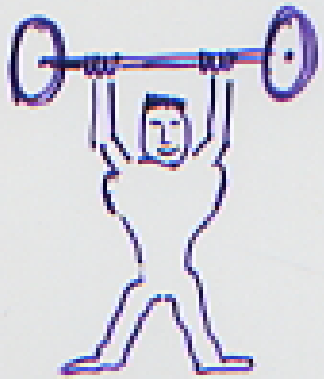
STRONG!



TRAIN-UNIT ASSIGNMENT

(SERVICES HAVE SEAT REQUIREMENTS)

$$\min \sum_{i \in I} \sum_{p \in P_i} c_{i,p} \, x_{i,p}$$

$$\sum_{i \in I} \sum_{\substack{p \in P_i: \\ p \text{ COVERS } j}} q_i \, x_{i,p} \geq n_j, \quad j \in J$$

$$\sum_{p \in P_i} x_{i,p} \leq 1, \quad i \in I$$

J = SET OF SERVICES TO BE COVERED

q_i = # SEATS AVAILABLE FOR COMMODITY i

n_j = # SEATS REQUIRED BY SERVICE j

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KNAPSACK CONSTRAINTS

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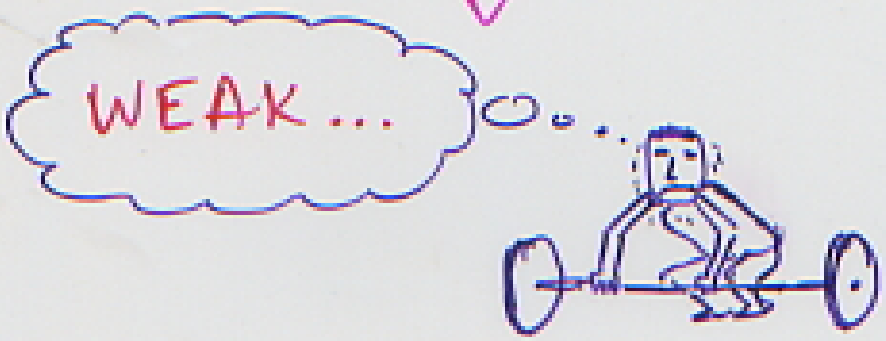

STRENGTHENING THE KNAPSACK CONSTR.

SPECIAL CASE :

$$\sum_{i \in I} \sum_{\substack{p \in P_i: \\ p \text{ covers } j}} x_{i,p} \leq 2$$

(AT MOST TWO COMMODITIES TO COVER EACH SERVICE)

Ex. $n_j = 1000$ (# SEATS REQUIRED)

$q_1 = 1200$, $q_2 = 750$, $q_3 = 400$ (# SEATS AVAIL.)

ORIGINAL (WEAK) CONSTRAINT:

$$\sum_{p \in P_1} 1200\, x_{1,p} + \sum_{p \in P_2} 750\, x_{2,p} + \sum_{p \in P_3} 400\, x_{3,p} \geq 1000$$

STRONG ("COMBINATORIAL") CONSTRAINTS:

$$2 \sum_{p \in P_1} x_{1,p} + \sum_{p \in P_2} x_{2,p} + \sum_{p \in P_3} x_{3,p} \geq 2$$

$$\sum_{p \in P_1} x_{1,p} + \sum_{p \in P_2} x_{2,p} \geq 1$$

FOR REAL-WORLD INSTANCES, IMPROVEMENT OVER PRACTITIONER'S SOLUTION ONLY WITH STRONG CONSTRAINTS !

TRAIN TIMETABLING / PLATFORMING

(FIND SCHEDULE FOR EACH TRAIN)

$$\min \sum_{i \in I} \sum_{p \in P_i} c_{i,p} \, x_{i,p}$$

$$\sum_{i \in I} \sum_{\substack{p \in P_i : \\ p \text{ USES } r}} x_{i,p} \leq 1, \quad r \in R$$

$$\sum_{p \in P_i} x_{i,p} = 1, \quad i \in I$$

R = SET OF RESOURCES (TRACKS, PLATFORMS, ... AT A GIVEN TIMEPOINT)

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CLIQUE CONSTRAINTS

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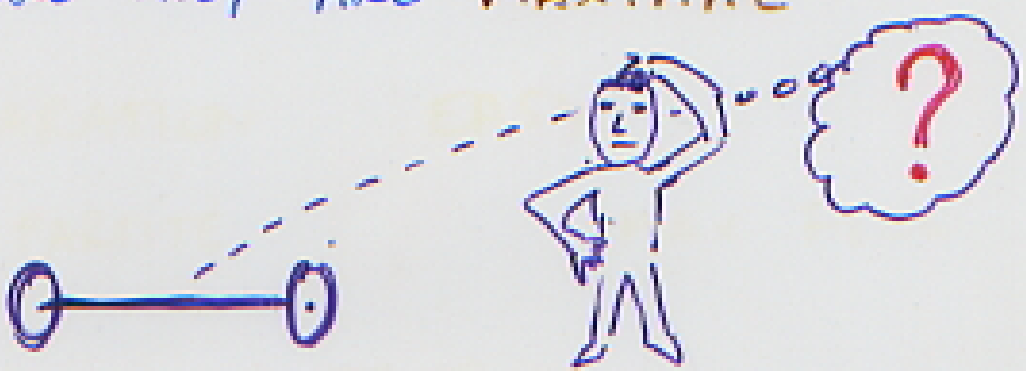
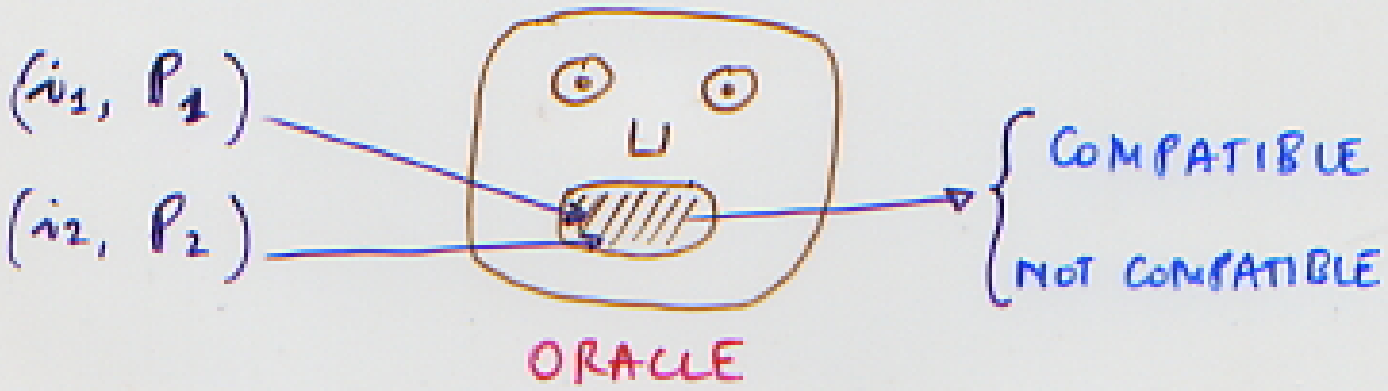
MAKE SURE THEY ARE MAXIMAL



ADDITIONAL CONSTRAINTS (PLATFORMING)

INCOMPATIBILITIES NOT RELATED WITH A SPECIFIC RESOURCE



WEAK CONSTRAINTS :

$$x_{i_1,P_1} + x_{i_2,P_2} \leq 1 \,, \qquad (i_1,P_1) \text{ NOT COMPATIBLE WITH } (i_2,P_2)$$

$$\sum_{P \in \overline{P}_1} x_{i_1,P} + \sum_{P \in \overline{P}_2} x_{i_2,P} \leq 1 \,,$$

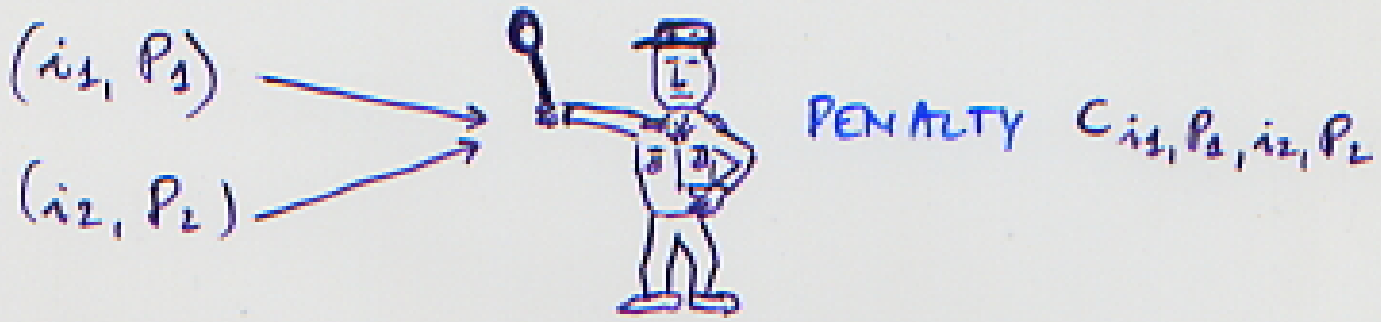
WHERE $\overline{P}_1$, $\overline{P}_2$ MAXIMAL S.T. CONSTRAINT VALID

SEPARATION IS EASY (MAX CLIQUE IN BIPARTITE GRAPH $\Rightarrow$ MAX FLOW)

MIGHT CAUSE TROUBLES WHEN COMBINED WITH COLUMN GENERATION

ADDITIONAL CONSTRAINTS (PLATFORMING)

RESOURCE THAT CAN BE USED BY TWO COMMODITIES WITH PENALTY



TRIVIAL FORMULATION:

$$\min \ \ldots + c_{i_1,P_1,i_2,P_2}\, y_{i_1,P_1,i_2,P_2} + \ldots$$

$$y_{i_1,P_1,i_2,P_2} \geq x_{i_1,P_1} + x_{i_2,P_2} - 1$$

z_{i_1,i_2} PENALTY VARIABLE

$$\min \ \ldots + z_{i_1,i_2} + \ldots$$

$$z_{i_1,i_2} \geq \sum_{P \in \mathcal{P}_1} \alpha_P\, x_{i_1,P} + \sum_{P \in \mathcal{P}_2} \beta_P\, x_{i_2,P} - \gamma$$

WHERE COEFFICIENTS α_P, β_P ARE MAXIMAL

SEPARATION IS "EASY" (POLYTOPE WITH $|\mathcal{P}_1| \cdot |\mathcal{P}_2|$ VERTICES)

SUMMARY

* (UN)FORTUNATELY, NOWADAYS LP SOLVERS CAN BE USED TO TACKLE THESE PROBLEMS

* (EXPONENTIALLY) MANY VARIABLES IN THE "GOOD" FORMULATIONS FOR LP SOLVERS

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COLUMN GENERATION

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FIND A "GOOD" ILP MODEL YOURSELF

* BY BEING CAREFUL AND PATIENT THE PRACTITIONERS' SOLUTIONS CAN BE (SUBSTANTIALLY) IMPROVED

* ALGORITHM BUSINESSMANNING

HOW TO CONVINCE THE PRACTITIONERS TO USE THE ENGINEERED ALGORITHMS?

ITALIAN POLITICS UPDATE

APRIL 15, 2008:

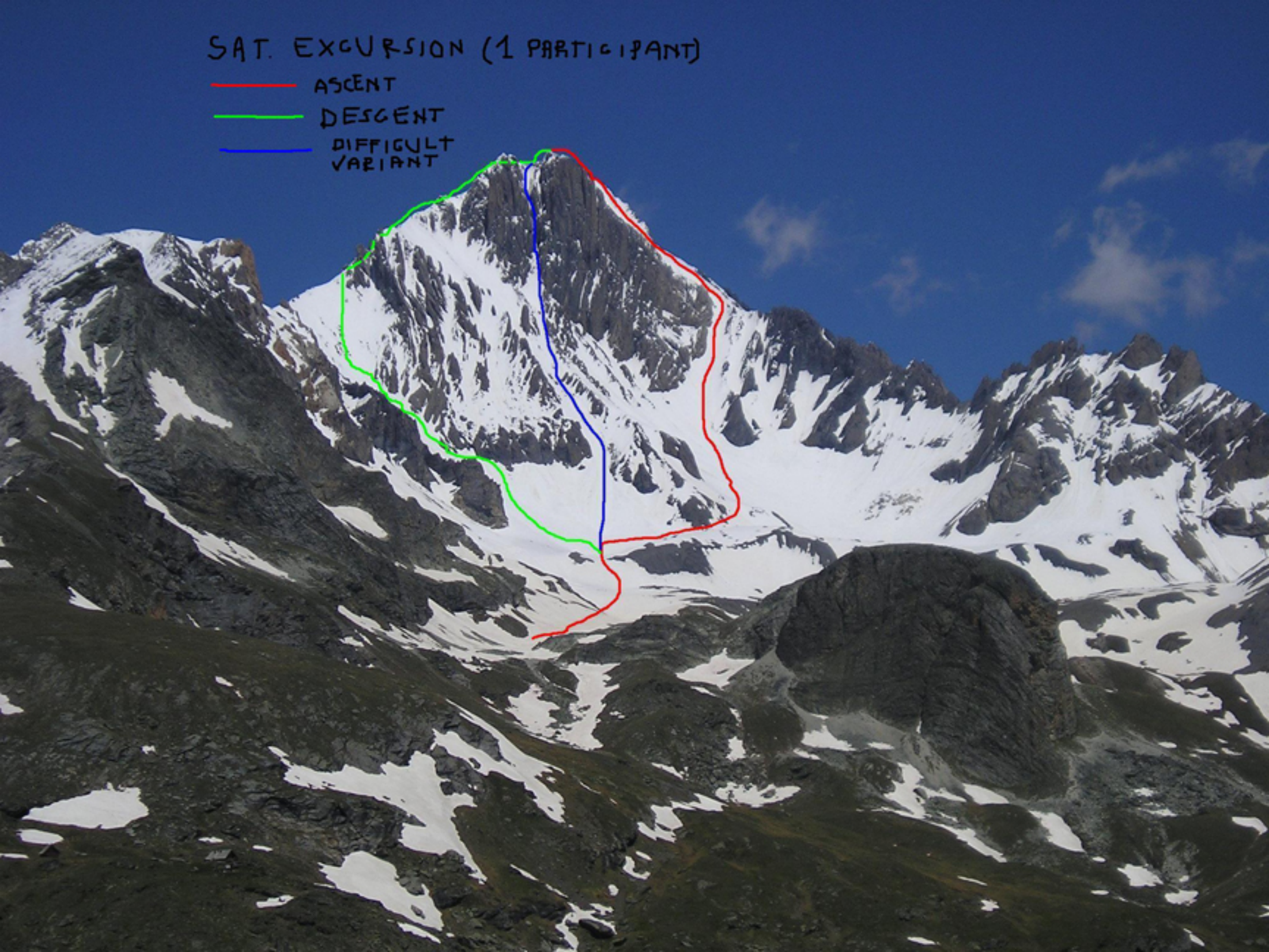
THE ENTERING GOVERNMENT AND THE OPPOSITION AGREED NOT TO FIGHT EACH OTHER SYSTEMATICALLY BUT TO COOPERATE (!!!) TO ACHIEVE IMPORTANT OBJECTIVES

JUNE 17, 2008:

ACCORDING TO THE PUBLIC OPINION, THE MAIN OBJECTIVE HAS BEEN ACHIEVED:

QUALIFICATION TO QUARTER FINALS, EURO2008

SO, GOVERNMENT AND OPPOSITION CAN START FIGHTING AGAIN...


SAT. EXCURSION (1 PARTICIPANT)
ASCENT
DESCENT
DIFFICULT VARIANT