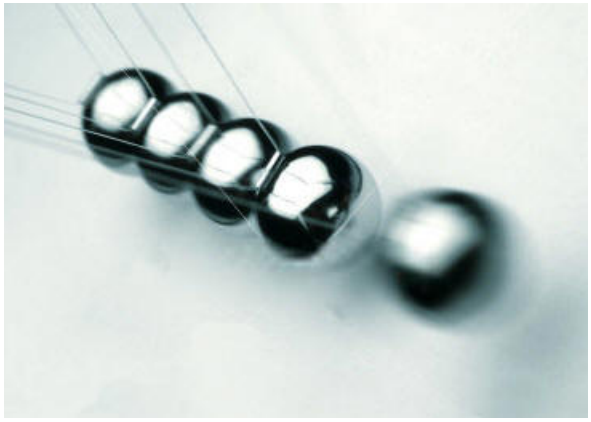
Neuvième atelier sur le contrôle optimal, les jeux dynamiques et la dynamique non-linéaire

7-9 mai 2007

Ninth Workshop on Optimal Control, Dynamic Games and Nonlinear Dynamics

May 7-9, 2007



Organisateurs / *Organizers*

Michèle Breton, GERAD - HEC Montréal

Sihem Taboubi, GERAD - HEC Montréal

Georges Zaccour, GERAD - HEC Montréal et Chaire de théorie des jeux et gestion

Le programme en bref / *Overview of Events*

Lundi, 7 mai 2007 / *Monday, May 7, 2007*

08:45–09:00 Séance d'ouverture / *Opening Session*
Roland Malhamé, directeur du GERAD / *Director of GERAD*
Pierre Hansen, membre du comité organisateur / *member of the Organizing Committee*
Georges Zaccour, organisateur du Neuvième atelier / *organizer of the Ninth Workshop*

09:00–10:00 Séance plénière MAP / *Plenary Session MAP*
Alain Haurie, Université de Genève, Suisse

10:00–10:30 Pause café / *Coffee Break*

10:30–12:10 Séances en parallèle MA / *Parallel Sessions MA*

12:10–14:00 Déjeuner / *Lunch*

14:00–15:00 Séance plénière MBP / *Plenary Session MBP*
Gustav Feichtinger, Vienna Institute of Demography, Austria

15:00–15:30 Pause café / *Coffee Break*

15:30–17:10 Séances en parallèle MB / *Parallel Sessions MB*

17:30–20:00 **Réception "Vins et fromages" / *Wine and Cheese Party***

Mardi, 8 mai 2007 / *Tuesday, May 8, 2007*

09:00–10:00 Séance plénière TAP / *Plenary Session TAP*
Dimitri Bertsekas, MIT, USA

10:00–10:30 Pause Café / *Coffee Break*

10:30–12:10 Séances en parallèle TA / *Parallel Sessions TA*

12:10–13:30 Déjeuner / *Lunch*

13:30–15:10 Séances en parallèle TB / *Parallel Sessions TB*

15:10–15:30 Pause café / *Coffee Break*

15:30–17:10 Exposé magistral TC1 / *Tutorial TC1*
Georges Zaccour, HEC Montréal, Canada

15:30–17:10 Séances en parallèle TC / *Parallel Sessions TC*

Mercredi, 9 mai 2007 / *Wednesday, May 9, 2007*

09:00–10:00 Séance plénière WAP / *Plenary Session WAP*
Richard Hartl, University of Vienna, Austria
Peter Kort, Tilburg University, Netherlands

10:00–10:30 Pause café / *Coffee Break*

10:30–12:10 Séances en parallèle WA / *Parallel Sessions WA*

Emplacement des activités / *Location of activities*

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◇ Pauses café : Société générale de financement du Québec (rez-de-jardin)
◇ Courrier électronique : salle Cogeco (l^{er} étage)
◇ Séances plénières : amphithéâtre IBM (rez-de-jardin)
◇ Autres séances : l^{er} étage
◇ Vins et fromage : Salon L'Oréal (rez-de-jardin)

◇ *Coffee break: Société générale de financement du Québec (Garden Level)*
◇ *Email facilities: room Cogeco (1st floor)*
◇ *Plenary Sessions: IBM Lecture Hall (Garden Level)*
◇ *Other Sessions: 1st floor*
◇ *Wine and Cheese Party: Salon L'Oréal (Garden Level)*

MAP Plenary I

Salle / Room: Amphithéâtre IBM
Président / Chairperson: Roland Malhamé

09:00 **Metamodeling: The Contribution of Large Scale Optimization to Environmental Integrated Assessment**

- **Haurie, Alain**, Ordecsys et GERAD, <ahaurie@ordecsys.com>

This presentation deals with the design of integrated assessment models for the analysis of air quality or climate change policies. One proposes to use oracle based optimization techniques (OBOT) to realize different sorts of coupling between economic and environment sub-models. One will present three recent developments: (i) the integration of a local pollution (ozone) model with a regional energy/technology model; (ii) the integration of an economic growth model with a global circulation model of climate change and; (iii) the design of a burden sharing game between different economic regions of the world in a post-Kyoto perspective.

MA1 Dynamic Games I

Salle / Room: Hélène-Desmarais
Président / Chairperson: Anna Jaskiewicz

10:30 **Search control optimization of electromechanical systemat minimal expense of light energy**

- **Avetisyan, Vahan**, Institute of Mechanics, NAS, Armenia, 375019, 24 b Marshal Baghramyan, Yerevan, Armenia, 375019, <vanavet@yahoo.com>
- **Martirosyan, S.R.**, Institute of Mechanics, 24-b, Marshal Baghramyan Avenue, Yerevan, Armenia, 375019

The presentation is devoted to the problem of optimal guaranteed control of an electromechanical manipulator with the purpose of searching an immobile object. In difference to [1-3], the functional representing the energy expenditure of the light source located on the gripper of the manipulator is considered. The searched object is detected if it appears in a circle of given light intensity. An algorithm for the manipulator's gripper motion control and a low for the variation of the electric current in the chain of the light source are constructed, under which the motion in the corresponding search trajectory provides detection of the searched object in the guaranteed time, with minimal expense of light energy. The economy gain of the light expense attained by the manipulator's motion along the trajectory corresponding to the constructed optimal control is evaluated. References: 1. Melikyan A.A. "The problem of time-optimal control with the search for a target point", PMM USSR, 1990, vol. 54, no 1, pp. 1-7. 2. Avetisyan V.V. "Controlled search with subsequent catch of immobile object by a robot-manipulator", in

Proceedings of the 11-th International Sym¬po¬si¬um on Dynamic Games and Applications. Tucson, USA. AZ, 2004, vol. 1, pp. 24-29. 3. Avetisyan V.V. “Control of search for an immobile object aimed at its capture”, Journal of Computer and Systems Sciences International, 2006, vol. 45, no. 6, pp. 997-1005.

10:55 **Pursuit-evasion problems of two cars in an ellipsoid under gravity**

- **Imado, Fumiaki**, Shinshu University, 4-17-1 Wakasato, Nagano, Nagano, Japan, 380-8557, <imado@imado1.shinshu-u.ac.jp>
- **Melikyan, Arik**, Russian Academy of Sciences, 101-1 Vernadsky Ave., Moscow, Moscow, Russia, 119526, <melik@ipmnet.ru>

Authors have agreed to collaborate to solve the problem with different approaches, then compare the results and discuss later. The study is progressed with following steps.
(a) Given arbitrary initial points and final points on the surface in an ellipsoid. Find the minimum time controls and trajectories from initial points to final points. The velocities at the final time are free.
(b) In above problems, final velocities are constrained to be 0.
(c) Let us consider two cars A(Pursuer)and B(Evader). Assume their masses are same, but the control force of B, F_B is less than that of A, F_A. B tries to evade with its maximum force, and the direction of F_B is parallel to the projection of the straight line A to B to the curved surface and in opposite direction to A. Find the minimum time capture solution and trajectory of A by solving the one-sided optimal control problem. Note that in this problem, if the surface is a flat horizontal plane, the solution becomes the solution of this differential game.
(d) Let the solution of (c) as nominal controls of A and B, and find the differential game solution of A and B, that is, A tries to minimize while B tries to maximize the capture time. The problem will be very difficult, however, if the curvature is very small, it may be possible to obtain the game solution.
The geometry is shown in Fig.1. The Eq. of ellipsoid is given by (1). Imado employed subsidiary $R(x,y,z) = x^2/d^2 + y^2/e^2 + z^2/f^2 - 1 = 0$ (1) parameters u,v and derivations are as follows. $x = d\cos u\cos v \quad y = e\cos u\sin v \quad z = f\sin u$ (2) The tangential lines of u,v directions at $p(x,y,z)$ on (1) is given by $\{p(u,v) + sp_v(u,v) + tp_v(u,v) \,|\, s,t \in R\}$ (3) where $p_u = \partial p/\partial u = (x_u, y_u, z_u)$ $p_v = \partial p/\partial v = (x_v, y_v, z_v)$ where s and t are real numbers and treated as control parameters. Let a car is located at p .The control force F is in the tangential plane, which is assumed to change its direction instantaneously in the plane. The normal vector at p is given by v, where $v = p_u \times p_v / \left| p_u \times p_v \right|$ (4) Let F_g be the component of the gravity force mg in the plane, then $F_g = mg - F_v = mg - (v \cdot mg)v$ (5) while, the control force is expressed by $f/m = sp_u + tp_v$ (6) $F \cdot F \le m^2 a^2$ (7) Then the problem is expressed as $\ddot{u} = k(\dot{u},\dot{v},u,v,s,t), \ddot{v} = l(\dot{u},\dot{v},u,v,s,t)$ (8) with initial and final values of $\dot{u},\dot{v},u,v$ are specified, and obtain optimal controls s and t which minimize final time under the constraint of (7). Another car model is introduced in the same way, and the dynamic game problems are solved. Currently, problems (a) and (b) are perfectly solved and

some cases of (c) are already obtained by Imado. A steepest ascent method is employed to solve them. Melikyan's derivation is as follows. $R(x,y,z)=0$ (1), $\dot{R}=n\cdot v=k_1x\dot{x}+k_2y\dot{y}+k_3z\dot{z}=0$ (8) $\ddot{R}=k_1\dot{x}^2+k_2\dot{y}^2+k_3\dot{z}^2+k_1xu_1+k_2yu_2+k_3z(u_3+g)=0$ (9) here *v* is the velocity vector, $n=(n_1,n_2,n_3)$ is a normal vector to the surface, they are given by $v=(\dot{x},\dot{y},\dot{z}),n=\nabla R=(k_1x,k_2y,k_3z),(k_1=2/d^2,k_2=2/e^2,k_3=2/f^2)$ (10) The Eqs take the form $\ddot{x}=N_1+u_1,\ddot{y}=N_2+u_2,\ddot{z}=N_3+u_3+g$ (11) where $N=\lambda n$.The control force is assumed to be tangent to the surface, which gives the restriction: $u\cdot n=u_1n_1+u_2n_2+u_3n_3=0$ (11) The optimal control and dynamic game problems will be solved following to the above steps (a)-(d) and the results will be discussed in comparing to the results of Imado. In the presentation, the results obtained by Imado up to now will be shown.

A part of the study has been conducted with the assistance of Japan Society of Promotion for Science (JSPS S-05209).

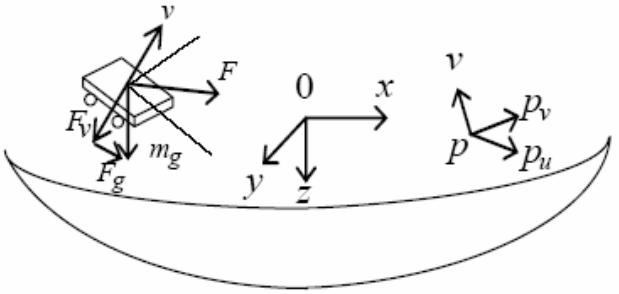

Fig.1 A car in an ellipsoid under gravity

11:20 **Zero-sum ergodic semi-Markov games**

- **Jaskiewicz, Anna**, Wroclaw University of Technology, Wybrzeze Wyspianskiego 27, Wroclaw, Poland, 50-370, <ajaskiew@im.pwr.wroc.pl>

Zero-sum ergodic semi-Markov games with the ratio-average and time-average payoff criteria are considered. Assuming that the transition probability is weakly continuous (Feller) and the one-step payoff function is lower semicontinuous and may be unbounded, we show that the optimality equation has a lower semicontinuous solution. The proof is based on a fixed point argument with regulation (smoothing) of appropriate functions. Moreover, we prove that the ratio-average as well as time-average payoff stochastic games have the same value. Next we show that player 1 has an ε-optimal stationary strategy (ε > 0), whereas player 2 has an optimal stationary strategy.

MA2 Environmental Applications I

Salle / Room: Mary-Husny
Président / Chairperson: Guiomar Martín-Herrán

10:30 **Carbon sequestration with reforestations and biodiversity-scenic values**

- **Caparrós, Alejandro**, Consejo Superior de Investigaciones Cientificas, Calle el pinar, Madrid, Spain, <acaparros@ieg.csic.es >
- **Cerdá Tena, Emilio**, Universidad Complutense, Campus de Somosaguas, Madrid, Sapin, 28223, <ecerdate@ccee.ucm.es >
- **Ovando, Paola**, Consejo Superior de Investigaciones Científicas, Calle el Pinar, Madrid, Spain, <egaop40@ieg.csic.es>
- **Campos, Pablo**, Consejo Superior de Investigaciones Científicas, Calle el pinar, Madrid, Spain, <pcampos@ieg.csic.es>

This paper presents an optimal control model to analyze reforestations with two different species, including commercial values, carbon sequestration and biodiversity or scenic values. We first solve the model qualitatively with general functions and discuss the implications of partial or total internalization of environmental values, showing that internalizing only carbon sequestration may have negative impacts on biodiversity-scenic values. To evaluate the practical relevance, reforestations in the South-west of Spain with cork-oaks (a slow growing native species) and with eucalyptus (a fast growing alien species) are compared. We do the analysis with two different carbon crediting methods, the Carbon Flow Method and the Ton Year Accounting Method, showing that the first implies to increase more the forest surface than the second. However, with the first method more eucalyptus are used while with second most of the additional reforestations are done with cork-oaks. A stated preferences study is used to value the impact on visitors of cork-oak and eucalyptus reforestations. We show that if these values were internalized the equilibrium values would imply a significantly higher amount of surface devoted to cork-oaks than to eucalyptus, even after internalizing carbon sequestration. Key words: optimal control, forests, carbon, sequestration, biodiversity, scenic, stated preferences, carbon accounting.

10:55 **Investment in tourism market and reputation**

- **Claude, Denis**, HEC Montréal, 3000 chemin de la Côte-Sainte-Catherine, Montréal, Québec, Canada, H3T 2A7, <denis_claude_economics@yahoo.fr>
- **Zaccour, Georges**, HEC Montréal, GERAD, 3000, Chemin de la Côte-Sainte-Catherine, Montréal, Québec, Canada, H3T 2A7, <georges.zaccour@gerad.ca>

Recent contributions in tourism economics acknowledge that the tourism market is imperfectly competitive and, as such, should be studied from an industrial organization perspective. This approach seems especially relevant to shed lights on one issue of importance for tourism destinations: how to achieve sustainable tourism development? Indeed, it has long been empirically observed that tourism development follows a life-cycle. After a period of growth, the development of touristic (mountain and seaside) resorts usually stagnate and decline. At least part of

the explanation for this pattern is to be found in the evolution of destinations' reputation over time. The present paper investigates the incentives for adjacent tourist resorts to invest in quality in order to maintain their collective reputation. We propose a dynamic model where i) several adjacent tourist resorts select their tourist flows and ii) invest in order to remedy to the detrimental effects tourism flows have on local environmental amenities. The overall tourist presence and the sum of investments made by tourist resorts jointly define the quality of the touristic product offered by this tourism destination. We assume that this quality cannot be observed by consumers at the time of purchase. However, in this situation of imperfect information, consumers form expectations about the quality of the touristic product offered at any point of time. These expectations define the collective reputation of tourist resorts, determine the position of the tourist resorts' demand curve and constitute the state variable in the differential game. We characterize and compare equilibrium time paths of tourist presence, investment and reputation depending on whether or not tourist resorts take their investment decisions cooperatively.

11:20 **On the Effect of Resource Exploitation on Growth: Domestic Innovation vs. Foreign Direct Investment**

- **Cabo, Francisco**, Universidad de Valladolid, Avda. Valle Esgueva, 6, Valladolid, Valladolid, Espagne, 47011, <pcabo@eco.uva.es>
- **Martín-Herrán, Guiomar**, Universidad de Valladolid, Avda. Valle Esgueva, 6, Valladolid, Valladolid, Espagne, 47011, <guiomar@eco.uva.es>
- **Martínez-García, María Pilar**, Universidad de Murcia, Campus del Espinardo, Murcia, Murcia, Espagne, 30100, <pilarmg@um.es>

We introduce a renewable natural resource sector into an endogenous growth model with an expanding variety of productive inputs. We first study the sustainability of growth in an economy that carries out domestic innovation. This hypothesis is suitable for industrialized countries, but it is less consistent with the realities of developing economies, whose technological progress can be weak or even non-existent. A second model takes this fact into account and relies on trade and on foreign direct investment in technology to solve the problem of growth sustainability. Technology is diffused from a technologically leading country to the country endowed with the natural resource. The existence, uniqueness and stability of a sustainable growth path are proved for both models. The growth rates and welfares under both scenarios, that is, under domestic innovation and under direct foreign investment, are compared.

MBP Plenary II

Salle / Room: Hélène-Desmarais
Président / Chairperson: Georges Zaccour

14:00 **Dynamics and control of age-structured population**

- **Feichtinger, Gustav**, <or@server.eos.tuwien.ac.at>

The practical motivation for this paper is provided by the recruitment problem faced by many organizations of fixed size: to keep average age young (and thus keep innovation and productivity high) while at the same time keeping levels of recruitment high. A typical example is an academy of sciences. The problem is formalized by an infinite-horizon optimal control model for a first order PDE with a non-local dynamics (a McKendrick type equation). Based on the non-standard necessary optimality condition proved in the paper, the following results are established: (i) stationarity of the optimal recruitment density; (ii) strong ergodicity of the optimal solution; (iii) principle of "bi-polar" recruitment in the case where the productivity of the organization is measured by the average age of the members. An interesting interpretation of this result is that an academy of awards should focus on awarding relatively young talents for recent outstanding achievements, and, on the other hand, old persons for their all-life-long contributions. In the second part of the paper some applications in manpower planning (Person-flow models) are given.

MB1 Marketing I

Salle / Room: Hélène-Desmarais
Président / Chairperson: Alessandra Buratto

15:30 **Games of Stackelberg Type in Bilevel Toll Design Problem**

- **Stankova, Katerina**, Delft University of Technology, Mekelweg 4, Delft, the Netherlands, 2628CD, <K.Stankova@tudelft.nl>
- **Bliemer, Michiel C.J.**, Delft University of Technology, Stevinweg 1, Delft, the Netherlands, 2628CN, <M.C.J.Bliemer@tudelft.nl>
- **Olsder, Geert Jan**, Delft University of Technology, Mekelweg 4, Delft, the Netherlands, 2628CD, <G.J.Olsder@tudelft.nl>

Optimal toll design problem with second-best tolling is a game of Stackelberg type with the road authority as a leader and drivers as followers. The game is defined on a given network with a finite nonempty set of nodes and a finite nonempty set of directed links (arcs). Some of the links are tollable. Some of the nodes are origin nodes (origins), others are destination nodes (destinations). The drivers making trips between their origins and destinations choose among alternative routes and possible departure times so as to minimize their perceived travel costs. Their decisions determine the route and link flows in the network. We assume that in an optimal state the logit-based stochastic equilibrium applies (LBSE). The road authority minimizes the total travel time of the system by setting tolls on tollable links. Two possible strategies for setting these tolls are considered: 1. For each time period the

toll on each tollable links is set as a constant. The aim of the road authority is to find optimal toll values minimizing the total travel time of the system provided that the travelers are driven by LBSE. The game within this setting is a Stackelberg game. 2. For each time period the toll on each tollable links is set as a function of the traffic flows in the network, e.g., as a linear function of the link flow on this tolled link. In this case the aim of the road authority is to find an optimal toll function minimizing the total travel time with travelers driven by LBSE. The game within this setting is an inverse Stackelberg game. After introduction of the games and discussion about general properties of the problem we consider two case studies: a 2-link network with one link tollable solved analytically and a 'real size' network (56 links, 8 links tollable) solved numerically. In the case studies we compare outcomes of inverse Stackelberg and Stackelberg games. We show that the use of traffic-flow dependent tolling can markedly improve the system performance. For the road authority the traffic-flow dependent tolling is always at least as good as the constant tolling possibility. Keywords: Stackelberg game, inverse Stackelberg game, toll, optimal toll design problem, logit-based stochastic equilibrium, traffic flow.

15:55 **Optimal control of trade discounts in a vertical distribution channel**

- **Ellero, Andrea**, Università Ca' Foscari di Venezia, Dorsoduro 3825/E, Venezia, Italy, 30123, <ellero@unive.it>
- **Bykadorov, Igor**, Siberian Branch Russian Academy of Sciences, Acad. Koptyug prospect 4, Novosibirsk, Russia, 630090, <bykad@math.nsc.ru>
- **Moretti, Elena**, Università Ca' Foscari di Venezia, dorsoduro 3825/E, Venezia, Italy, 30123, <emoretti@unive.it>

To earn a reasonable profit the members of a distribution channel often adopt rather simple pricing techniques. For example, manufacturers may use cost-plus pricing, simply defining the price adding a desired profit margin to (variable) production costs; in a similar fashion, retailers very often use to determine shelf prices adding a fixed percentage markup to the wholesale price. In this paper we focus on the effects of trade promotions, a widely used dynamic pricing strategy that manufacturers can exploit to raise sales. Trade discounts have usually a double positive effect on sales since part of the wholesale price reduction may be transferred to the shelf price (pass-through) and part of the discount will be kept by the retailer who will be more motivated, and higher motivation means higher effort in selling the product. We consider explicitly retailer’s motivation as a state variable and retailer's performance as a function of retailer's skill and motivation [1], [3]. The trade-off between manufacturer's and retailer's goals, each of the two firms aims at maximizing its own profit, leads us to consider channel pricing policies in a differential game framework [2]. We assume that the manufacturer can decide the discount on wholesale price (trade discount) while the retailer controls the amount of pass-through. Depending on the particular market structure manufacturer and retailer can both become leader of the channel, depending on the particular market structure: For particular feasible control sets we are able compare the profits of the members of the distribution channel at Nash equilibrium points and at Stackelberg equilibria. In particular we describe the different cases in which manufacturer and/or retailer prefer to be leader

or to be follower analyzing the values of the model's parameters. References: [1] Bykadorov, I., Ellero, A. and Moretti, E., Optimal control of trade discount in a vertical distribution channel, Rendiconti per gli Studi Economici Quantitativi, Numero speciale, (2005) 121-129. [2] Jœrgensen, S., Zaccour, G., Differential games in marketing, Kluwer Ac. Pub., London (2004). [3] Mitchell T. R., Motivation: new directions for theory, research and practice, Academy of management review, 7 (1982) 80-88.

16:20 **Integrated marketing communication programs in licensing**

- **Buratto, Alessandra A.**, University of Padova, Via Trieste, 63, Padova, Italy, 35131, <buratto@math.unipd.it>

Licensing is the process of leasing a legally protected entity (brand, name, logo, ...) in conjunction with a product or a product line. It is usually based on a contractual agreement between two business entities: the owner of the property, called licensor, and the renter of the rights, called licensee. Licensing is used as a marketing tool with the aim of generating brand awareness and reinforcing brand image. Both the owner of the properties and the manufacturer take benefits from such an agreement and marketing communication plays an important role in order to achieve the desired objectives. Furthermore, integrating the actions of the two parts becomes crucial for a successful licensing. We formulate and analyze a differential game in which the licensor and the licensee are the players, with the respective communication programs as actions, and with their profits as the natural objectives. We consider some variants of the game associated with different types of agreement in the licensing contract and we discuss possible equilibria.

MB2 Optimal Control and System Dynamics (DOCSD) I

Salle / Room: Mary-Husny
Président / Chairperson: Peter Kort

15:30 **Using Artificial Neural Networks to Model Nonlinear Dynamics in the Employee Turnover Process**

- **Somers, Mark J**., New Jersey Institute of Technology, School of Management, NJIT, Newark, New Jersey, USA, 07102, <somers@adm.njit.edu>

Although artificial neural networks (ANN's) are widely used in business applications in disciplines such as finance and operations research, there is comparatively little interest in using to them to model employee behaviors at work such as turnover, absenteeism and job performance. This is unfortunate because the few applications of ANN's to modeling employee turnover) and job performance have yielded far better predictive accuracy than have linear multivariate statistics with results indicating strong evidence of nonlinearity with implications for both theory building and model building. This study models voluntary turnover among 245 staff nurses using a well established constellation of predictor variables including: job satisfaction, job involvement, job tenure, job search behavior, role conflict, role ambiguity, age, and level of education. It has two purposes. The first is

build an optimal model among these predictors using two neural network paradigms: multi-layer perceptrons (MLP's) and radial basis functions (RBF's). The second is to explore nonlinear dynamics in employee turnover using the neural networks that produced the best results. Results indicated that all of the predictor variables are relevant to modeling employee turnover and that both neural networks outperformed the linear model. Specifically, binary logistic regression correctly classified 60.6% of the sample, but misclassified 100% of leavers (turnover being a binary variable comprised of stayers and leavers). The most accurate model was an RBF neural network with 4 neurons in the hidden layer. This model correctly classified 67% of the test data sample (data from which weights were not derived), 64% of stayers, and 73% of leavers. The next most accurate model was an MLP with 8 neurons in the hidden layer. This model correctly classified 63 percent of the test data sample, 61% of stayers and 68% of leavers. Detailed examination of relationships among selected predictors and turnover using response surfaces indicated clear evidence of nonlinearity. These nonlinear dynamics took the form of thresholds or "tipping points" where the likelihood of turnover was either greatly attenuated or potentiated. For example, role conflict and role ambiguity interact to a point to increase the likelihood of turnover, but then reach an abrupt ceiling where further increases have no effect. At this point, the process is driven by other factors. Similarly, high levels of job satisfaction and low intensity of job search inhibit turnover until job search reaches a certain level, at which point, decreases in job satisfaction greatly increase the probability of turnover. These findings suggest that ANN's are a valuable tool in modeling and managing employee turnover. Organizations seeking to predict overall turnover would do well to use neural networks in additional to conventional statistical methods. Researchers interested in understanding the psychological and economic dynamic of turnover are also likely to gain insights that are likely to be uncovered using linear models and methods.

15:55 **Dominant Firms, Barriers to Entry Capital and Antitrust Policy**

- **Kato, Mika**, Howard University, 2400 4th St NW ASB-B-319, Washington, DC, USA, 20059, <mkato@howard.edu>
- **Semmler, Willi**, New School for Social Research, 65 Fifth Ave, New York, NY, USA, 10003, <semmlerw@newschool.edu>

The main idea of our paper comes from earlier industrial organization literature that has shown that the threat of entry limits the price setting power of dominant firms and stimulates the incumbents to undertake innovations. We provide a theoretical framework of the dynamics of competition where incumbents attempt to restrict or inhibit competition through competition restricting investments such as advertisement, political lobbying, protection of innovations through patents, and creating excess capacity, etc.. Depending on how regulatory institutions respond to this type of investment, complex dynamics may emerge. Since the effectiveness of competition restricting investments depends on regulatory rules set and enforced by antitrust institutions, we show how an antitrust and competition policy can be designed that may prevent the build up of such a competition restricting capital, strengthening incentives for price and innovation competition.

16:20 **Two state capital accumulation with heterogenous products: disruptive vs. non-disruptive goods**

- **Kort, Peter**, Tilburg University, PO Box 90153, Tilburg, Netherlands, 5000le, <kort@uvt.nl>
- **Hartl, Richard**, University of Vienna, <richard.hartl@univie.ac.at>

This paper considers the problem of a firm having the option to produce different products for the same market. One product is an old fashioned "standard" one that has been around already for a long time. The other product is a recent innovation, which is qualitatively better than the older one. At the same time, producing this innovative product is more complicated for the firm, because it requires skilled labor, advanced technology, or other inputs that are in limited supply. A straightforward example is Philips, which has produced the "standard" CRT television sets for decades, while in 1999 it started a joint venture with its Korean partner LG to produce LCD (flat screen) television sets. Another example (the Economist, May 20th, 2006, p.75) is Neumann, a traditional microphone producer that is investing 1.4m Euro to develop digital microphones ("Someone will do it, so we decided it should be us"). The aim of this paper is to establish the optimal production levels of two different goods over time. The goods, one being old fashioned or "standard" and the other being more advanced or "high tech", are produced by the same firm and are sold on the same heterogeneous product market. This implies that an increased quantity of one good not only reduces the price of this good but also the price of the other good. This framework enables us to establish the optimal speed at which the firm should start taking a high tech product into production, given that it is already actively producing a standard good. In other words, we try to answer the question of whether and how quickly to introduce an upgraded product that will cannibalize part of an existing product's market. The paper is original in that it looks at the question in continuous time and with state variables for two capital stocks, each of them governing the production processes of the two products. We are interested in understanding not only whether a firm should introduce the new product but also what is the ideal rate at which to introduce it and how that affects sales of the standard product. Hence, we need to employ an optimal control model to focus on "dynamics". This inevitably raises complexities relative to a discrete, two-time period model, so we focus on the case of a monopolist or someone for whom pricing drives innovation but not competition. This approach yields an extra benefit. As the examples in this introduction reflect, competition is often invoked as the reason for introducing a new product, but we will see that even a monopolist can benefit from product innovation and, hence, face structurally similar choices. In the paper we make a distinction between disruptive and non-disruptive goods. We argue that, where non-disruptive goods may behave according to a usual linear demand structure, disruptive goods have the characteristic that small volumes have negligible influence while the standard product is completely disappeared when the disruptive good is dominantly present in the market. Hence, disruptive goods are more than just a high-tech good in the sense that when they are present in significant quantities, they wipe out the value of the standard good altogether. The basic punch line would be that non-disruptive goods can coexist or not, depending smoothly on the parameter values, but that firms need to be more

decisive about their strategy with respect to a disruptive good, and a monopolist may prefer not too let them into the market. It turns out that the presence of non-disruptive goods leads to saddle point behavior, while disruptive goods generate a history dependent (Skiba) solution. Our inquiry is motivated by the case of product innovation, in which case the initial stock of capital dedicated to producing the innovated product is zero. However, we will also provide solutions for other initial values of this capital stock. This generality is relevant if, for example, a firm producing two different products that are imperfect substitutes experiences unanticipated process innovation that alters one or more relevant problem parameters and needs to know how optimally to steer to a new equilibrium.

TAP Plenary III

Salle / Room: Amphithéâtre IBM
Président / Chairperson: Dominique Orban

09:00 **Convex Optimization: Old Themes and New Treatments**

- **Bertsekas, Dimitri P.**, Massachusetts Institute of Technology, <dimitrib@mit.edu>

Dimitri P. Bertsekas Convex optimization was radically transformed following Fenchel's work (1951), and was articulated by Rockafellar in his classic text (1970) and in the more recent variational/nonsmooth analysis text by Rockafellar and Wets 1998). Yet, some of the fundamental underpinnings of the field have remained poorly understood by a broad audience, because of the difficulty of the analysis and the lack of unification. This talk will review an effort by the author in a recent book to restructure the subject using a handful of unifying principles that can be easily visualized and readily understood. Several new lines of research and analysis were necessary for this, including: (1) A unified development of minimax theory and constrained optimization duality as special cases of duality between two simple geometrical problems. (2) A unified development of conditions for existence of solutions of convex optimization problems, conditions for the minimax equality to hold, and conditions for the absence of a duality gap in constrained optimization. (3) A unification of the major constraint qualifications that guarantee the existence of Lagrange multipliers for nonconvex constrained optimization, using the notion of constraint pseudonormality and an enhanced form of the Fritz John necessary optimality conditions.

TA1 Dynamic Games II

Salle / Room: Hélène-Desmarais
Président / Chairperson: Florian Wagener

10:30 **Optimal cooperation and trust over time**

- **El Ouardighi, Fouad**, ESSEC Business School, <elouardighi@essec.fr>

In this paper, the problem of optimal cooperation among two players is addressed in a dynamic setting. Following El Ouardighi (2002), a partnership is described as the interplay between a joint production activity and a mutual trust process. In order to study each player's optimal contribution to the partnership, an objective function is specified both for non-cooperative and cooperative settings. Explicit solutions are provided for various configurations under a finite decision horizon. A ranking between the solutions obtained is then established to determine the impact of mutual trust on each player's contribution.

10:55 **A Model of Dynamic Stable Joint Venture**

- **Zenkevich, Nikolay A**, St. Petersburg State University, 3, Volkhovskiy per., St. Petersburg, Russia, 199004, <zenkevich@som.pu.ru>

Consider the case when there are 3 companies involved in joint venture and share their joint profit according to the dynamic Shapley. Through knowledge diffusion participating firms can gain core skills and technology that would be very difficult for them to obtain on their own. The evolution of the technology level of company under joint venture is known. The profit of the joint venture is the sum of the participating firms' profits. The member firms would maximize their joint profit and share their cooperative profits according to the Shapley value. Applying the method of regularization for dynamic cooperation problem, we constructed the control in the form of special payments, paid at each time instant on the optimal trajectory. The dynamic stable solution is obtained for the joint venture dynamic model. Literature Petrosjan L. A., Zaccour G. 2003. Time-consistent Shapley value allocation of pollution cost reduction. Journal of economic dynamics and control 27 (3): 381-398. Petrosjan L. A., Zenkevich N. A. 1996. Game Theory. World Scientific Publishing Co. Pte. Ltd.: Singapore. Shapley L. S. 1953. A value for n-person games. In: Contributions to the Theory of Games II. Princeton University Press: Princeton; 307-317. Yeung D. W. K., Petrosyan L. A. 2006. Cooperative stochastic differential games. Springer.

11:20 **Markov-Perfect Nash Equilibria in Models With a Single Capital Stock**

- **Wagener, Florian FOO**, Universiteit van Amsterdam, Roetersstraat 11, Amsterdam, Netherlands, 1018 WB, <wagener@uva.nl>
- **Dockner, Engelbert EJ**, University of Vienna, Brünner Strasse 72, Wien, Austria, A-1210, <engelbert.dockner@univie.ac.at>

Many economic problems can be formulated as dynamic games in which strategically interacting agents choose actions that determine the current and future levels of a single capital stock. We study necessary conditions that allow us to characterize Markov perfect Nash equalibria (MPNE) for these games. These conditions result in an auxilary system of ordinary differential equations that helps us to explore stability, continuity and differentiability of MPNE. The techniques are used to derive detailed properties of MPNE for several games including the exploitation of a finite resource, the voluntary investment in a public capital stock, and the inter-temporal consumption of a reproductive asset.

TA2 Environmental Applications II

Salle / Room: Mary-Husny
Président / Chairperson: Lucia Sbragia

10:30 **A Robust Feedback Nash Equilibrium in a Climate Change Policy Differential Game**

- **Hennlock, Magnus**, School of Business, Economics and Law, Gothenburg University, P.O. Box 640, Gothenburg, Sweden, 405 30, <Magnus.Hennlock@economics.gu.se>

Climate change policies involve decision-making processes that are subject to fundamental uncertainties concerning the underlying scientific information available. Policy makers' decisions to take or not take measures today are based on scientists' long term projections, generated by physical climate models, in general Global Circulation Models (GCM), evaluated for different emissions scenarios (Harvey, 2000). This paper develops a differential game based on a physical model of global climate dynamics similar as in e.g. Nordhaus and Yang (1996), and using assumptions in the IPCC 2001 scientific report (Houghton, et al. 2001) for calculating radiative forcing and global mean temperature due to anthropogenic CO_2 emissions. Particular effort has been made to find analytical tractable solutions to feedback Nash Equilibria given this physical climate model dynamics as a constraint for $N > 2$ asymmetric players, acting as regional social planners in N regions, as they choose regional investment strategies (in regional production that generates CO_2) and regional abatement (reduction in CO_2 emissions) strategies. The scientific research on climate change has also made clear that the fundamental uncertainty about climate change concerns not only outcomes but also probability distributions of global mean temperature projections as true or inferred future probability distributions are not evident from historic samples as climate changes. In GCM simulations, which usually spans 100 years into future, probability distributions are usually results from scientists' ad hoc statement based on different assumptions, resulting in several models generating several scenarios for given emissions projections, see IPCC 2001 scientific report (Houghton, et al., 2001) Accordingly, the differential game model in this paper introduces uncertainty by perturbing the climate change dynamics such that radiative forcing, and hence, global mean temperature will have unknown outcomes and probability distributions. Specifically, each regional player is facing a set of dynamic climate models when choosing strategies (Hansen et al., 2001). The feedback Nash equilibrium strategies in investment and abatement are then extended to robust feedback Nash equilibrium strategies as each player solves a robust control maximin problem (Gilboa and Schmeidler, 1989) and (Hansen et al., 2001) given that every other player does the same, facing the same set of climate dynamic models. Pareto optimal strategies are compared to robust as well as non-robust feedback Nash equilibrium. A sensitivity analysis as well as a simulation for the three types of solutions are performed using 1990 baseline levels of atmospheric CO_2 concentration rate and global mean temperature level as initial conditions.

10:55 **Traditional and Emergent Environmental Regulations and the Policies of the Firm**

- **Sokri, Abderrahmane**, HEC Montréal, 3000 Côte-Sainte-Catherine, Montréal, Québec, Canada, H3T 2A7, <sokri@hotmail.com>
- **Zaccour, Georges**, HEC Montréal, GERAD, 3000, Chemin de la Côte-Sainte-Catherine, Montréal, Québec, Canada, H3T 2A7, <georges.zaccour@gerad.ca>

We study in this paper the impact of a Public Disclosure Program (PDP) as well as traditional environmental regulation (tax/subsidy) on optimal policies of the firm. A PDP aims at forcing the firm to report its emissions. This information affects its image (goodwill), and ultimately its profit. In our model, this impact is endogenous, i.e., a firm polluting less than its prescribed target would win consumer's sympathy and raises its goodwill, whereas it is the other way around when the firm exceeds its emissions quota. The concept of goodwill (or brand equity) is inherently dynamic and so is our model. The evolution of this goodwill is assumed to depend also on advertising expenditures. We address the following research questions: (1) What are the optimal emissions, pricing and advertising policies of the firm under the different regulatory regimes? (2) How the different regulatory scenarios, i.e., PDP, tax/subsidy, both regulations, and no regulation (laissez-faire policy), compare in terms of the above policies? (3) Under which conditions, if any, a PDP can be profit improving with respect to a laissez-faire policy?

11:20 **Dynamic models for international environmental agreements**

- **Sbragia, Lucia**, HEC Montreal, 3000 Chemin de la Côte-Sainte-Catherine, Montreal, Quebec, Canada, H3T2A7, <lucia.sbragia@hec.ca>
- **Breton, Michèle**, GERAD, CREF, HEC Montreal, 3000 Chemin de la Côte-Sainte-Catherine, Montreal, Quebec, Canada, H3T 2A7, <michele.breton@hec.ca>

International environmental problems where many countries are involved are characterized by the fact that full participation to an international environmental agreement is very unlikely to be observed. In this paper, we develop a model to analyze, in a dynamic framework, how countries join environmental agreements. In the model, a non-signatory country decides of its emissions by maximizing its own welfare, whereas a signatory country decides of its emissions by maximizing the aggregate welfare of all signatory countries. All countries suffer from the same environmental damage. Signatory countries are assumed to be able to punish the non-signatories, proportionally to the number of signatory countries and to the level of pollution. This has a cost, which is proportional to the number of non-signatory countries and to the level of pollution. Two dynamics are considered: first, the evolution of the pollution overtime, which increases with the total emissions of countries; second the evolution of the number of signatory countries, which is described by a replicator dynamics, according to which the strategy that currently performs better will be imitated by a fraction of the players in next period. The study starts from the static game. We then consider a repeated game where (myopic) countries play the optimal static strategies over time. Finally, we assume that countries are able to account for the evolution of the state in their decisions. A local and global stability analysis is developed as well as a sensitivity analysis with

respect to the different parameters. The fundamental question is if this model is able to capture (realistic) situations where some countries are in and others are out of an international environmental agreement.

TB1 Dynamic Games III

Salle / Room: Hélène-Desmarais
Président / Chairperson: Leon Petrosyan

13:30 **Dynamic Games of Economic Growth**

- **Leong, Chee Kian**, <leong_chee_kian@pmail.ntu.edu.sg>
- **Huang, Weihong**, Nanyang Technological University, Division of Economics, School of Humanities and Social Sciences

This paper analyzes a dynamic game of economic growth. A vote maximizing government faces the trade-off between long run economic growth and distributional equity while the private sector consists of capitalowning firms which maximize the stream of dividend payments over time. Acceptable solutions do not exist except at the steady state. Even when acceptable solution exists, cooperation will only be sustained if none of the players are unfairly treated. Cheating revert cooperation to the feedback Nash equilibrium. The vote cost for distributional conflict is higher for developing countries.

13:55 **Lobbying and Public Good Provision in a Federal Economy:A Dynamic Approach**

- **Sengupta, Bodhisattva Mr**, McGill University, 443, Leacock Building, 855, Sherbrooke Street West, Montreal, Quebec, Canada, H3A 2T7, <bodhisattva.sengupta@mail.mcgill.ca>

In this paper, we address the issue of dynamic lobbying within a federation that consists of a central government and two regional governments. Regions lobby for more central funds to produce a pure public good such as an improvement in environmental quality. Thus, benefits of lobbying are public, unlike what is traditionally assumed in the public choice literature. Lobbyists capture a part of the central grant; the other part is used to produce the public good. Allocation of central funds is dictated by two considerations: the level of lobbying as well as the effective use of central funds. Welfare to the consumer equals utility from the public good, net of lobbying cost. Unlike the regional governments, the central government is non strategic. We investigate steady state behaviour as well as welfare implications of different lobbying protocols: e.g. open and closed loop cooperative and non cooperative behaviour. We show that, for symmetric provinces, the steady state lobbying level and stock of public good are greater than the case when lobbyists are benevolent and cannot capture the rent. Second, introducing a rent-appropriating lobbyist may increase the welfare of the consumers compared to the case where the lobbyist is 'benevolent'. Third, for a wide range of parameter values, non-cooperative lobbying results in higher welfare for the consumer vis-à-vis

cooperative lobbying protocols. Predictions of Fershtman and Nitzan (1991) emerge as limiting cases of our model.

14:20 **Strategically Supported Cooperative Solutions**

- **Petrosyan, Leon**, St.Petersburg State University, Universitetskaya nab. 7/9, Jelesnovodskaya ul. 27, app.20, St.Petersburg, Russia, 199155, <spbuoasis7@peterlink.ru>

Consider an N-person cooperative differential game G(x,T-t) with independent motions from the initial state x and with prescribed duration T-t. Suppose that y(s) is a cooperative trajectory maximising the sum of players payoffs. Suppose also that before starting the game players agree to allocate the joint cooperative payoff V(N,x,T-t) according to the imputation A, which is considered as a solution of the cooperative game G(x,T-t). It can be shown that there esists allways an nonnegative IDP (imputation distribution procedure) which garantees the time-consistency (dynamic stability) of the solution A. Using individual rationality of the imputation A we prove that if in the subgames G(y(s),T-s) along the cooperative trajectory y(s), the solution will be derived from the imputation A with the use of the above mentioned IDP there exists an "weak" Nash equilibrium in G(x,T-t) for which the payoffs of the players in the game will be equal exactly to the components of the imputation A (solution of the cooperative game G(x,T-t)). In this sence we can say that the cooperative solution A is strategically supported by some specially constructed "weak" Nash equilibrium. Similar results can be obtained for cooperative differential games with non-trasferable payoffs and differential games with coalitional structures.

TB2 Deterministic Optimal Control and System Dynamics (DOCSD) II

Salle / Room: Mary-Husny
Président / Chairperson: Silvia Faggian

13:30 **The Pontryagin maximum principle and optimal economic growth problems**

- **Aseev, Sergey**, Steklov Institute, Gubkina str. 8, Moscow, Russia, 119991, <aseev@mi.ras.ru>
- **Kryazhimskiy, Arkady**, International Institute for Applied Systems Analysis, Slossplatz 1, Laxenburg, Austria, A-2361, <kryazhim@iiasa.ac.at>

The paper is devoted to the theory of first-order necessary optimality conditions for optimal control problems with infinite time horizon. Such problems naturally arise in numerous studies on optimization of economic growth. We develop an approach for approximation of infinite horizon optimal control problems by finite horizon ones and use it to prove new versions of the Pontryagin maximum principle in general nonlinear and non convex setting. A special attention is paid to the transversality conditions at infinity. Using developed approximation approach we describe typical cases, in which some complementary conditions on the adjoint variable and the Hamiltonian take place. In some situations these conditions imply

validity of the standard transversality conditions at infinity, which commonly used in economic applications. Several significant earlier results are generalized. Examples, demonstrating possible pathologies of bahavior of adjoint variable at infinity in discounted infinite horizon optimal control problems are presented. Applications of the results obtained to particular optimal economic growth problems are demonstrated as well. Keywords: optimal control, infinite horizon, the Pontryagin maximum principle, transversality conditions, optimal economic growth Bibliography: 1. S.M. Aseev, A.V. Kryazhimskiy, The Pontryagin maximum principle for an optimal control problem with a functional specified by an improper integral, Doklady Mathematics, 69, No. 1, pp. 89-91, 2004. 2. S.M. Aseev, A.V. Kryazhimskiy, The Pontryagin maximum principle and transversality conditions for a class of optimal cintrol problems with infinite time horizons}, SIAM J. on Control and Optimization, 43, pp. 1094-1119, 2004.

13:55 **Cake Eating and Life Cycle Saving: Existence Theorems for a Class of Continuous Time Allocation Models**

- **Leung, Siu Fai**, Hong Kong University of Science and Technology, <sfleung@ust.hk>

This paper investigates the existence of an optimal solution for a class of continuous time allocation problems which include models of cake eating and life cycle saving. We offer examples of these models for which an optimal solution does not exist and study the reasons for the non-existence. We provide two theorems to show that the existence can be guaranteed if some simple conditions are satisfied. Several examples are offered to illustrate the usefulness of the conditions.

14:20 **Dynamic programming for infinitehorizon boundary control problems of PDE's with age structure**

- **Faggian, Silvia**, LUM "Jean Monnet", c/o Baricentro, SS 100, km18, Casamassima, BA, Italy, 70010, <faggian@lum.it>
- **Gozzi, Fausto**, LUISS "Guido Carli", Viale Pola, Roma, Roma, Italy, <fgozzi@luiss.it>

We develop the dynamic programming approach for a family of infinite horizon boundary control problems with linear state equation and convex cost. We prove that the value function of the problem is the unique regular solution of the associated stationary Hamilton-Jacobi-Bellman equation and use this to prove existence and uniqueness of feedback controls. The idea of studying this kind of problem comes from economic applications, in particular from models of optimal investment with vintage capital. Such family of problems has already been studied in the finite horizon case in some previous papers of these authors. The infinite horizon case is more difficult to treat and it is more interesting from the point of view of economic applications, where what mainly matters is the behavior of optimal trajectories and controls in the long run. The study of infinite horizon is here performed through a nontrivial limiting procedure from the corresponding finite horizon problem.

TC1 Tutorial - Zaccour

Salle / Room: Banque Scotia
Président / Chairperson: Sihem Taboubi

15:30 **Sustaining Cooperation Over Time in Environmental Differential Games**

- **Zaccour, Georges**, HEC Montréal, GERAD, 3000, Chemin de la Côte-Sainte-Catherine, Montréal, Québec, Canada, H3T 2A7, <georges.zaccour@gerad.ca>

The objective of this tutorial is to review the different approaches put forward in the literature to sustain cooperation over time in differential games. After introducing the terminology and the nature of the problem, I will focus on the concepts of cooperative equilibria, time-consistency, agreeability, and incentive strategies. All approaches will be illustrated using environmental-economics examples.

TC2 Stochastic Control

Salle / Room: Mary-Husny
Président / Chairperson: Kuno Huisman

15:30 **An application of piecewise deterministic optimal control problem to brand management**

- **Rubel, Olivier**, GERAD - HEC Montréal, <olivier.rubel@hec.ca>

A stochastic optimal control problem faced by a firm subject to product-harm crises is analyzed. The problem is of the piecewise deterministic optimal control variety. The firm controls the product's price as well as corporate and brand advertising investments. The crisis may occur with a fixed probability at any instant of time. Once the crisis has occurred, the corporate and brand goodwill stocks are decreased according to a damage function. We analyze the optimal strategies and consider two cases, whether or not there is a "re-build" period after the crisis where the firm does not sell the product but invests in corporate and brand advertising in order to mitigate the crisis effects. Managerial conclusions are derived.

15:55 **The worst case for real options**

- **Trojanowska, Magdalena**, University of Antwerp, Prinsstraat 13, Antwerp, Belgium, 2000, <Magdalena.Trojanowska@ua.ac.be>
- **Kort, Peter**, Tilburg University, PO Box 90153, Tilburg, Netherlands, 5000le, <kort@uvt.nl>

We examine the problem of timing of an investment decision under uncertainty in the presence of high frequency shocks, which arise to, e.g., forthcoming innovations. Because the application of the rational expectations equilibrium concept endows the investor with too much knowledge about the future growth prospects, we adopt a framework where the decision maker is ambiguity averse and the possibility of model misspecification is explicitly taken into account. We derive a robust decision rule for an investment in a finite life project in presence of a

stochastic instantaneous return. We find that ambiguity aversion (fear of model misspecification) accelerates investment in the short-run, while the opposite holds in the long-run. Moreover, a large degree of ambiguity results in foregoing investment with greater probability than in the absence of ambiguity (model misspecification). We apply ex post analysis to validate our decision criterion. We derive the stochastic endogenous discount factor implied by ambiguity aversion and we analyze the investment lag caused by the ex ante lack of certainty about the dynamics of the profit flow. The investment threshold for short life projects is found to be more fragile to model misspecification than the threshold for long life or perpetual projects.

16:20 **A stochastic version of the signaling game**

- **Gryglewicz, Sebastian**, Tilburg University, Warandelaan 2, PO Box 90153, Tilburg, The Netherlands, 5000 LE, <s.gryglewicz@uvt.nl>

Many important economic situations are set in dynamic stochastic environments, yet such a feature has been absent from the models based on signalling games. This paper studies a new class of signalling games in continuous time in which the stake contested by the uninformed player is a diffusion process. The informed player's type is either strong or weak and is fixed, while it is costly to keep the other player uninformed. The uniformed player gets the stake if he contests it from the weak type, but receives nothing if the other player is strong. Our primary insight is that, as the stake evolves in a stochastic environment, at some point of time the incentive constraints may stop being binding. In particular, provided that the game starts at the pooling equilibrium, the uninformed player wants to enter as the stake gets high enough given his belief about the other player's type. On the other hand, if the stake gets low enough, the weak informed player does not want to send costly signals anymore and prefers to reveal her type. The decisions of the two players are strategically interrelated. The best responses are optimal stopping decisions, with the uninformed (resp. informed) player seeking a critical high state (resp. low state) to stop the signalling game. We prove that the stopping time game has no Markov perfect equilibrium in pure strategies. Using the updated beliefs about the informed player's type as a second state variable, we characterize an equilibrium in which the informed player applies a mixed strategy and the uniformed one solves a free-boundary problem. The uninformed player's problem is non-trivial and involves path-dependent payoffs and learning from the path of the diffusion process. Yet the characterization of the equilibrium is relatively basic and is a solution to two ordinary differential equations. Our model provides new insights into standard signalling situations in industrial organization and corporate finance. We discuss several of such applications. In industrial organization, the standard static signalling models of limit pricing provide little in terms of policy instruments to detect anti-competitive pricing practices. The stochastic version of the limit pricing game implies that price dynamics may reveal limit pricing of incumbents. In the corporate finance theory, both signalling and continuous-time diffusion processes play prominent roles. The stochastic signalling game merges these two, so far independent, modelling environments. The model can, for instance, explain and

endogenize the random default triggers from some structural credit risk models to obtain positive credit spreads for debt contracts with very short maturities.

16:45 **Investment in Hightech Industries: an Example from the LCD Industry**

- **Huisman, Kuno**, Tilburg University, Post Office Box 90153, Tilburg, The Netherlands, 5000 LE, <k.j.m.huisman@uvt.nl>
- **Kort, Peter**, Tilburg University, PO Box 90153, Tilburg, Netherlands, 5000le, <kort@uvt.nl>
- **Plasmans, Joseph**, University of Antwerp, Prinsstraat 13, Antwerp, Belgium, 2000, <joseph.plasmans@ua.ac.be>

In this paper we analyze investment decisions of firms in hightech industries. Typical examples of hightech industries are industries for electronic (consumer) products such as dvd players, LCD (liquid-crystal display) television sets, personal computers, MP3 players, photo cameras, mobile phones, and personal digital assistants. Prices for personal computers dropped very fast during the last decades. Delaying a purchase decision with one year would thus imply that the same or even a better personal computer will be available for less money. The same holds for other products such as digital photo cameras, dvd players, and LCD television sets. Another feature of this kind of industries is that hightech products become obsolete more quickly, i.e. the economic lifetime of these products becomes shorter and shorter. As an example think of the quick increase in the number of megapixels in a digital photo camera. Every new generation of this product has more megapixels and makes the previous generations on the one hand less attractive for the high-end consumers, but on the other hand reachable for the more moderate consumers due to the lower prices. From the production side it is known that there is an enormous learning effect in the production process, implying that production costs are decreasing over time. We conclude that hightech firms face sharply decreasing prices, rapid product changes, and decreasing production costs. A lot of hightech industries face a phenomenon that is called the crystal cycle in the LCD industry (see also Mathews (2005)). Due to investing capacity increases. Then, to leave no capacity idle, firms reduce the price in order to attract consumers. After a while demand exceeds the available capacity, which makes that firms increase their prices. The result is that demand drops and becomes smaller than capacity. This in turn implies that firms will lower their prices again and the next cycle starts. Mathews (2005) shows that in the period 1990 to 2003 there have been five cycles. Real options theory is the appropriate tool to analyze investment decisions under uncertainty. For an excellent introduction to the real options literature we refer to Dixit and Pindyck (1994). In most real options models uncertainty is incorporated via a geometric Brownian motion process (GBM). Departing from this theory, this paper analyzes the investment problem of the hightech firm. After that we confront this theoretical framework with real life data and find that, mainly due to the crystal cycle, the price development in the LCD market does not follow geometric Brownian but a vector autoregressive (VAR) process instead. Unfortunately, it is not possible to derive analytical solutions for dynamic programming applied to VAR models. Therefore, we employ simulation to analyze the impact of the different

stochastic processes for the prices on the investment decision. Applying the VAR model shows that at the time of an investment, the resulting capacity increase goes along with a decreasing output price and an increasing unit cost. The decreasing output price is a result of the fact that the firms need to attract additional customers in order to keep on using a considerable part of the increased capacity. Unit costs are higher because the learning effect is prominently present in these industries. This implies that unit costs are high just after the firm starts producing with new capital stock. A main difference with GBM is that under VAR a decision to invest is not only based on current prices and costs. In addition the (recent) development of prices and costs is also taken into account, so that the decision whether or not to invest can depend on the place of the current price in the crystal cycle. It turns out that this difference in approach leads to a different investment decision for two generations of LCD plants: under GBM it is not optimal to invest where the NPV is even negative, while under VAR investing turns out to be optimal. Confronting our findings with the investment decisions taken in practice, we conclude that only two out of five decisions are supported by the GBM approach. In three cases the GBM model would have advised not to invest, while in practice the firm did invest. For VAR the score is four out of five: in one case the firm invested while according to our VAR model it should not have done that. References: Dixit and Pindyck (1994). Investment Under Uncertainty. Princeton University Press, Princeton, New Jersey, United States of America. Mathews (2005). Corporate strategy - strategy and the crystal cycle. California Management Review, 47, 6-32.

WAP Plenary IV

Salle / Room: Hélène-Desmarais
Président / Chairperson: Bruno Viscolani

09:00 **Technological progress in capital investment models**

- **Hartl, Richard**, University of Vienna, <richard.hartl@univie.ac.at>
- **Kort, Peter**, Tilburg University, PO Box 90153, Tilburg, Netherlands, 5000le, <kort@uvt.nl>

This talk surveys both existing and new work about the effect of technological progress on investment timing and capital accumulation. For investment timing we review some recent real option models, while capital accumulation combined with (embodied) technological progress will be studied employing either a two-state, a vintage or a multi-stage optimal control framework.

WA1 Computation

Salle / Room: Hélène-Desmarais
Président / Chairperson: Nicolas Hudon

10:30 **A Descent Method for Differential Variational Inequalities Arising in Computations of Non-Cooperative Differential Nash Equilibrium**

- **Kwon, Changhyun**, The Pennsylvania State University, 244 Leonhard Building, University Park, Pennsylvania, USA, 16802, <chkwon@psu.edu>
- **Friesz, Terry L**, The Pennsylvania State University, 305 Leonhard Building, University Park, Pennsylvania, USA, 16802, <tfriesz@psu.edu>
- **Lin, Cheng-Chang**, National Cheng Kung University, 1 University Road, Tainan, Taiwan, 701, <cclin@mail.ncku.edu.tw>

The differential variational inequalities, or DVIs, are infinite-dimensional variational inequalities with a special structure involving ordinary differential equations; in optimal control theory, these ordinary differential equations represent the state dynamics. The importance and application of DVIs are rising in mechanics, mathematical economics, transportation research, and many other complex engineering systems. In particular, a non-cooperative differential Nash game may be placed in the form of a differential variational inequality wherein each individual player develops for itself a strategy that is based on current and future knowledge of other players' strategy patterns. The assumption that game agents are non-cooperative allows the optimal control problems for individual agents to be combined to obtain a single DVI. In this paper, we are interested in a numerical method to solve DVI problems. Among the algorithms to solve the variational inequality problems, descent methods with gap functions have special structures in

themselves; a variational inequality problem can be converted to an equivalent optimization problem whose optimal objective value is zero if and only if the optimal solution solves the original variational inequality problem. For the infinite-dimensional variational inequality problems, Zhu and Marcotte (1998) and Konnov et al. (2002) devised descent methods using gap functions in Banach spaces and Hilbert spaces, respectively. In this paper, we propose a descent method for DVIs. We show that the framework of gap functions considered by Konnov et al. (2002) can be extended to the DVI setting. Due to the presence of the state dynamics, the gap functions will form an optimal control problem, whose optimum also solves the original DVI when the optimal objective is zero. We show how to formulate a non-cooperative differential Nash game as a DVI. In addition we will provide a pseudo code for the gap function algorithm as well as numerical examples drawn from applications like dynamic traffic assignment, revenue management and supply chain management.

10:55 **The max-plus finite element method for solving deterministic optimal control problems**

- **Lakhoua, Asma**, INRIA Rocquencourt, Domaine de Voluceau, Le Chesnay, France, 78153, <asma.lakhoua@inria.fr>

We are interested in the numerical computation of the value function of a finite horizon deterministic optimal control problem. It is known that under regularity assumptions, the value function is solution of a Hamilton-Jacobi equation. Maslov observed that the evolution semigroup, or Lax-Oleinik semigroup, associated to this equation is linear in the max-plus semiring. We recall that the max-plus semiring is the set $\R\cup\{-\infty\}$, equipped with the addition $a\oplus b=\max(a,b)$ and the multiplication $a\otimes b=a+b$. The method that we present can be seen as the max-plus analogue of the finite element method. It exploits the max-plus linearity of the semigroup and relies on a max-plus variational formulation. The value function is approximated by a max-plus linear combination. We obtain a discrete semigroup which can be interpreted as the dynamic programming operator of a deterministic zero-sum two-player game, with finite action and state spaces. The state space of the game corresponds to the set of finite elements. To each test function corresponds one possible action of the first player, and to each finite element corresponds one possible action of the second player. Hence, our method is different from the max-plus based method introduced by Fleming and McEneaney (2000), which leads to a discrete max-plus linear operator corresponding to a control problem. One interest of our method is to provide, as in the case of the classical finite element method, a systematic way to compute error estimates, which can be interpreted geometrically as ``projection'' errors. We will show that the total error order varies between $\sqrt\delta+\frac{\Delta x}{\delta}$ and $\delta+\frac{\Delta x^2}{\delta}$ depending on the regularity of the value function and on the approximation method of the ``stiffness'' matrix (which is necessary except in special cases). Here δ and Δx denote the time and space discretization steps. ince the ``mass'' and ``stiffness'' matrices are full, we will give a result allowing a semi-sparse implementation of the method. Finally, we will discuss discounted infinite horizon

deterministic optimal control problems. In this case, the max-plus finite element method leads to a discounted stationnary dynamic programming equation of a zero-sum two-player game. This fixed point problem can be solved by fast algorithms of policy iteration type. This is a joint work with Marianne Akian et Stephane Gaubert. More details can be found in [1,2,3] [1] M. Akian and S. Gaubert and A. Lakhoua, A max-plus finite element method for solving finite horizon deterministic optimal control problems, in proceedings of the sixteenth international symposium on mathematical theory of networks and systems (MTNS'04), Leuven, Belgium, 2004, and arXiv:math.OC/0404184. [2] M. Akian and S. Gaubert and A. Lakhoua, The max-plus finite element method for optimal control problems: further approximation results, in proceedings of the 44th IEEE conference on decision and control and European control conference (CDC-ECC'05), Seville, Spain, 2005, and arXiv:math.OC/0509250 [3] M. Akian and S. Gaubert and A. Lakhoua, The max-plus finite element method for solving deterministic optimal control problems: basic properties and convergence analysis, accepted for publication (after minor revision), SIAM J. Control Optim, and arXiv:math.OC/0603619

11:20 **Geometric Construction of Optimal Periodic Trajectories**

- **Hoeffner, Kai**, Queens University, Dupuis Hall, Division St., Kingston, ON, Canada, K7L 3N6, <kai@hoeffner.org>
- **Hudon, Nicolas**, Queens University, Dupuis Hall, Division St., Kingston, ON, Canada, K7L 3N6, <nick.hudon@chee.queensu.ca>
- **Guay, Martin**, Queens University, Dupuis Hall, Division St., Kingston, ON, Canada, K7L 3N6, <martin.guay@chee.queensu.ca>

A method for generating optimal periodic trajectories to a class of Hamiltonian systems is presented. A periodic Hamiltonian system is defined as the target equilibrium for nonlinear systems in strict feedback form. Parameter dependent nonlinear periodic system description is introduced for a two dimensional subsystem and extended to higher dimensions using backstepping. Extremum-seeking is used as a parameter update law that guarantees the convergence of the system to the optimal periodic orbit for a given parametrization. The procedure resulted in an on-line feedback control implementation. A typical example from optimal periodic control literature (the triple integrator) is used to illustrate the application of this approach.

WA2 Marketing II

Salle / Room: Mary-Husny
Président / Chairperson: Simon-Pierre Sigué

10:30 **Duopolistic advertising differential games with communication interference**

- **Viscolani, Bruno**, University of Padova, <viscolani@math.unipd.it>

Two manufacturers produce and advertise similar goods for a homogeneous market. Their advertising efforts interfere negatively with each other and the net results affect the evolution of the two manufacturers' goodwill variables according to the linear Nerlove and Arrow's model. As a consequence, one manufacturer's goodwill may become negative. Assuming that a positive demand is associated to a positive goodwill only, we obtain the Nash equilibrium advertising policies. Either player decision is the solution of a nonsmooth optimal control problem. We characterize the distinct situations where either both manufacturers remain active and approach a definite pair of equilibrium market shares, or one manufacturer only remains active over the infinite horizon, whereas the other goes out of business at a finite time. We investigate also the feasibility of a cooperative behaviour, finding an optimal pair of cooperative advertising policies under precise credibility conditions. For cooperation to be feasible it is necessary that one manufacturer's advertising effectiveness is high enough to overcome his opponent's interference. If such a condition does not hold for either player, then only the competitive behavior can be observed.

10:55 **Pricing and Advertising of Private and National Brands in a Dynamic Marketing Channel**

- **Amrouche, Nawel**, HEC Montréal, GERAD, 3000, Chemin de la Côte-Sainte-Catherine, Montréal, Québec, Canada, H3T 2A7, <nawel.amrouche@gerad.ca>
- **Martín-Herrán, Guiomar**, Universidad de Valladolid, Avda. Valle Esgueva, 6, Valladolid, Valladolid, Espagne, 47011, <guiomar@eco.uva.es>
- **Zaccour, Georges**, HEC Montréal, GERAD, 3000, Chemin de la Côte-Sainte-Catherine, Montréal, Québec, Canada, H3T 2A7, <georges.zaccour@gerad.ca>

We consider a marketing channel where a retailer sells, along the manufacturer's brand, her own private label. We assume that each player invests in advertising in order to build the brand's goodwill. One distinctive feature of this paper is the introduction of the negative effect of own advertising on other player's goodwill stock evolution. We characterize Feedback-Nash pricing and advertising strategies and assess the impact of the efficiency of other player's advertising on these strategies.

11:20 **Price Competition During and After Promotions**

- **Sigué, Simon-Pierre**, Athabasca University, 1 University Drive, Athabasca, AB, Canada, T9S 3A3, <simons@athabascau.ca>
- **Karray, Salma**, University of Ontario Institute of Technology, 2000 Simcoe Street North, Oshawa, Ontario, Canada, L1H 7L7, <salma.karray@uoit.ca>

This paper provides a framework for setting regular prices and promotional discounts in a duopoly in the presence of long-term promotional effects and when the firms' pricing and promotional strategies are common knowledge as in On-line markets. We show that at the equilibrium, the two firms may not promote and adopt an Everyday Low Price (EDLP) strategy. The consumers tendency to stockpile promoted products, the level of band loyalty and product differentiation, and the possibility of post-promotional sales increase critically influence regular prices, price discount rates, and profits. Particularly, under some specified conditions, consumer stockpiling intensifies promotional competition and reduces firms' profits, while the possibility to attract new consumers reduces the need to heavily promote and ensures better profits. Finally, managerial implications are discussed.

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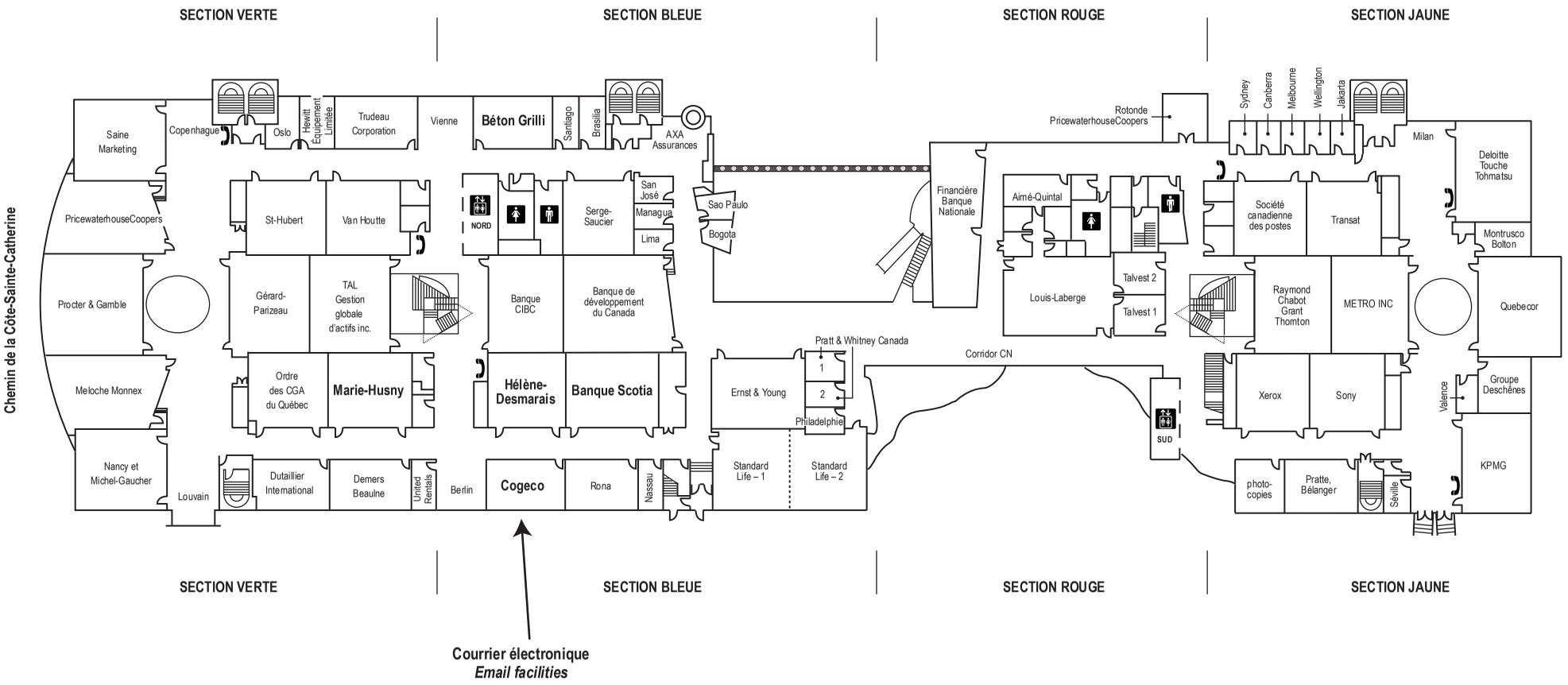
PLAN DES SALLES DE COURS ET CUBICULES – 1er ÉTAGE



Programme / *Program*

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