An electricity market incentive game based on time-of-use tariff

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***Abstract*—In this paper we model an electricity market game in which producer acts as profit taker and consumer is a follower bounded to a cost function related to comfort of load shifting from day time to night time. We consider a time-of-use (TOU) tariff scheme where the night and day pricing differs. We first analyze the interaction between a single retailer and consumers and then extend the framework to a two retailer case.**

***Index Terms*—Dynamic Game Theory, Games, Incentive, Competition, Electricity Market, Time of Use Pricing**

I. Introduction

A smart electricity meter identifies consumption with far greater granularity compared to a conventional meter. Moreover with integrated communication capabilities, such information can be transmitted back to the utility via a communication network for monitoring and billing purposes. Pricing information from the utility, e.g. due to incentives from retailers, can be further communicated back to the consumers. Smart metering, coupled with other information and communication technologies enables new pricing schemes such as Real Time Pricing (RTP), Critical Peak Pricing (CPP) and Time of Use Pricing (TOU) which is the main concern of this study.

In this paper a game between retailers, consumers and producers is discussed from a deregulation perspective. TOU tariff is modeled together with incentive models for shifting load from day time to night time constitutes a game. At first, a monopoly retailer case is discussed and later two retailers case is considered. Effect of deregulation of monopoly and competition is discussed from a game-theoretic perspective, which is contrary to neo-classical approach.

II. Application and Literature Review

Today, a number of countries such as Canada, Australia, Italy, Netherlands and Japan offer smart metering and time varying tariff pricing. A number of incumbent utilities are considering new necessary product and service options (i.e. TOU contracts, RTP contracts, curtailable service menus, price risk protection, economic development rates, fixed bill rate options, two-part tariffs and cross-product bundling) [4].

Utilities have experimented with time differentiated pricing models for some time now [10]. Hardware availability for real-time electricity monitoring was considered as a challenge in early implementations [10]. Despite this fact, both consumer and utility experience with dynamic pricing was considered to be positive early on. Today, a number of smart metering solutions are available in market today enabling dynamic tariff schemes to be implemented [3].

Equilibrium models based on mixed complementarity to estimate ex ante TOU prices were proposed in [2]. In [6], an analysis of pricing and investment decisions on multiple power plants by a utility under TOU tariff was presented. A multi-agent simulation approach was considered in [11] to understand response of different customers to TOU pricing.

In [8] a simple supply chain in an hypothetical electricity market is modeled and an incentive game is set up. Consumers, Retailer, Network Operator and Producer are considered as the stakeholders of the market. An incentive game is described and the free rider problem arising from the fact that Network Operator and Retailer share similar objectives is discussed. The market model is similar to the model used in [8]. The market is examined from the perspective of competition though.

III. Player Descriptions

In this section the basic stakeholders of the Electricity market model are considered, which can be listed as consumers, retailers and producers. Electricity is produced in various ways and acknowledged as a commodity. We assume monopoly for production. Retailers purchase electricity from a producer and sell to the consumer.

We assume different retailers discriminate themselves based on pricing of the electricity. Homogenous consumers who react to load shifting incentives based on their comfort is assumed.

A. Producer

The market is cleared at all times, which is indeed the case in electricity markets because of the fact that storage of energy is not an interim process yet in electricity transmission and distribution systems [5]. In order to meet the demand, the producer may have to produce expensive electricity. Thus load balancing is to the advantage of the generator. Inherently this situation results in an incentive scheme for retailers to push consumers to shift their load to low-demand times, e.g. from day time to night time. The inherent incentive to the retailer, which is due to the above described supply-demand relation, will be modeled in retailer description.

In our model we assume a fully liberalized wholesale market. The producer is interested in balancing the load. They are concerned with the *total* shifted load for load balancing purposes. Typically load is accumulated during day time in the network. As more consumers shift their load from day-time to night-time, the retailer can have cheaper electricity based on the *total* load on the network. We model this phenomenon in the form of incentives for retailers for the load shifted to night-time. This phenomenon has practical correspondence too. In many countries the free market offers a similar incentive for retailers in case the total load is shifted to night time although peak times may vary [9].

B. TOU Retailer

TOU retailer applies two kinds of tariffs during a day, namely ”night” and ”day” tariff. Typically a very large load during day is undesirable. During day, electricity consumption approaches critical levels, which makes the price at night cheaper.

Then the interest of TOU retailer is in maximizing its revenue that is bounded to the consumers' consumption pattern and the whole sale price of electricity. It sets the night and day prices, denoted by p_N and p_D respectively. The retailer is interested in setting an optimal incentive payment, which is in the form of discount, r, for night time load. In this paper we assume the day tariff p_D is fixed and the night tariff differs according to the incentive, r applied by the retailer. Thus $r = p_D - p_N$.

Example: The producer's incentive to retailers for total shifted load is modeled as $f(L)$:

$$f(L) = \begin{cases} \gamma(1 - e^{-\frac{L}{\theta}}) & \text{if } L > 0 \\ 0 & otherwise \end{cases} \quad (1)$$

where L is the total shifted load on the network and $f(L)$ is the incentive price per consumer payed to the retailer. Model parameters γ and θ are positive constants. The total shifted load, which can be expressed as $Q_T u$ (assuming all the Q_T number of consumers shift u amount of load), is in the interest of the producer.

Numerical Example: Suppose we take $\gamma = 30$ and $\theta = 600$. Then we have the incentive function in terms of the total shifted load as depicted in Figure 1.

Note that in our model, we assume each consumer consumes the same, fixed amount of electricity, although they may prefer to shift some load from day time to night time given the right amount of incentive to compensate the burden of shifting the load (e.g. running washing machine at night instead of day). Let x_D and x_N be the amounts of consumption of one typical consumer during day-time and night-time respectively. The function $f(L)$ corresponds to the decrease in procurement price of a unit of electricity as seen by the retailer. We assume a fixed procurement price for retailer, i.e. for both day and night times the price is assumed to be the same. If we call this price P_p, the price of total generated electricity for the retailer is $P_p(x_P + x_N)Q_T$, where $x_P + x_N$ is the total consumption of one consumer, at status quo. In case some load is shifted by the consumers, the total price becomes $P_{p_{new}}(x_P + x_N)Q_T$, where $P_{p_{new}} < P_p$ due to efficient generation. In our model $f(L)$ represents the difference between these two total prices per consumer, that is $(P_p - P_{p_{new}})(x_P + x_N)$.

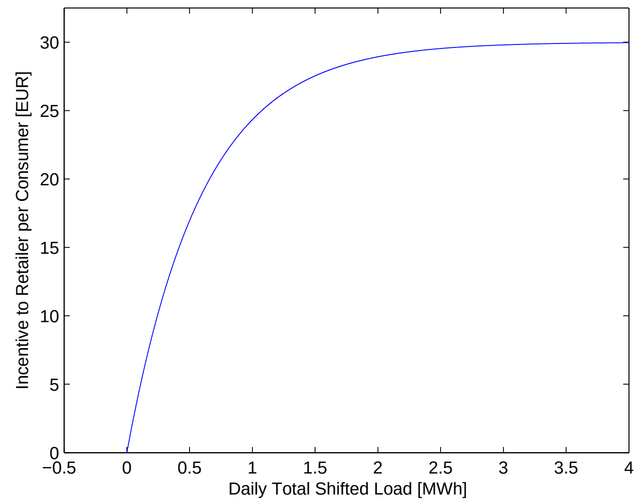

Fig. 1. Incentive for total shifted load for retailer

Since all the consumers consume the same amount, here we model the incentive per “consumer” instead of per “unit load consumed by one consumer”. For a more general model, it may be more appropriate to use the latter though.

This incentive paid to the retailer can be considered as “the discount of electricity whole sale price”. This function is quite intuitive in this sense. By shifting the load the most expensive electricity production is cut off firstly. Hence the discount on electricity price is steeper for the firstly shifted load amounts. This discount is reflected as an average discount to the retailer though. This discount behavior can last up to a point where the load is balanced. Thus this discount function has an horizontal asymptote at γ.

Of course one can argue that if load is shifted beyond “balance point”, which can be defined as the load shift which brings the night-day balance to the system optimum then the day time becomes the cheaper period. Hence the discount function has to drop after the balance point. We assume this would never be the case in our model. This makes sense since the cost of shifting too much load to night would be costly for consumers and even infeasible from a comfort perspective. This is discussed further in the model for consumer behavior.

C. Consumer

We assume Q_T number of homogeneous consumers, who behave the same in terms of reaction to incentives. They follow the retailer's demands. The cost of shifting load from day to night, which is due to comfort and reluctance, is modeled as a strictly increasing convex function, $g(.)$.

Example: In our example the cost of shifting load can be modeled as in [8]:

$$g(u) = \begin{cases} -\beta \log \frac{\alpha - u}{\alpha} & \text{if } u > 0 \\ 0 & otherwise. \end{cases} \quad (2)$$

Here u is the day to night shift of the load of one consumer and $g(u)$ is the associated cost to one consumer. Model coefficients α and β are positive constants.

The asymptote at α has a character that matches real behavior. In practice the total shifted load can never be beyond a certain amount of load. This is true because of the fact that some activities such as watching European Champions League final, using electric oven when you have guests over for dinner or keeping a reading light on while learning game theory can never be shifted to night time for a particular consumer. The steep behavior in the $g(.)$ function gives the corresponding flavor of reality in our model.

As suggested in the previous subsection, this asymptote also explains the logic behind the assumption, which says that the night time usage never exceeds day time usage.

Suppose we have a $g(.)$ as in equation (2) with $\alpha = 2$ and $\beta = 4$. This function is depicted in Figure 2.

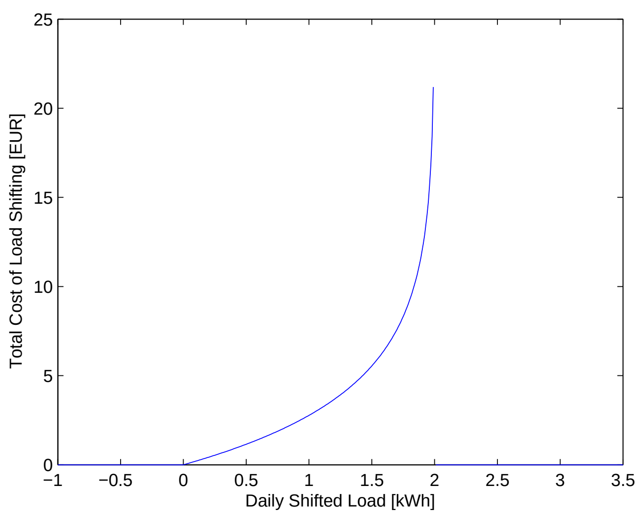

Fig. 2. Consumer cost for load shifting

Numerical Example: We assume the total consumption of the consumer, assumed to be constant, is 19 kWh. Furthermore we assume the day and night consumptions are 16 kWh and 3 kWh respectively. Day time to night time shift of this total load matters in terms of marginal costs of generation, distribution and so on. The shift is the main focus of this study.

IV. Decision Model

The market works such that the producer produces and serves electricity to the retailer, which in turn transfers it to the consumers.

First let us consider the producer. In our market model, the producer is the monopoly providing a commodity. It sells electricity for one fixed price regardless of the time of the day. However deviation from the standard price can occur depending on the load balance. The change is modeled as an incentive or discount function as explained in the previous section.

We can also consider the producer as a ”wholesale market”. By wholesale market, here we mean a mechanism, which offers electricity for a price.

Now let us consider the consumer. The consumer's interest is to minimize her cost function, resulting in the following optimization problem

$$\min_u \{g(u) + p_D(x_D - u) + p_N(x_N + u)\}. \quad (3)$$

$x_D + x_N$ is assumed to be constant. The shifted load u can be adjusted by the consumer for her own interest. At all cases we assume the consumer is perfectly rational and follows the offered incentive. Although this assumption of rational consumer seems unreasonable at first sight, one can consider it as an approximation of the collective consumer behavior, coupled with the comfort cost mentioned in equation (2).

Now if we write the relation between p_D and p_N as follows

$$p_N = p_D - r \quad (4)$$

where we can consider r as a form of incentive to push consumer to shift the load to night time, then we can cast equation (4) into the following form for a given r:

$$\min_u \{g(u) - ru + p_D x_D + (p_D - r)x_N\}. \quad (5)$$

Then the optimal amount of shifted load, $\bar{u}$, satisfies the following equation:

$$\frac{\partial g(u)}{\partial u}|_{u=\bar{u}} - r = 0 \quad (6)$$

If we take $g(.)$ as in equation (5), following our **Example** the optimal shift for a given r is found as

$$\bar{u} = \frac{-\beta + r\alpha}{r}. \quad (7)$$

Now consider the TOU retailer. The producer imposes an incentive for the load shifted by consumer, which is related to load balancing purposes of the producer as previously explained. For our **Example**, we assume an $f(L)$ as in equation (4).

Then a TOU retailer, assuming it is the sole retailer in the market, is interested in maximizing the following profit function

$$\max_{p_N} \{f(\bar{u}(r)Q_T)Q_T + p_D(x_D - \bar{u}(r))Q_T + p_N(x_N + \bar{u}(r))Q_T\} \quad (8)$$

under the constraint $p_N = p_D - r$, with p_D being constant. The equation (8) can be cast in the following form

$$\max_r \{f(\bar{u}(r)Q_T)Q_T - r\bar{u}(r)Q_T + p_D x_D Q_T + (p_D - r)x_N Q_T)\} \quad (9)$$

or equivalently,

$$\max_r \{f(\bar{u}(r)Q_T) - r(\bar{u}(r) + x_N) + p_D(x_D + x_N)\}. \quad (10)$$

Equation (10) results in the following optimality condition:

$$[\frac{\partial f(\bar{u}Q_T)}{\partial u}Q_T - r]\frac{\partial \bar{u}}{\partial r} - (\bar{u} + x_N) = 0. \quad (11)$$

At this point if we consider (6) and take derivative with respect to r, we have

$$\frac{\partial^2 g(\bar{u})}{\partial u^2}\frac{\partial \bar{u}}{\partial r} = 1. \quad (12)$$

Following equation (12), equation (11) can be written as

$$\frac{\partial f(\bar{u}Q_T)}{\partial u}Q_T - r = \frac{\partial d^2 g(\bar{u})}{\partial u^2}(\bar{u} + x_N). \quad (13)$$

Proposition 1. *For equation* (13) *only one solution may exist.*

Proof: $f(.)$ is a concave function, which implies $\frac{\partial^2 f()}{\partial u^2} < 0$; hence $\frac{\partial f(\bar{u}Q_T)}{\partial u}$ is a decreasing function of $\bar{u}$. $-r$ is also a decreasing function of $\bar{u}$ by definition. Thus the left hand side of the equation (13) is strictly decreasing.

On the other hand since $\frac{\partial g(.)}{\partial u}$ is assumed as a convex function, the right hand term $\frac{\partial^2 g(\bar{u})}{\partial u^2}(\bar{u} + x_N)$ is strictly increasing. Thus these two curves can intersect at most at one single point. ■

For illustration purposes we follow our **Example** and take $f(.)$ as in equation (1) and $g(.)$ as in equation (2).

Twice differentiating $g(u)$ in equation (2) for $u > 0$ we have

$$\frac{\partial^2 g(\bar{u})}{\partial u^2} = \frac{\beta}{(\alpha - \bar{u})^2}. \quad (14)$$

Using equations (14), (13) we come up with the $\bar{u}$, r pair satisfying following two equations

$$r = \frac{\beta}{\alpha - \bar{u}} \quad (15)$$

$$\frac{\gamma Q_T}{\theta}e^{-\frac{uQ_T}{\theta}} - r = \frac{\beta}{(\alpha - \bar{u})^2}(\bar{u} + x_N). \quad (16)$$

Numerical Example:
Continuing our *Numerical Example* and assuming the consumer nominally consume $x_N = 3$ kWh at night, we find $r = 3.29$ EUR and $u = 0.78$ kWh for a single retailer and a population of $Q_T = 1000$ consumers using equations (15) and (16).

If we compare this state to the no-incentive state, for which all the time electricity is priced with p_D, the consumer's profit can be calculated as

$$\begin{aligned} \text{ConProfit} =& -\{g(\bar{u}) + p_D(x_D - \bar{u}) + (p_D - r)(x_N + \bar{u})\} \\ & + \{p_D(x_D + x_N)\} \quad (17) \\ =& r(x_N + \bar{u}) - g(\bar{u}) \quad (18) \end{aligned}$$

which would be 10.46 EUR. The retailer's profit per consumer would be 9.43 EUR/consumer. These values are the optimal values both retailer and consumer would get for the described market. The producer also enjoys a load shift of $u = 0.78$ kWh in this case. These results are obtained in case there is a retailer monopoly. However in the current liberalized electricity market, the retailer no longer has monopoly. Thus in the next section we will describe a game where two retailers are interested in pushing their customers shift to profit from generator's incentive.

V. Game

In the previous section, we discussed the tactical maneuvers of consumers and a single retailer in the modeled electricity market. The consumer is interested in minimizing electricity cost by shifting load to night time. However while doing this, she has to find a compromise between her comfort and gained surplus. From the perspective of the retailer, the situation is a bit more complicated as it has to consider the amount of incentive coming from the producer while taking the reaction from the consumer into account.

In this section, we assume two identical TOU retailers, namely *retA* and *retB*, each of which has half of the market share, i.e. $Q_A = \frac{Q_T}{2}$ and $Q_B = \frac{Q_T}{2}$. These retailers share the same electricity pool to procure electricity. Thus the incentive that is expected from the producer is dependent on the rival retailer's actions due to the equation (1).

A. Producer and Consumer

In this case the producer behaves according to the shifted load as previously described in equation (1). The producer is interested in the *total* load shift and pays the incentive based on that.

The situation for a single consumer also does not change. The consumer follows her associated retailer's incentive based on the cost function (2). In this respect we can consider her as a follower in a Stackelberg game [1].

B. Retailers

From the perspective of the retailers, the incentive expected from the producer varies according to the opponent's move. Thus the *retA*, without loss of generality, confronts the following optimization problem:

$$\max_{r_I}\{f(L)Q_A - r_I\bar{u}_I Q_I + p_D x_D Q_I + (p_D - r_I)x_N Q_I\} \quad (19)$$

where the total shifted load L is

$$L = \sum_{I \in A,B}(\bar{u}_I Q_I). \quad (20)$$

r_I is the strategy of *retI* for this game, where $I \in \{A, B\}$.

The strategies r_A and r_B are two strategies chosen from the interval $[0, \alpha)$ according to our **Example**. p_D is fixed as in the one retailer case. The game is a continuous non-cooperative game.

In the next sub-section we will find a Nash Equilibrium (NE) for this game.

C. Nash Equilibrium

In this section we assume the retailers have perfect information about their cost and incentive functions and the customers' cost and incentive functions, but do not know about their opponent's strategy. The consumers of both retailers behave in the same manner as in the first part since from the consumer's perspective the situation has not changed at all. Then the consumer group for each retailer reacts to a particular u_I in the following way by choosing

$$r_I = \frac{\beta}{\alpha - \bar{u}_I}. \tag{21}$$

With a similar reasoning, we can inherit the equations (12) and (14) directly as

$$\frac{\partial^2 g(\bar{u}_I)}{\partial u_I^2}\frac{\partial \bar{u}_I}{\partial r_I} = 1, \tag{22}$$

$$\frac{\partial^2 g(\bar{u}_I)}{\partial u_I^2} = \frac{\beta}{(\alpha - \bar{u}_I)^2}. \tag{23}$$

However the situation for retailers is different than in the monopoly retailer case. As the retailers share the same producer, the rival retailer's action affects the payoff. The incentive per consumer, $f(L)$, that the producer would pay can be written analytically as follows using equation (20)

$$f(L) = \gamma(1 - e^{-\frac{(\bar{u}_A Q_A + \bar{u}_B Q_B)}{\theta}}). \tag{24}$$

retA wishes to choose r_A that solves the equation (19), which gives the following condition

$$\frac{\partial f(L)}{\partial r_A} - (\bar{u}_A + x_N) - r_A\frac{\partial \bar{u}_A}{\partial r} = 0, \tag{25}$$

where

$$\frac{\partial f(L)}{\partial r_A} = \frac{\gamma Q_A}{\theta}(e^{-\frac{\bar{u}_A Q_A + \bar{u}_B Q_B}{\theta}})\frac{\partial \bar{u}_A}{\partial r}. \tag{26}$$

Then equation (25) becomes

$$(\frac{\gamma Q_A}{\theta}e^{-\frac{\bar{u}_A Q_A + \bar{u}_B Q_B}{\theta}} - r_A)\frac{\partial \bar{u}_A}{\partial r} = \bar{u}_A + x_N. \tag{27}$$

Using equation (27), (22) and (23) we get

$$\frac{\gamma Q_A}{\theta}e^{-\frac{\bar{u}_A Q_A + \bar{u}_B Q_B}{\theta}} - r_A = \frac{\beta}{(\alpha - \bar{u}_A)^2}(\bar{u}_A + x_N). \tag{28}$$

Similarly, for *retB*, we obtain

$$\frac{\gamma Q_B}{\theta}e^{-\frac{\bar{u}_A Q_A + \bar{u}_B Q_B}{\theta}} - r_B = \frac{\beta}{(\alpha - \bar{u}_B)^2}(\bar{u}_B + x_N). \tag{29}$$

Then the NE solution for both retailers must satisfy the following four equations

$$r_A = \frac{\beta}{\alpha - \bar{u}_A} \tag{30}$$

$$\frac{\gamma Q_A}{\theta}e^{-\frac{\bar{u}_A Q_A + \bar{u}_B Q_B}{\theta}} - r_A = \frac{\beta}{(\alpha - \bar{u}_A)^2}(\bar{u}_A + x_N) \tag{31}$$

$$r_B = \frac{\beta}{\alpha - \bar{u}_B} \tag{32}$$

$$\frac{\gamma Q_B}{\theta}e^{-\frac{\bar{u}_A Q_A + \bar{u}_B Q_B}{\theta}} - r_B = \frac{\beta}{(\alpha - \bar{u}_B)^2}(\bar{u}_B + x_N). \tag{33}$$

Numerical Example:

If we continue with our Numerical Example with $Q_A = Q_B = \frac{Q_T}{2} = 500$ then we have $u_i = 0.57$ and $r_i = 2.79$ for both consumers as the Nash Equilibrium of this game by solving the corresponding equations in the previous section. Each consumer's profit becomes 8.62 EUR and profit per consumer of each retailer becomes 8.38 EUR/consumer

This point is indeed the NE [7] as unilateral deviations from the Equilibrium constitutes loss for the corresponding player. To illustrate this, we can consider the total profit function of *RetA* in terms of r_A in case *RetB* plays NE, i.e. for $r_B = 2.79$.

The total profit of *RetA* in terms of *RetA*:

$$\begin{aligned} Prof_A =& \{p_D(x_D - \bar{u}_A) + (p_D - r_A)(x_N + \bar{u}_A)) + f(L)\} \\ & - p_D\{(x_D + x_N)\} \qquad (34) \\ =& - r_A(x_N + u_A) + f(L) \qquad (35) \end{aligned}$$

which is shown in Figure 3.

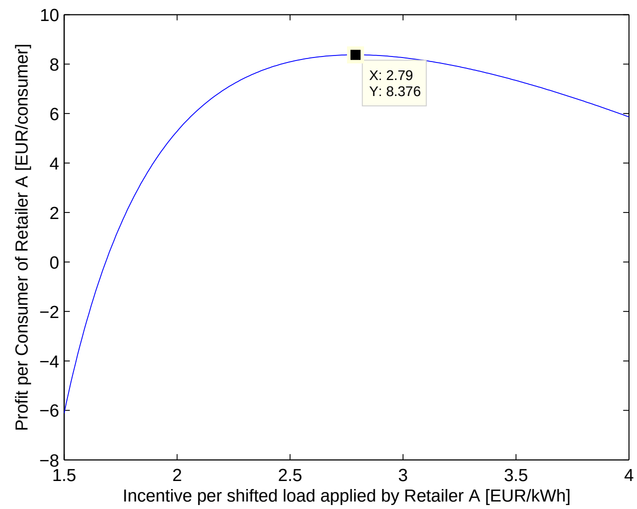

Fig. 3. RetailerA Total profit function if RetailerB plays NE

This result shows that when monopoly is divided into two identical retailers, the global welfare gets worse since the optimal incentive drops in this case. The decrease in the retailer incentive causes less load shift, which is not beneficial for generators.

One can also analyze collusion situation where both retailers offer an incentive on agreement. In this case both retailers apply $r = 3.29$ EUR. Then we obtain the load shift of $u = 0.78$ kWh. If the retailers defect and break up the agreement, the incentive in equilibrium drops to $r = 2.79$ EUR for both retailers. This analysis s the subject of further research.

VI. Conclusions and Remarks

Current ongoing deregulation of Electricity markets in many Western countries face many challenges due to transition. Many researchers argue the effect of the change from various perspectives. In this paper the effect of competition in a hypothetical electricity retail market for load balancing purposes is examined from a game-theoretic perspective.

An incentive game between fundamental stakeholders of a hypothetical supply chain in an electricity market is set up. Firstly the decision model in which only one retailer operates is formed. In this decision model, the optimal incentive for load shifting for a monopolistic retailer is determined. Secondly we divided the monopolistic retailer into two equal retailers, each of which is contracted with half the number of total consumers and examined the Nash equilibrium. The game revealed that the Nash Equilibrium solution gives a worse global welfare in the oligopolistic case compared to the monopolistic case. All the stake holders leave worse off in case the number of retailers are increased.

Classically competition in a market is encouraged from an economic perspective. The pillars of this thought stem from the idea that competition increases efficiency and allocates productive resources to their most highly-valued uses. In our model we did not consider labor efficiency or price competition but ignoring these phenomenon we observe the effect on an incentive scheme for generation efficiency. These incentives are the true determinants of the global welfare since load balancing is directly proportional to these incentives.

The aforementioned results provide new insights into efficiency discussions in deregulation of electricity markets. The current work can be extended to perfect competition case in which large number of retailers share the same market. The possible shifts of the consumer profile among the retailers can be modeled which would imply taking the price efficiency into account. The consumers are modeled uniformly which could be extended to various types of consumers.

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