A Differential Game of International Pollution Control with Evolving Environmental Costs

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Introduction

- Give more time to developing countries (DCs) to make serious efforts to reduce their greenhouse gas (GHG) emissions.
- Reasons:
  1. Industrialized countries (ICs) are the main responsible for current state of environment.
  2. Other priorities (eradicating extreme poverty, education, health care, building infrastructure, etc.).
- Anyway "As incomes rise, the demand for improvements in environmental quality will increase, as will the resources available for investment" (The World Bank (1992, p. 39)).
- Idea of a relationship between welfare (prosperity or development) and environmental concerns has been around for some time.

Introduction

- Suppose DCs: (i) need $[0, T]$ to accomplish a desired level of development; and (ii) fully internalize the environmental externalities after T.
- Then:
  1. How do coop. and non-coop. emissions strategies compare during $[0, T]$ and in the long run?
  2. Can cooperation between DCs and ICs reduce T?
  3. Under which conditions cooperation is collectively better than noncooperation during the time interval $[0, T]$?
- Important questions in negotiations of IEA.

Introduction

- Differential game: player 1 represents ICs, and player 2 DCs.
- Natural choice (strategic issues + dynamic processes).
- Literature
  - Differential-games literature in environmental economics; van der Ploeg and de Zeeuw (1992) and Long (1992). Surveys in Jørgensen, Martín-Herrán and Zaccour (2010) and Long (2010).
  - Subset: international environmental agreements (IEA);
    - Cooperative approach (Germain, Toint, Tulkens and de Zeeuw (2003), Petrosjan and Zaccour (2003));
    - Noncooperative approach (Rubio and Casino (2005), Rubio and Ulph (2007), de Zeeuw (2008), Breton, Sbragia and Zaccour (2010))

Introduction

- Main contribution: different treatment of the two players in terms of their **environmental concerns (EC)**;
- Exploratory study: EC (modestly) translated into damage cost.
- Main Results:

1. ICs emit less under coop. than non-cooperation, $\forall t \in [0, \infty)$.
2. DCs emit less under coop. than non-cooperation, $\forall t \in [T, \infty)$. For $t \in [0, T]$, the result depends on the degree of EC in DCs.
3. For a large region of the parameter space, T^C (cooperation) $>$ T^N (non coop).
4. Cooperation may not create enough dividend (IN THE SHORT RUN) to be attractive.
5. It may not be the best option for ICs to press DCs to engage in abatement efforts in the short term.

Model I: Players and dynamics

- Two-player differential game, $t \in [0, \infty)$.
- Player 1 represents ICs, characterized by high EC levels.
- Player 2 represents DCs for whom environmental issues are not yet at the top of their economic agenda.
- $e_i(t)$: emissions of player i at time t; by-product of production $y_i(t)$, with $e_i(t) = h_i(y_i(t))$.
- Assume $h_i(\cdot)$ is smooth, and write $y_i(t) = h_i^{-1}(e_i(t)) \triangleq f_i(e_i(t))$.
- Revenues: $f_i(e_i)$ is concave and increasing.
- $S(t)$ the stock of pollution

$$\dot{S}(t) = \mu(e_1(t) + e_2(t)) - \delta S(t), \qquad S(0) = S_0,$$

Model II: Environmental costs

- Typical assumption: Damage cost is a convex increasing function in pollution stock, $D_i(S)$. Assume this for ICs.
- DCs internalize gradually the environmental cost. Process will become complete when a threshold level of economic development has been achieved.
- Measuring economic development; a tough job (available infrastructure, consumption per capita, life expectancy, etc.)
- Cumulative revenues (could add environmental awareness)
- Shafik and Bandyopadhyay (1992) (149 countries; 1960-90): "income has the most consistently significant effect on all indicators of environmental quality."

Model II: Environmental costs

- Discount rate $\rho, 0 < \rho < 1$.
- $Y_2(t)$: cumulative discounted revenues by t,

$$Y_2(t) = \int_0^t e^{-\rho z} f_i(e_i(z))dz,$$

- $\bar{Y}_2$:threshold value of cumulative revenues before player 2 fully accounts for environmental damage.
- Damage-cost function

$$D_2(S(t), Y_2(t)) = \begin{cases} d_2(S(t), Y_2(t)), & \forall Y_2(t) < \bar{Y}_2, \\ D_2(S(t)) & \forall Y_2(t) \geq \bar{Y}_2, \end{cases}$$

with

$$\frac{\partial D_2(S, Y_2)}{\partial S} > 0, \quad \frac{\partial^2 D_2(S, Y_2)}{\partial S^2} \geq 0, \quad \frac{\partial D_2(S, Y_2)}{\partial Y_2} \geq 0.$$

Model III: Payoffs

- Optimization problems:

$$\begin{aligned}
\max_{e_1} W_1 &= \int_0^{\infty} e^{-\rho t} \left(f_1 \left(e_1 \right) - D_1 \left(S \right) \right) dt, \\
\max_{e_2} W_2 &= \int_0^{T} e^{-\rho t} \left(f_2 \left(e_2 \right) - d_2 \left(S, Y_2 \right) \right) dt \\
&\quad + \int_T^{\infty} e^{-\rho (t-T)} \left(f_2 \left(e_2 \right) - D_2 \left(S \right) \right) dt, \\
\text{subject to} &: \quad \dot{S} = \mu(e_1 + e_2) - \delta S, \qquad S\left(0\right) = S_0,
\end{aligned}$$

where T is the date satisfying the following equality:

$$\int_0^{T} e^{-\rho t} f_i(e_i) dt = \bar{Y}_2.$$

Model IV: Functional Forms

- One-to-one correspondence between T and $\bar{Y}_2$, then

$$D_2\left(S\left(t\right),t\right)=\begin{cases} d_2\left(S\left(t\right),t\right), & \forall t<T, \\ D_2\left(S\left(t\right)\right), & \forall t\geq T. \end{cases}$$

- Specific functional forms:

$$\begin{aligned} f_i(e_i) &= \alpha_i e_i-\frac{1}{2}e_i^2, \qquad D_1(S)=\beta_1 S, \\ D_2\left(S\left(t\right),Y_2\left(t\right)\right) &= \begin{cases} \frac{t}{T}\gamma\beta_2 S, & \forall Y_2\left(t\right)<\bar{Y}_2, \\ \beta_2 S & \forall Y_2\left(t\right)\geq\bar{Y}_2. \end{cases} \\ \dot{S}\left(t\right) &= \mu(e_1\left(t\right)+e_2\left(t\right))-\delta S\left(t\right), \qquad S\left(0\right)=S_0, \end{aligned}$$

α_i and $\beta_i>0$, $i=1,2$, and $\gamma\in\{0,1\}$.
Linear damage cost: Labriet and Loulou (2003), Radner (yesterday).

Solutions

- Cooperative (C) and non-cooperative (N) solutions.
- Cooperation: IEA, joint optimization.
- Feedback information structure: pollution emissions (pollution stock).
- Complication due to assumption on damage cost of DCs.
- We proceed backward:
  - Given $T^j, j = C, N$, solve the infinite-horizon problem defined on $[T^j, \infty)$.
  - Use second-period value functions as salvage values in overall optimization problem defined on $[0, \infty)$.
  - We compute T^j and $S\left(T^j\right)$.
  - T^j depends on (i) mode of play; and (ii) on $\gamma \in \{0, 1\}$. We write T^j_γ, for $j = C, N$, and $\gamma = 0, 1$.

Feedback-Nash Equilibrium

Assuming an interior solution, the FNE emissions are given by

$$e_1^N(t) = \alpha_1 - \frac{\mu\beta_1}{\rho+\delta}, \quad \forall t \in [0,\infty),$$

$$e_2^N(t) = \begin{cases} \alpha_2 - \frac{\mu\beta_2\left(\gamma F\left(t;T^N\right)+e^{(\rho+\delta)\left(t-T^N\right)}(1-\gamma)\right)}{(\rho+\delta)}, & \text{for } t \in \left[0, T^N\right], \\ \alpha_2 - \frac{\mu\beta_2}{\rho+\delta}, & \text{for } t \in \left[T^N, \infty\right), \end{cases}$$

where T^N is the solution to the equation

$$\int_0^{T^N} \left(\alpha_2 e_2^N - \frac{1}{2}(e_2^N)^2\right) e^{-\rho t} dt = \bar{Y}_2.$$

$$F\left(t; T^N\right) = \frac{1 + t(\rho+\delta) - e^{(\rho+\delta)\left(t-T^N\right)}}{(\rho+\delta)\, T^N}$$

Cooperative Solution

Assuming an interior solution, the optimal emissions of player i, $i = 1, 2$, are given by

$$e_i^C(t) = \begin{cases} \alpha_i - \frac{\mu\beta_1}{\rho+\delta} - \frac{\mu\beta_2\left(\gamma F\left(t; T^C\right) + e^{(\rho+\delta)\left(t-T^C\right)}(1-\gamma)\right)}{(\rho+\delta)}, & \text{for } t \in \left[0, T^C\right], \\ \alpha_i - \mu\frac{\beta_1+\beta_2}{\rho+\delta}, & \text{for } t \in [T^C, \infty), \end{cases}$$

where T^C is the solution to equation

$$\int_0^{T^C} \left(\alpha_2 e_2^C - \frac{1}{2}(e_2^C)^2\right) e^{-\rho t} dt = \bar{Y}_2.$$

$$F\left(t; T^C\right) = \frac{1 + t(\rho+\delta) - e^{(\rho+\delta)\left(t-T^C\right)}}{(\rho+\delta)\, T^C}$$

Comparison: Player 1 (ICs)

Player 1 emits more in the non-cooperative game than under cooperation at all instants of time, that is,

$$e_1^N(t) > e_1^C(t), \quad \forall t \in [0, \infty).$$

Comparison: Player 2 (DCs)

Player 2's cooperative and non-cooperative emissions compare as follows:

For all $t \geq \max\left\{T^N, T^C\right\}$, $e_2^N(t) - e_2^C(t) > 0$.

For all $t \in \left[\min\left\{T^N, T^C\right\}, \max\left\{T^N, T^C\right\}\right]$,

1. If $T^N < T^C$, then $\forall t \in \left[T^N, T^C\right]$, we have

$$e_2^N(t) - e_2^C(t) \quad \text{is} \quad \begin{cases} > 0 \Leftrightarrow \frac{\beta_1}{\beta_2} > 1 - F\left(t; T^C\right), & \gamma = 1, \\ > 0 \Leftrightarrow \frac{\beta_1}{\beta_2} > G\left(t; T^C\right), & \gamma = 0. \end{cases}$$

2. If $T^N > T^C$, then for all $t \in \left[T^C, T^N\right]$, we have

$$e_2^N(t) - e_2^C(t) \quad \text{is} \quad \begin{cases} > 0, & \gamma = 1, \\ > 0, & \gamma = 0. \end{cases}$$

Comparison: Player 2 (DCs)

For all $t \in \left[0, \min\left\{T^N, T^C\right\}\right]$, we have

$$e_2^N(t) - e_2^C(t) \quad \text{is} \begin{cases} \ldots, & \gamma = 1, \\ > 0, & \gamma = 0. \end{cases}$$

if $\gamma = 1$, then

1. If $\min\left\{T^N, T^C\right\} = T^N$, we get $e_2^N(t) - e_2^C(t) > 0$
2. If $\min\left\{T^N, T^C\right\} = T^C$, then

$$e_2^N(t) - e_2^C(t) > 0 \Leftrightarrow \frac{\beta_1}{\beta_2} > LCE.$$

Comparison: Special Cases

- Suppose $\gamma = 0$. If $\beta_1 > \beta_2$, then $e_2^N(t) - e_2^C(t) > 0, \forall t \in [0, \infty)$.

- If $\gamma = 0$, then $T^N < T^C$.

- CLAIM: If $\gamma = 1$, then $T^N < T^C$.

Comparison of Outcomes

Difference between total non-cooperative payoffs when $\gamma = 0$ and $\gamma = 1$,

$$\Delta_\gamma = \sum_{i=1}^{2} \left(W_i^N (\gamma = 0) - W_i^N (\gamma = 1) \right).$$

$\Delta_\gamma > 0$ when

1. Discount rate and marginal damage cost of player 1 are large, while the marginal damage cost of player 2 is small
2. Discount rate and α_i, $i = 1, 2$, are large enough
3. Scaling parameter μ is large enough and the absorption rate δ is small enough
4. For a large enough initial pollution stock

First conclusion: $\Delta_\gamma > 0$ for very different reasons, e.g., high asymmetry in marginal damage cost, high revenues from production (emissions), and fast accumulation of pollution (high initial stock, or large μ and small δ).

Comparison of Outcomes

Under which conditions is cooperation collectively better than noncooperation in the short run?

- T^C and T^N do not coincide in general. Which one?
- We found (analytically) that $T^N < T^C$ for $\gamma = 0$, and (numerically) that $T^N < T^C$ for $\gamma = 1$.
- Retain the interval $\left[0, T^N\right]$ as a basis for comparing the total outcomes.
- No worry for $[T^N, \infty)$.
- Idem for $[0, \infty)$, but...

Comparison of Outcomes

The difference in payoffs is given by

$$\triangle_W = \int_0^{T^N} e^{-\rho t} \left(f_1\left(e_1^C\right) - f_1\left(e_1^N\right) + f_2\left(e_2^C\right) + f_2\left(e_2^N\right) + D_1\left(S^C\right) - D_1\left(S^N\right) + d_2\left(S^N, Y_2\right) - d_2\left(S^C, Y_2\right)\right) dt \lesseqgtr 0$$

- $\triangle_W < 0$, when
  1. T^N and the marginal damage cost β_1 of player 1 are large enough, while β_2 is small enough
  2. Rate of discount, ρ is large enough
  3. Increasing absorption parameter δ enlarges (a little bit) the region where $\triangle_W$ is positive.
- Interestingly, these results are in line with $\Delta_\gamma > 0$.
- However, varying the revenue parameters $\alpha_i, i = 1, 2$, and μ, does not seem to change the region where $\triangle_W$ is positive.

Conclusion

- Still a lot to be done...