Information Disclosure in the Commons

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November 30th, 2012

4th Workshop on Game Theory in Energy, Resources and the Environment

Context

Exploitation under the threat of dramatic events:

- Ecological services
- Climate change (thermohaline circulation shutdown, permafrost meltdown)
- Epidemiological outbreaks
- Resistance to antibiotics
- Any **threshold** common-pool resource...

Strategic behavior when the value of the threshold is not well known.

The Nash Demand Game

Several agents simultaneously claim a share x_i of a resource of size r. The payoffs, y_i, are determined by:

$$\begin{cases} y_i = x_i & \text{if } \sum_i x_i \leq r \\ y_i = 0 & \text{if } \sum_i x_i > r \end{cases}$$

The Nash Demand Game embodies the notion of strategic interaction in a context of threshold effects.

When r is known, any claims profile such that $\sum_i x_i = r$ is a Nash equilibrium. In what follows, the value of r will not be known with certainty.

The nature of the uncertainty matters

In many practical situations, **experts may disagree**:
(Laurent-Luchetti, Leroux and Sinclair-Desgagné; 2012, 2013)

- "Cautious" equilibria may coexist with "Dangerous" equilibria even if **all** agents are risk averse
- Cautious equilibria Pareto-dominate Dangerous equilibria
- Multimodal distributions lead to coordination problems
- Argument in favor of coordination devices like international environmental agreements (IEAs)
- Contrasts sharply with earlier "unimodal" models (Eso & White, 2004; White, 2008; Bramoullé and Treich, 2009), where risk aversion leads to self discipline.

An experiment

(Bochet, Laurent-Luchetti, Leroux and Sinclair-Desgagné; 2012)

- Subjects coordinate easily on symmetric Dangerous equilibria even if all agents are risk averse
- Subjects never coordinate on Pareto-dominating symmetric Cautious equilibria even if all agents are risk averse
- Coordination is more difficult as the probability of a high value of r decreases
- Sheds light on an anomaly uncovered by Rapoport et al. (1992) on the likelihood of overshooting in the (unimodal) NDG as uncertainty increases

Today's topic

Step away from the coordination issue, and focus on the diffusion of information.

- Is the diffusion of information immune to strategic considerations?
- Does more information improve social welfare?
- If so, do the informed agents always benefit from having access to the information?
- What are the implications in terms of information acquisition (research, etc.)?

The Model

Two agents with a common (unimodal) prior on r.

Information disclosure game (**not** a cheap talk game): Agent 1 receives a perfect signal $s = r \in \mathbb{R}_+$ with probability $q \in [0, 1]$ and can choose to disclose it to agent 2 before playing the NDG.

Three cases:

- Agent 1 is uninformed ($q = 0$)
- Agent 1 is informed: ($q = 1$)
- Agent 1 is possibly informed: $q \in (0, 1)$

The Uninformed Case, $q = 0$

Given x_j, agent i's best response is the result of:

$$\max_{x_i} u_i(x_i)\left[1 - F(x_i + x_j)\right]$$

First-order condition:

$$\frac{u_i'(x_i)}{u_i(x_i)} = \frac{f(x_i + x_j)}{1 - F(x_i + x_j)}.$$

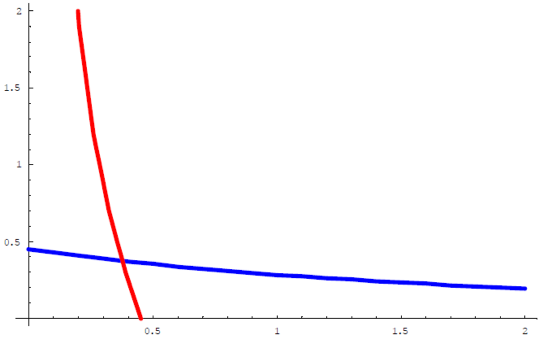
Assume u_i increasing and concave, and $\frac{f}{1-F}$ increasing

$\Longrightarrow$ Single-crossing condition.

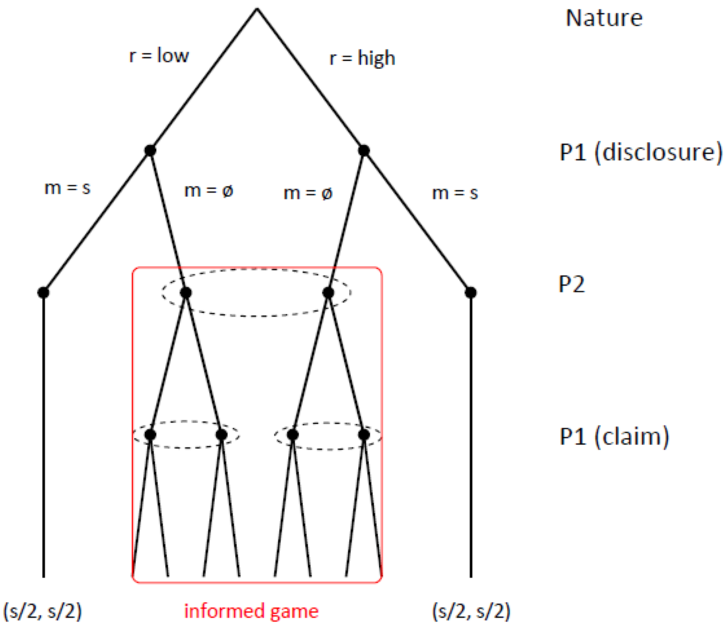
The Uninformed Case, $q = 0$

Denote by $\hat{x}_i(x_j)$ agent i's unique best response. One easily shows that:

- $\hat{x}_i$ is decreasing in x_j (strategic substitutes)
- $\hat{x}_i(x_j) + x_j$ is increasing in x_j.



The Informed Case, $q = 1$


Nature
r = low
r = high
P1 (disclosure)
m = s
m = ø
m = ø
m = s
P2
P1 (claim)
(s/2, s/2)
informed game
(s/2, s/2)

Agent 1 can disclose her signal ($m = s$) or conceal it ($m = \emptyset$). When agent 1 discloses, we assume agents claim $(s/2, s/2)$.

Agent 2's strategy is of the form:

$$\sigma_2 = \begin{cases} x_2 = s/2 & \text{if } m = s \\ x_2 = x_2^u & \text{if } m = \emptyset \end{cases}$$

for some $x_2^u \in \mathbb{R}_+$.

Given σ_2, agent 1's best response is the following:

$$\sigma_1(\sigma_2) = \begin{cases} \text{Send } m = s \text{ and claim } x_1 = s/2 & \text{if } s/2 \geq s - x_2^u \\ \text{Send } m = \emptyset \text{ and claim } x_1 = s - x_2^u & \text{if } s/2 < s - x_2^u \end{cases}$$

No profitable deviations for agent 2

Agent 2's expected utility:

$$\mathbb{E}v_2\left(\sigma_1, x_2^u\right) = \int_0^{2x_2^u} u_2\left(\frac{s}{2}\right) dF\left(s\right) + \left[1 - F\left(2x_2^u\right)\right] u_2\left(x_2^u\right).$$

Consider a deviation $x_2' \neq x_2^u$. Agent 2's expected utility becomes:

$$Ev_2\left(\sigma_1, x_2'\right) = \begin{cases} \int_0^{2x_2^u} u_2\left(\frac{s}{2}\right) dF\left(s\right) + 0 & \text{if } x_2' > x_2^u \\ \int_0^{2x_2^u} u_2\left(\frac{s}{2}\right) dF\left(s\right) + \left[1 - F\left(2x_2^u\right)\right] u_2\left(x_2'\right) & \text{if } x_2' < x_2^u \end{cases}$$

No profitable deviations are available for agent 2.

Result

Proposition: *The following pair of strategies forms a Perfect Bayesian Equilbria (PBE) of the informed game if and only if it is of the form:*

$$\sigma_1 = \begin{cases} \text{Send } m = s \text{ and claim } x_1 = s/2 & \text{if } s \leq 2x_2^u \\ \text{Send } m = \emptyset \text{ and claim } x_1 = s - x_2^u & \text{if } s > 2x_2^u \end{cases}$$

and

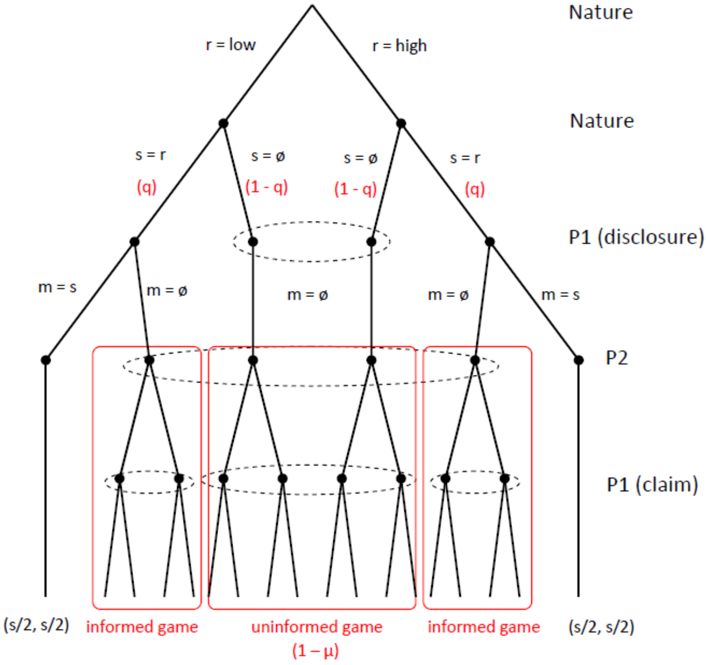
$$\sigma_2 = \begin{cases} x_2 = s/2 & \text{if } m = s \\ x_2 = x_2^u & \text{if } m = \emptyset \end{cases}$$

for some $x_2^u \in \mathbb{R}_+$. Agent 2 believes that agent 1 conceals information whenever $s > 2x_2^u$.

Remarks

- The range of values of x_2^u supporting a PBE is $[0, +\infty)$.
- However, the threshold is never exceeded. *Ex post*, we always have $x_1 + x_2 = r$ in all equilibria.
- All equilibria of the game are both *ex ante* and *ex post* Pareto efficient.

The Possibly Informed Case, $0 < q < 1$


Nature
r = low
r = high
Nature
s = r
(q)
s = ø
(1 - q)
s = ø
(1 - q)
s = r
(q)
P1 (disclosure)
m = s
m = ø
m = ø
m = ø
m = s
P2
P1 (claim)
(s/2, s/2)
informed game
uninformed game
(1 – μ)
informed game
(s/2, s/2)

Possibly Informed Agent, $0 < q < 1$

Now, upon receiving no message ($m = \emptyset$), agent 2 wonders whether agent 1...

- ...has concealed information, with belief $\mu \in [0, 1]$
- ...has simply not received any signal, with belief $(1 - \mu)$

Agent 2's strategy space is still of the form:

$$\sigma_2 = \begin{cases} x_2 = s/2 & \text{if } m = s \\ x_2 = x_2^u & \text{if } m = \emptyset \end{cases}$$

Agent 1's best response strategy

Given agent 2's strategy:

$$\sigma_1(\sigma_2) = \begin{cases} \text{Send } m = \emptyset \text{ and claim } \hat{x}_1(x_2^u) & \text{if } s = \emptyset \\ \text{Send } m = \emptyset \text{ and claim } x_1 = s - x_2^u & \text{if } s > 2x_2^u \\ \text{Send } m = s \text{ and claim } x_1 = s/2 & \text{if } s \leq 2x_2^u \end{cases}$$

Recall that $\hat{x}_1(x_2^u)$ is agent 1's best response in the uninformed game:

$$\hat{x}_1(x_2^u) = \arg\max_{x_1} u_1(x_1)\left[1 - F(x_1 + x_2^u)\right]$$

Payoff of agent 2

Agent 2's expected payoff when $m = \emptyset$ is:

$$\mathbb{E}[v_2|m = \emptyset] = \mu u_2\left(x_2^u\right) + (1-\mu)\, u_2\left(x_2^u\right)\left[1 - F\left(\hat{x}_1\left(x_2^u\right) + x_2^u\right)\right]$$

Consider a deviation $x_2' \neq x_2^u$. Then, $Ev_2\left(\sigma_1, x_2' | m = \emptyset\right) =$

$$\begin{cases} 0 + (1-\mu)\, u_2\left(x_2'\right)\left[1 - F\left(\hat{x}_1\left(x_2^u\right) + x_2'\right)\right] & \text{if } x_2' > x_2^u \\ \mu u_2\left(x_2'\right) + (1-\mu)\left[1 - F\left(\hat{x}_1\left(x_2^u\right) + x_2'\right)\right] u_2\left(x_2'\right) & \text{if } x_2' < x_2^u \end{cases}$$

(Note: The expected utility if $m = s$ is unchanged)

Profitable deviations for agent 2

$$Ev_2\left(\sigma_1, x_2' | m = \emptyset\right) =$$

$$\begin{cases} 0 + (1-\mu)\, u_2\left(x_2'\right)\left[1 - F\left(\hat{x}_1\left(x_2^u\right) + x_2'\right)\right] & \text{if } x_2' > x_2^u \\ \mu u_2\left(x_2'\right) + (1-\mu)\left[1 - F\left(\hat{x}_1\left(x_2^u\right) + x_2'\right)\right] u_2\left(x_2'\right) & \text{if } x_2' < x_2^u \end{cases}$$

Profitable deviations may exist for agent 2, who faces a payout/probability tradeoff:

- $x_2' > x_2^u$: loss of the term in μ, but larger and less likely payout in the $(1-\mu)$ term
- $x_2' < x_2^u$: smaller outcome in the μ term, and more likely but smaller payout in the $(1-\mu)$ term

Applying belief consistency

By belief consistency, Agent 2's beliefs are correct in equilibrium:

$$\begin{aligned} \mu &= \frac{\mathbb{P}\left[s \neq \emptyset | m = \emptyset\right]}{\mathbb{P}\left[m = \emptyset\right]} \\ &= \frac{q\left[1 - F\left(2x_2^u\right)\right]}{q\left[1 - F\left(2x_2^u\right)\right] + (1-q)} \end{aligned}$$

Result

Proposition: *The following pair of strategies forms a Perfect Bayesian Equilbria (PBE) of the informed game only if it is of the form:*

$$\sigma_1(\sigma_2) = \begin{cases} \text{Send } m = \emptyset \text{ and claim } \hat{x}_1(x_2^u) & \text{if } s = \emptyset \\ \text{Send } m = \emptyset \text{ and claim } x_1 = s - x_2^u & \text{if } s > 2x_2^u \\ \text{Send } m = s \text{ and claim } x_1 = s/2 & \text{if } s \leq 2x_2^u \end{cases}$$

and

$$\sigma_2 = \begin{cases} x_2 = s/2 & \text{if } m = s \\ x_2 = x_2^u & \text{if } m = \emptyset \end{cases}$$

for some $x_2^u \in [\underline{x}_2, \bar{x}_2]$*. Agent 2 believes that agent 1 conceals information whenever* $s > 2x_2^u$.

Remarks

- The values of x_2^u supporting a PBE are included in $[\underline{x_2}, \bar{x}_2] \subset [0, +\infty)$.
- $\underline{x_2} \leq \hat{x}_2 \leq \bar{x}_2$, where $\hat{x}_2$ is 2's unique equilibrium strategy of the uninformed game
- Not (yet?) able to prove that all values in $[\underline{x_2}, \bar{x}_2]$ support an equilibrium, but $\underline{x_2}, \hat{x}_2$, and $\bar{x}_2$ do.

Numerical example

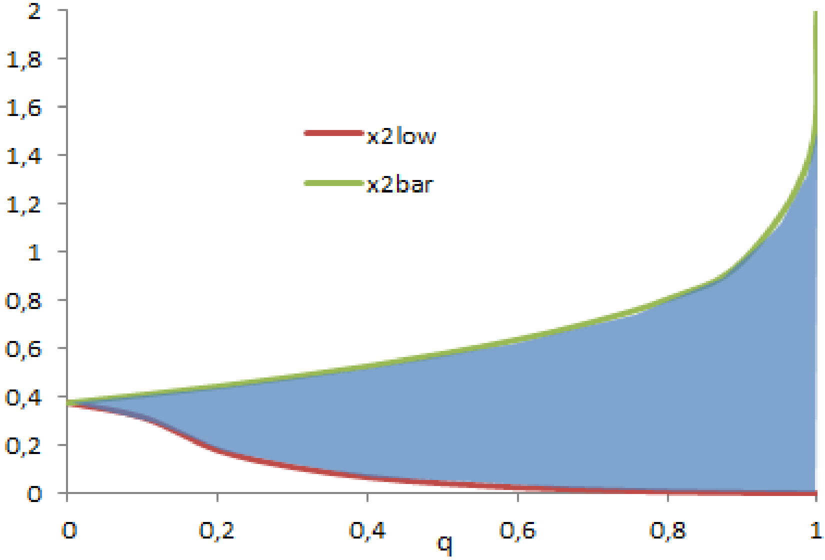
- "Positive" standard normal:

$$f(s) = \frac{2}{s\sqrt{2\pi}} \exp\left(-\frac{s^2}{2}\right) \qquad \text{defined on } \mathbb{R}_+$$

- vNM Utility functions:

$$u_1(y) = u_2(y) = \sqrt{y}$$

Supported equilibria

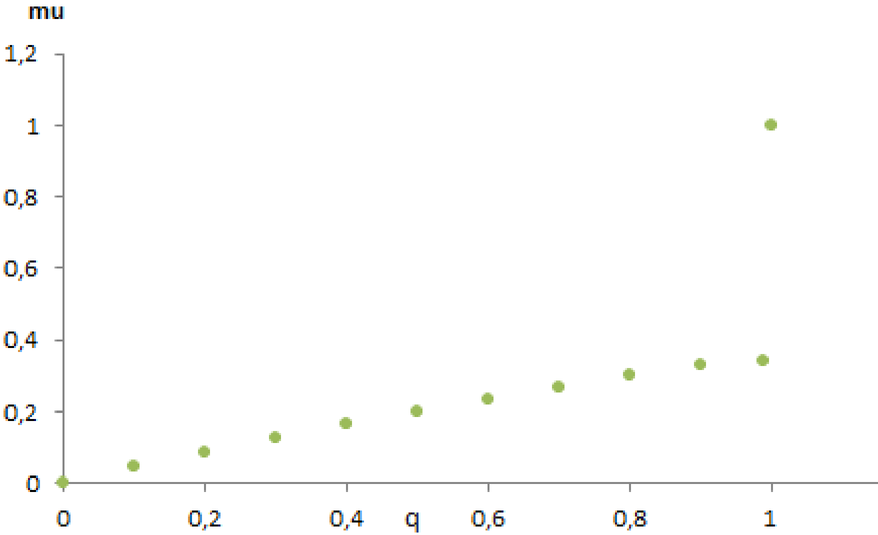


Equilibrium selection

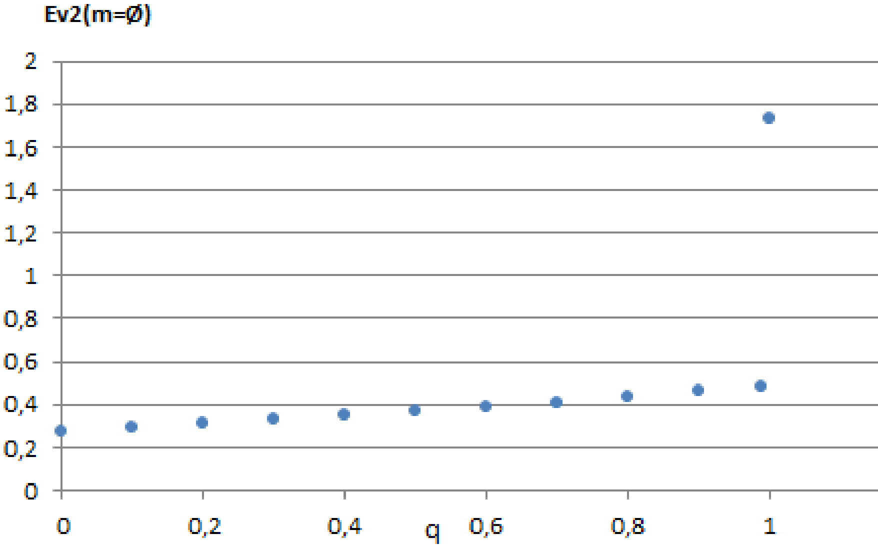
We track the equilibrium supported by $\bar{x}_2$:

- It is agent 2's largest credible threat
- It is the most dangerous equilibrium: the most prone to overshooting

Agent 2's belief that agent 1 has concealed the information



Expected v_2 when no message: $m = \emptyset$

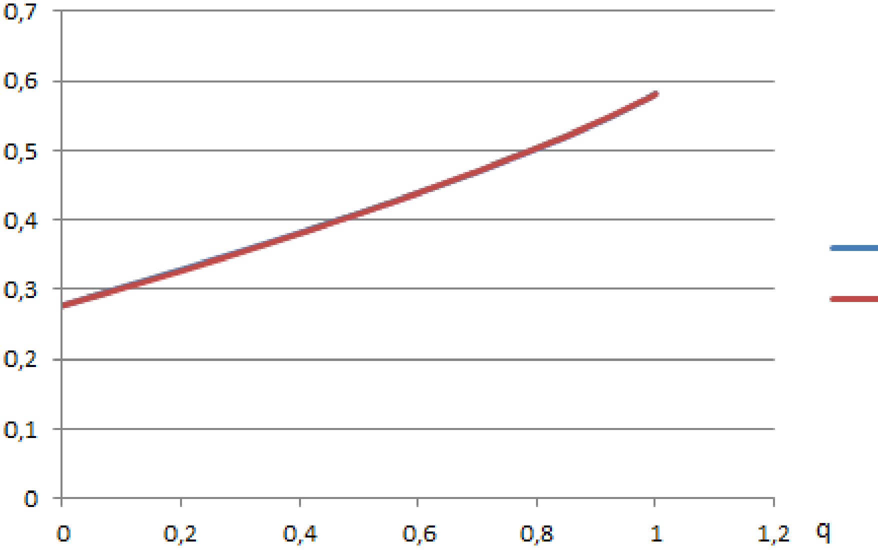


Remarks: $m = \emptyset$ is informative

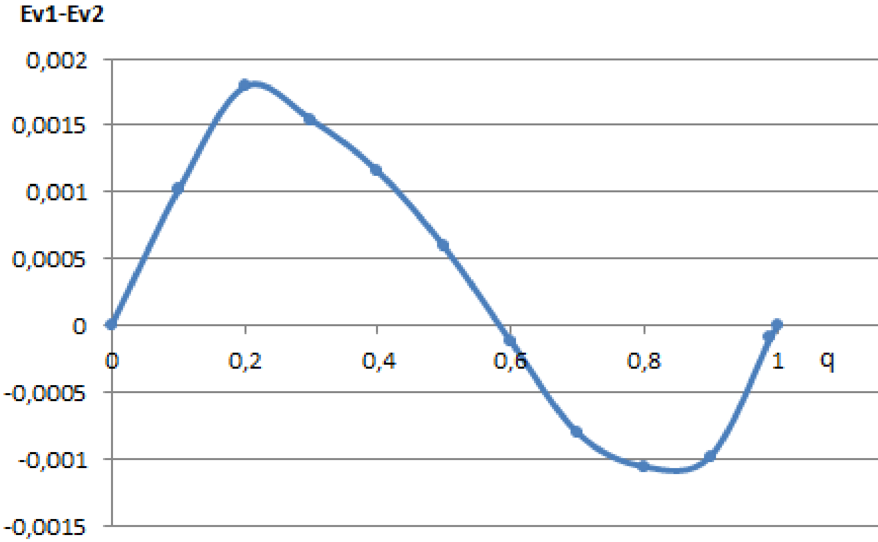
Even when $m = \emptyset$, agent 2 can benefit from the availability of information:

- Given agent 1's strategy, $m = \emptyset$ may mean that the resource is plentiful ("no news is good news")

Expected welfare



Zooming in: The value of information



Conjectures on implications for information acquisition

$\mathbb{E}[v_1] > \mathbb{E}[v_2]$ for low values of q, and $\mathbb{E}[v_1] < \mathbb{E}[v_2]$ for high values of q:

- Efforts to gather information are worthwhile if the success probability is not too high
- Otherwise, one is better off being the uninformed agent and respond with a large claim if $m = \emptyset$ (due to optimism or distrust)

In an R&D stage, prior to playing the disclosure game, it may be worthwhile to:

- Fund bad research (?!)
- Purposely not engage in research, or downplay the quality of the research being carried out (e.g. the US and Climate Change?)

Conclusion

We analysed a game of information disclosure in a threshold demand game. We find that:

- Strategic considerations affect the disclosure of information when $q \neq 1$
- More information improves welfare *ex ante*
- The informed agent does not always benefit the most from (possibly) being informed
- Strategic forces may discourage information acquisition (to be investigated more carefully)

Sheds some light on the management of common-property resources:

- Coasian-like argument in Ostrom (AER 2010): Efficiency can be achieved by sharing information.