Credibility in Closed-loop Supply Chain Coordination with a Revenue-Sharing Contract

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Practical examples
Research motivations

Coordination:
Players in CLSC have misaligned incentives to close the loop (Guide and van Wassenhove, 2009).

Coordination -> “…implementation of a mechanism (e.g., a contract) that leads players to a payoff-Pareto improving situation” (Cachon, 2003).

Nature of the incentive:

Endogenous: Ferguson et al. (2005), Majumber and Gronevelt (2001), Guide et al. (2003), Ray et al. (2005), (Agrawal and Toktay (2009)

The general framework

Non-coordinated chain

(Re)Manufacturer (Leader) \( A_M(t) \) \( \omega^B(t) \) Retailer (Follower) \( A_R(t) \) \( p^B(t) \) Customer

Endogenously Coordinated chain

(Re)Manufacturer (Leader) \( A_M^I(t) \) \( \omega^B(t) \) \( \phi^I(t) \) Retailer (Follower) \( A_R^I(t) \) \( p^I(t) \) Customer

Strategy

Exogenously Coordinated chain

(Re)Manufacturer (Leader) \( A_M^H(t) \) \( \omega^B(t) \) \( \phi^H \) Retailer (Follower) \( A_R^H(t) \) \( p^H(t) \) Customer

Fix parameter

Forward flow \hspace{1cm} \text{Reverse flow}
The (green) goodwill dynamics

\[
\dot{G} = \frac{dG(t)}{dt} = aA_M(t) + bA_R(t) - \delta G(t), \quad G(0) = G_0 \geq 0
\]

where:
- \(A_M(t)\) and \(A_R(t)\) are the Manufacturer’s and Retailer’s green activity programs efforts, respectively, while \(a\) and \(b\) are two constant parameters representing their effectiveness.
- \(\delta > 0\) is the forgetting effect.
The marketing role of goodwill: increasing the demand

\[ D(G(t), p(t)) = \theta \sqrt{G(t)} - \beta p(t) \]

where \( p(t) \) is the price that is controlled by the retailer, while \( \theta > 0 \) and \( \beta > 0 \) represent the demand sensitivity of goodwill and price, respectively.

The operational role of goodwill: increasing the return rate

\[ \tau(G(t)) = s \sqrt{G(t)} \]

\[ \pi_{\text{Return Management}}(G(t)) = \tau(G(t))[\Delta - c_O] = \tau(G(t))K \]

where \( \tau \geq 0 \) is the return rate and represents the percentage of products sold in the past that turns back to the manufacturer’s plant to be remanufactured, while \( s > 0 \) is a positive scaling parameter. \( \Delta \) is the marginal return’s residual value while \( c_O \) is the marginal operational cost.
Objective functions

Non-coordinated scenario

\[
J_M = \int_0^{+\infty} e^{-\rho t} \left\{ \left( \theta \sqrt{G} - \beta p \right) \left( \omega + K \sqrt{G} \right) - \frac{\mu_M A_M^2}{2} \right\} dt
\]

\[
J_R = \int_0^{+\infty} e^{-\rho t} \left\{ \left( \sqrt{G} - \beta p \right) \left( p - \omega \right) - \frac{\mu_R A_R^2}{2} \right\} dt
\]

Endogenous coordinated scenario

\[
J_M^I = \int_0^{+\infty} e^{-\rho t} \left\{ \left( \theta \sqrt{G} - \beta p \right) \left( \omega + K \sqrt{G} \right) \left( 1 - \phi^I \right) - \frac{\mu_M A_M^2}{2} \right\} dt
\]

\[
J_R^I = \int_0^{+\infty} e^{-\rho t} \left\{ \left( \sqrt{G} - \beta p \right) \left( p - \omega + \left( \omega + K \sqrt{G} \right) \phi^I - c_L \tau G \right) - \frac{\mu_R A_R^2}{2} \right\} dt
\]

Exogenous coordinated scenario

\[
J_M^II = \int_0^{+\infty} e^{-\rho t} \left\{ \left( \theta \sqrt{G} - \beta p \right) \left( \omega + K \sqrt{G} \right) \left( 1 - \phi^{II} \right) - \frac{\mu_M A_M^2}{2} \right\} dt
\]

\[
J_R^II = \int_0^{+\infty} e^{-\rho t} \left\{ \left( \sqrt{G} - \beta p \right) \left( p - \omega + \left( \omega + K \sqrt{G} \right) \phi^{II} - c_L \tau G \right) - \frac{\mu_R A_R^2}{2} \right\} dt
\]
The HJB equations

**Non-coordinated scenario**

\[
\rho V_M = \left( \theta \sqrt{G} - \beta p \right) \left( \omega + K \sqrt{G} \right) - \frac{\mu_M A_M^2}{2} + V_M ' \left[ aA_M + bA_R - \delta G \right]
\]

\[
\rho V_R = \left( \theta \sqrt{G} - \beta p \right) \left( p - \omega \right) - \frac{\mu_R A_R^2}{2} + V_R ' \left[ aA_M + bA_R - \delta G \right]
\]

**Endogenous coordinated scenario**

\[
\rho V_M^I = \left( \theta \sqrt{G^I} - \beta p^I \right) \left( \omega + K^I \sqrt{G^I} \right) \left( 1 - \phi \right) - \frac{\mu_M A_M^{I^2}}{2} + V_M '^I \left[ aA_M^I + bA_R^I - \delta G^I \right]
\]

\[
\rho V_R^I = \left( \theta \sqrt{G^I} - \beta p^I \right) \left( p^I - \omega + \phi \left( \omega + K^I \sqrt{G^I} \right) - sc_L \sqrt{G^I} \right) - \frac{\mu_R A_R^{I^2}}{2} + V_R '^I \left[ aA_M^I + bA_R^I - \delta G^I \right]
\]

**Exogenous coordinated scenario**

\[
\rho V_M^II = \left( \theta \sqrt{G^{II}} - \beta p^{II} \right) \left( \omega + K^{II} \sqrt{G^{II}} \right) \left( 1 - \phi \right) - \frac{\mu_M A_M^{II^2}}{2} + V_M '^{II} \left[ aA_M^{II} + bA_R^{II} - \delta G^{II} \right]
\]

\[
\rho V_R^II = \left( \theta \sqrt{G^{II}} - \beta p^{II} \right) \left( p^{II} - \omega + \phi \left( \omega + K^{II} \sqrt{G^{II}} \right) - sc_L \sqrt{G^{II}} \right) - \frac{\mu_R A_R^{II^2}}{2} + V_R '^{II} \left[ aA_M^{II} + bA_R^{II} - \delta G^{II} \right]
\]
The HJB equations

Non-coordinated scenario

\[ 8 \rho \beta \mu_m \mu_R \left( l_1 G + l_2 \right) = \mu_m \mu_R \left( \theta + \beta K \right)^2 G + 4 \beta l_1 \left( \mu_R a^2 l_1 + 2 \mu_m b^2 l_3 - 2 \delta \mu_m \mu_R G \right) \]

\[ 16 \rho \beta \mu_m \mu_R \left( l_3 G + l_4 \right) = \mu_m \mu_R \left( \theta + \beta K \right)^2 G + 8 \beta l_3 \left( 2 \mu_R a^2 l_1 + \mu_m b^2 l_3 - 2 \delta \mu_m \mu_R G \right) \]

Endogenous coordinated scenario

\[ 8 \rho \beta \mu_m \mu_R \left( n_1 G^I + n_2 \right) = \mu_m \mu_R \left[ \theta + \beta (K - sc_L) \right]^2 G^I + 4 n_1 \left( \mu_R a^2 n_1 + 2 \mu_m b^2 n_3 - 2 \delta \mu_m \mu_R G^I \right) \]

\[ 16 \rho \beta \mu_m \mu_R \left( n_3 G^I + n_4 \right) = \mu_m \mu_R \left[ \theta + \beta (K - sc_L) \right]^2 G^I + 8 \beta n_3 \left( 2 \mu_R a^2 n_1 + \mu_m b^2 n_3 - 2 \delta \mu_m \mu_R G^I \right) \]

Exogenous coordinated scenario

\[ 8 \rho \beta \mu_m \mu_R \left( m_1 G'' + m_2 \right) = \mu_m \mu_R \left[ \left( \theta + \beta K \right) \left( 1 + \phi_\epsilon \right) - 2 \beta sc_L \right] \left( \theta + \beta K \right) \left( 1 - \phi_\epsilon \right) G'' \]

\[ + 4 \beta m_1 \left( \mu_R a^2 m_1 + 2 \mu_m b^2 m_3 - 2 \delta \mu_m \mu_R G'' \right) \]

\[ 16 \rho \beta \mu_m \mu_R \left( m_3 G'' + m_4 \right) = \left[ \left( \theta + \beta K \right) \left( 1 + \phi_\epsilon \right) - 2 \beta sc_L \right]^2 G'' + 8 \beta m_3 \left( 2 \mu_R a^2 m_1 + \mu_m b^2 m_3 - 2 \delta \mu_m \mu_R G'' \right) \]
Manufacturer’s Green Activity Programs (GAP) Strategy comparison

**Proposition.** The manufacturer always invests in GAP efforts more under a non-coordinated scenario independently of the exogenous or endogenous nature of the sharing parameter. Under coordination, the manufacturer invests more in GAP when asymmetric information occurs.

\[ A_M - A_M^I = \frac{ac LS [2(\theta + \beta K) - sc L \beta]}{8\mu_M (\rho + \delta)} > 0 \]

\[ A_M - A_M^{II} = \frac{a(\theta + \beta K)\left[(\theta + (\Delta s - c_E h) \beta)\phi_\varepsilon + c_L s \beta (2 - 2\phi_\varepsilon - \phi_\varepsilon^2)\right]}{8\beta \mu_M (\rho + \delta)} > 0\forall \phi_\varepsilon \in (0,1] \]

\[ A_M^I - A_M^{II} = \frac{a((\theta + \beta K)\phi_\varepsilon - c_L s \beta)^2}{8\beta \mu_M (\rho + \delta)} > 0\forall \phi_\varepsilon \in (0,1] \]

Does a manufacturer’s GAP strategy depend on his decisional power?
Retailer’s GAP Strategy comparison

**Proposition.** The retailer always invests more in GAP under a non-coordinate than under an endogenous coordination. Under an exogenous coordination scenario the retailer invests more in GAP only when the sharing parameter is sufficiently high.

\[ A_R - A_R^{\prime} = \frac{b c_L s [2(\theta + \beta K) - s c_L \beta]}{16 \mu_M (\rho + \delta)} > 0 \]

\[ A_R - A_R^{\prime\prime} = \begin{cases} b \left[ (\theta + K \beta)^2 - \left[ (1 + \phi^{\prime\prime}) (\theta + K \beta) + 2 c_L s \beta \right]^2 \right] & \leq 0 \quad \text{if} \quad \phi^{\prime\prime} \in \left[ \frac{2 c_L s \beta}{\theta + K \beta}, 1 \right] \\ > 0 & \text{otherwise.} \end{cases} \]

\[ A_R^{\prime} - A_R^{\prime\prime} = \begin{cases} b \left[ (\theta + K \beta - c_L s \beta)^2 - \left[ (1 + \phi^{\prime\prime}) (\theta + K \beta) + 2 c_L s \beta \right]^2 \right] & \leq 0 \quad \text{if} \quad \phi^{\prime\prime} \in \left[ \frac{c_L s \beta}{\theta + K \beta}, 1 \right] \\ > 0 & \text{otherwise.} \end{cases} \]
Green Goodwill comparison

**Proposition.** The accumulated stock of goodwill under a non-coordinated scenario is always higher than the stock under an endogenous coordination scenario. Under an exogenous coordination scenario the stock of goodwill is higher than the stock in the other scenarios only when the sharing parameter is sufficiently high.

\[
G - G' = \frac{c_L s \left(2a^2 \mu_R + b^2 \mu_M\right)[2(\theta + \beta K) - c_L s \beta]}{16\delta \mu_M \mu_R (\delta + \rho)} > 0
\]

\[
G - G'' = \frac{(2a^2 + b^2)(\theta + \beta K)^2 - [(\theta + \beta K)\left(a^2 (1 + \phi'') + b^2 (1 + \phi'')\right)] - c_L s \beta} {16\delta \mu_M \mu_R (\delta + \rho)} 
\begin{cases} 
  \leq 0 & \text{if } \phi'' \in \left(\frac{4c_L s \beta (2\psi - c_L s \beta)}{\theta - \beta K}, 1\right) \\
  > 0 & \text{otherwise.}
\end{cases}
\]

\[
G' - G''' = \frac{\phi'' (\theta + \beta K) - c_L s \beta \left\{2a^2 \mu_R \left[(\theta + \beta K)\phi'' - c_L s \beta\right] - b^2 \mu_M \left[(\theta + \beta K)(2 + \phi'') - 3c_L s \beta\right]\right\}} {16\delta \mu_M \mu_R (\delta + \rho)} 
\begin{cases} 
  \leq 0 & \text{if } \phi'' \in \left[\frac{c_L s \beta}{\theta + \beta K}, 1\right] \\
  > 0 & \text{otherwise.}
\end{cases}
\]
Price strategies and sales comparison

Claims:

1. Mitigation of the double marginalization effect is not followed by higher sales.

2. When the sharing parameter is low, coordination damages the sales.

3. A lower price due to coordination boosts the sales only for high sharing parameter values.
Claims:

1. A manufacturer should prefer an endogenous coordination.

2. When coordination is not an option, the manufacturer should select the coordination type according to the sharing parameter values.
Claims:

1. A retailer prefers an exogenous coordination.

2. When coordination is not an option, the retailer prefers an exogenous coordination mechanism only when the sharing parameter is sufficiently high.
Early Conclusions

1. There is a room for coordination only when the incentive is exogenous.

2. Coordination leads to the mitigation of the double marginalization effect but not an increasing sales also.

3. When coordination is compulsory rather than an option, the decision maker should select the type of incentive (endogenous vs. exogenous) according to the sharing parameter values.
Further analysis

1. Is there a payoff Pareto-improving region?

2. How does environmental performance influence the decision maker?

3. How will the results change if the value functions are not linear (e.g., quadratic)?