The Thin Green Line:
Transboundary Pollution Problems in Coupled Lake Systems

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2nd Workshop Game Theory in Energy, Resources and the Environment 2008

29 June 2008

- The Game
- Non Cooperative Solution
- Dynamic Programming
- Numerical Results
- Stochastic Elements & Droughts

The Single Lake Game

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- Costs: Phosphorus $\longrightarrow$ nutrients for weeds and algæ

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- Player's benefits are a function of his/her own loading

State Equations

- Upper lake

$$x_{t+1} = b_x x_t + \frac{x_t^2}{1 + x_t^2} + \sum_i a_{i,t}$$

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- Lower lake

$$y_{t+1} = b_y y_t + \frac{y_t^2}{1 + y_t^2} + \sum_i a_{i,t} + \mu x_t$$

Weighted Social Welfare Criteria

- The Planner's Problem

$$\max_{\{a_{i,t}\}} \sum_{t=0}^{\infty} \beta^t \Big(W_1(a_{1,t}, \cdots, a_{n,t}, x_t) - \lambda W_2(a_{1,t}, \cdots, a_{m,t}, y_t) \Big)$$

$$x_{t+1} = bx_t + \frac{x_t^2}{1+x_t^2} + \sum_i a_{i,t}$$

$$y_{t+1} = by_t + \frac{y_t^2}{1+y_t^2} + \sum_i a_{i,t} + \mu x_t$$

Objectives and the Game Potential

- Each upper lake player solves

$$\max_{\{a_{i,t}\}} \sum_{t=0}^{\infty} \beta^t \left(u_i(a_{i,t}) - k_i c(x_t) \right)$$

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- Potential function

$$\max_{\{a_{1,t} \ldots a_{n,t}\}} \sum_{t=0}^{\infty} \beta^t \left(\sum_i u_i(a_{i,t})/k_i - c(x_t) \right)$$

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Properties of Solutions

- Two possible optimal steady states:

Low P	oligotrophic	low in nutrients
High P	eutrophic	high in nutrients

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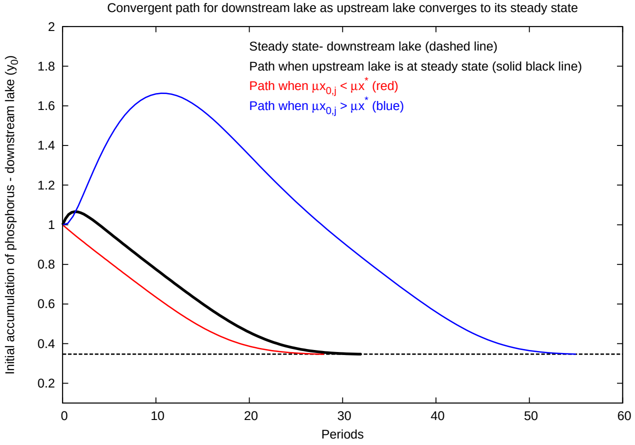
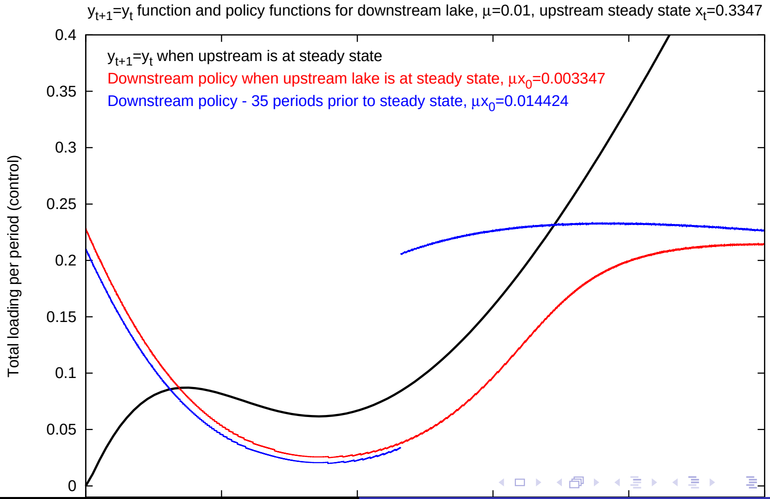
Properties of Solutions

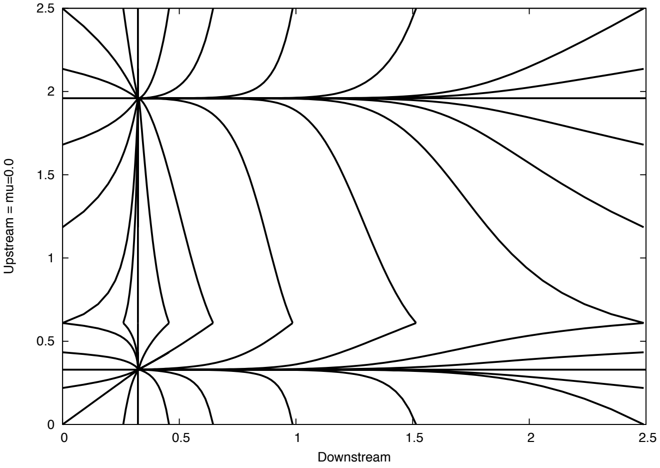
- Two possible optimal steady states:

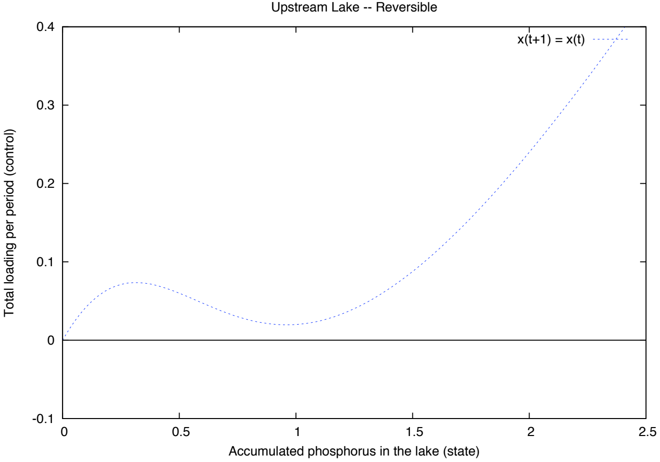
Low P	oligotrophic	low in nutrients
High P	eutrophic	high in nutrients

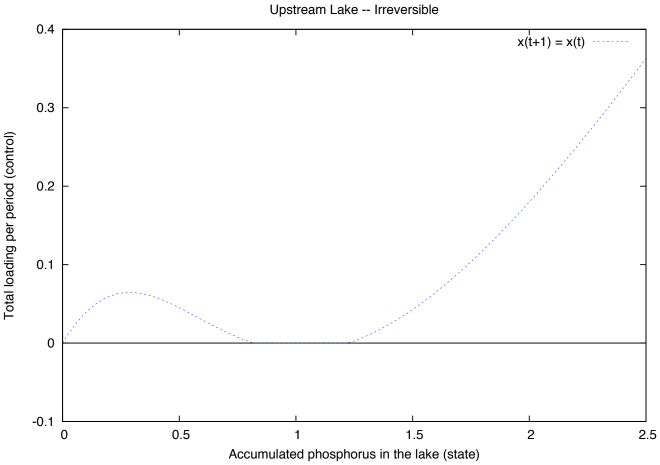
- Upper lake converges monotonically to SS
- Skiba points (both steady states are optimal)
- Lower lake very sensitive to μ

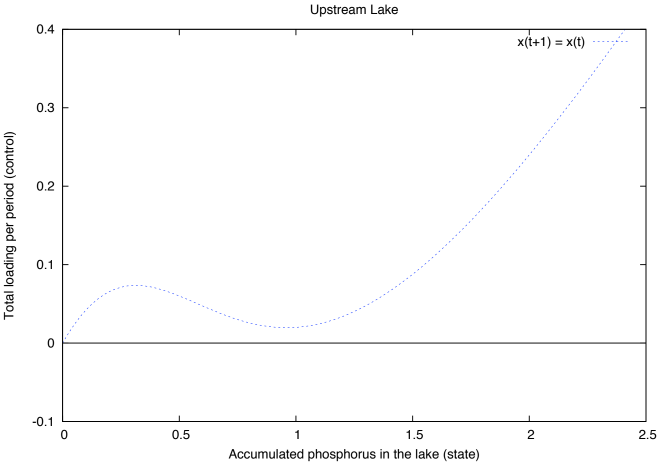

Convergent path for downstream lake as upstream lake converges to its steady state
Steady state- downstream lake (dashed line)
Path when upstream lake is at steady state (solid black line)
Path when $\mu x_{0,j} < \mu x^*$ (red)
Path when $\mu x_{0,j} > \mu x^*$ (blue)
Initial accumulation of phosphorus - downstream lake (y_0)
Periods
2
1.8
1.6
1.4
1.2
1
0.8
0.6
0.4
0.2
0
10
20
30
40
50
60

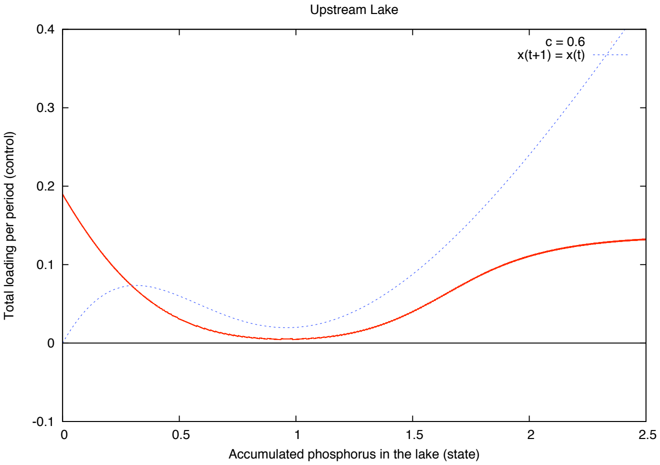

$y_{t+1}=y_t$ function and policy functions for downstream lake, μ=0.01, upstream steady state x_t=0.3347
$y_{t+1}=y_t$ when upstream is at steady state
Downstream policy when upstream lake is at steady state, μx_0=0.003347
Downstream policy - 35 periods prior to steady state, μx_0=0.014424
Total loading per period (control)
0.4
0.35
0.3
0.25
0.2
0.15
0.1
0.05
0

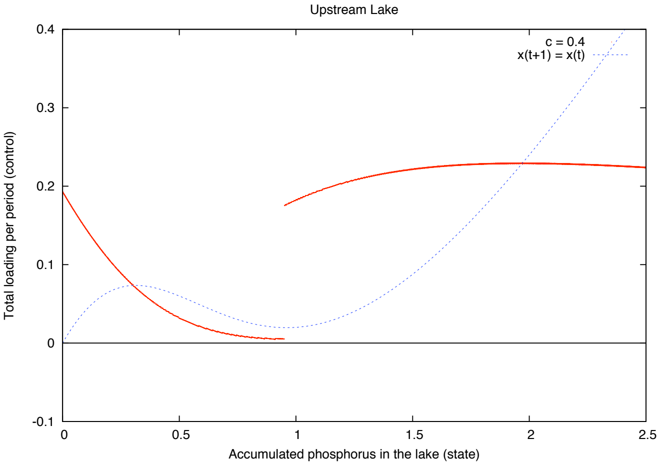

2.5
2
1.5
1
0.5
0
Upstream = mu=0.0
0
0.5
1
1.5
2
2.5
Downstream

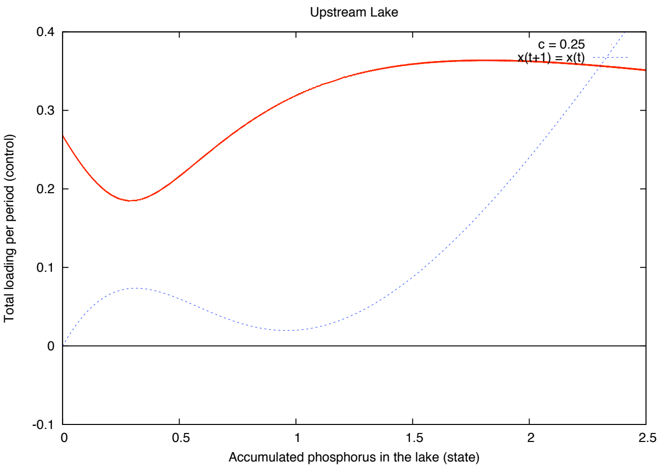

Upstream Lake -- Reversible
x(t+1) = x(t)
Total loading per period (control)
Accumulated phosphorus in the lake (state)
0.4
0.3
0.2
0.1
0
-0.1
0
0.5
1
1.5
2
2.5

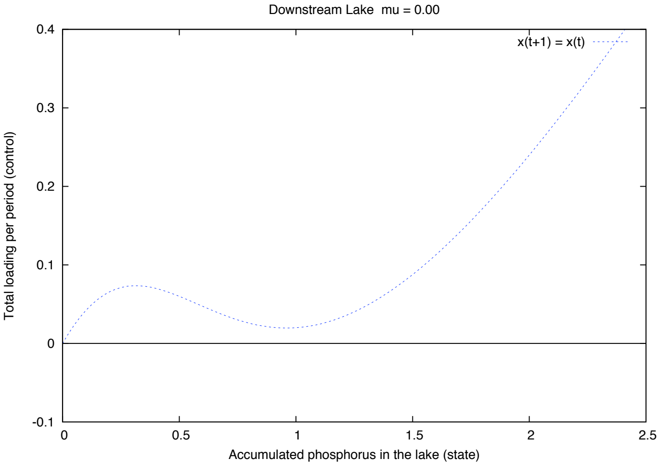

Upstream Lake -- Irreversible
x(t+1) = x(t)
Total loading per period (control)
Accumulated phosphorus in the lake (state)
0.4
0.3
0.2
0.1
0
-0.1
0
0.5
1
1.5
2
2.5

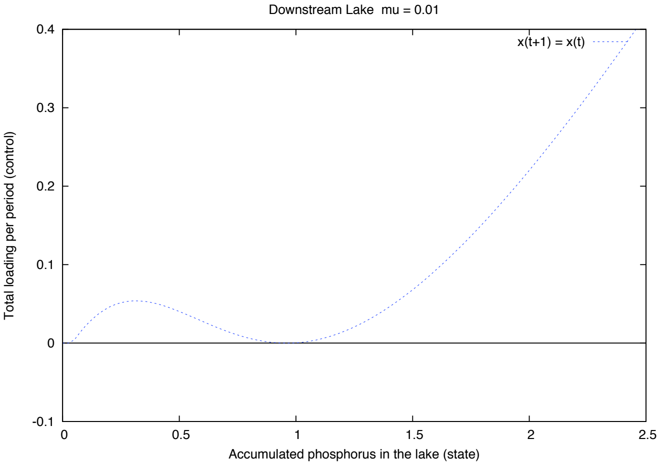

Upstream Lake
x(t+1) = x(t)
0.4
0.3
0.2
0.1
0
-0.1
Total loading per period (control)
0
0.5
1
1.5
2
2.5
Accumulated phosphorus in the lake (state)

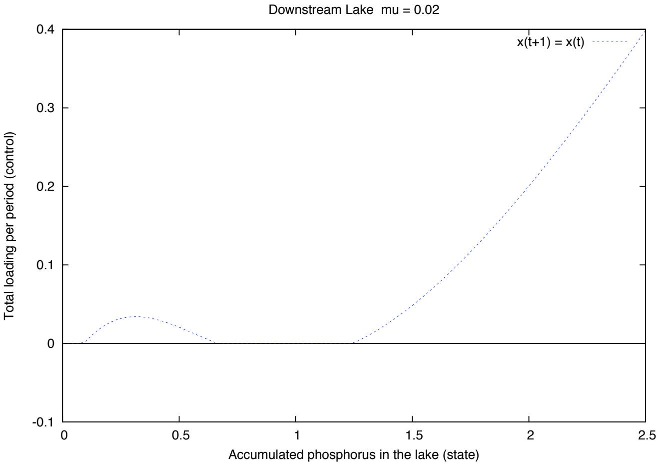

Upstream Lake
c = 0.6
x(t+1) = x(t)
0.4
0.3
0.2
0.1
0
-0.1
0
0.5
1
1.5
2
2.5
Total loading per period (control)
Accumulated phosphorus in the lake (state)

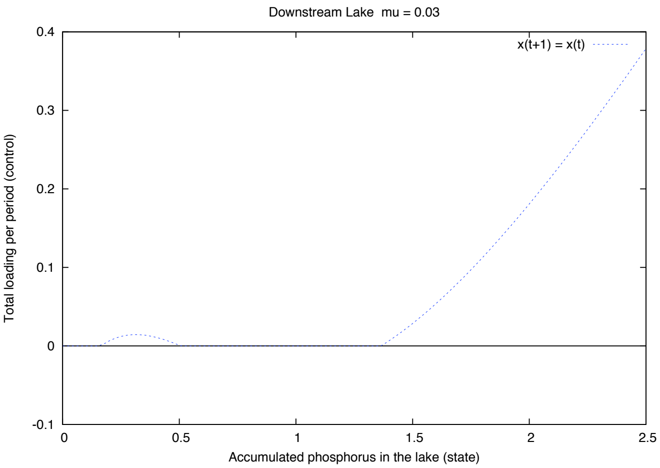

Upstream Lake
c = 0.4
x(t+1) = x(t)
Total loading per period (control)
Accumulated phosphorus in the lake (state)
0.4
0.3
0.2
0.1
0
-0.1
0
0.5
1
1.5
2
2.5

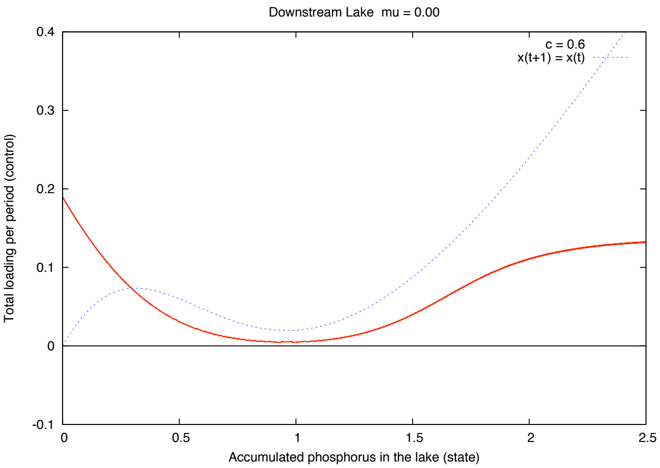

Upstream Lake
c = 0.25
x(t+1) = x(t)
Total loading per period (control)
Accumulated phosphorus in the lake (state)
0.4
0.3
0.2
0.1
0
-0.1
0
0.5
1
1.5
2
2.5

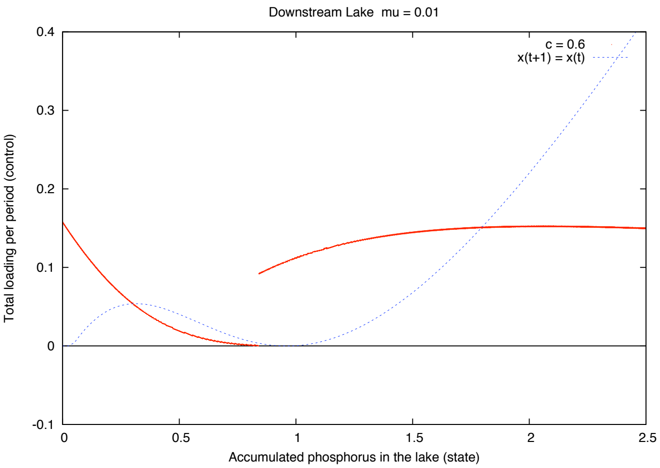

Downstream Lake mu = 0.00
x(t+1) = x(t)
0.4
0.3
0.2
0.1
0
-0.1
0
0.5
1
1.5
2
2.5
Total loading per period (control)
Accumulated phosphorus in the lake (state)

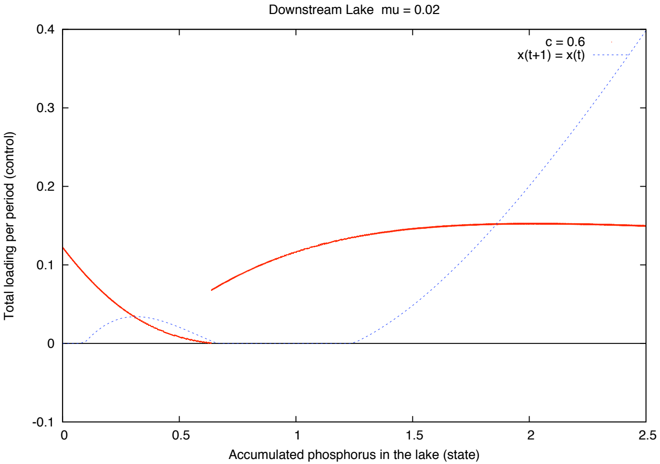

Downstream Lake mu = 0.01
x(t+1) = x(t)
Total loading per period (control)
Accumulated phosphorus in the lake (state)
0.4
0.3
0.2
0.1
0
-0.1
0
0.5
1
1.5
2
2.5


Downstream Lake mu = 0.02
x(t+1) = x(t)
Total loading per period (control)
Accumulated phosphorus in the lake (state)
0.4
0.3
0.2
0.1
0
-0.1
0
0.5
1
1.5
2
2.5


Downstream Lake mu = 0.03
x(t+1) = x(t)
Total loading per period (control)
Accumulated phosphorus in the lake (state)
0.4
0.3
0.2
0.1
0
-0.1
0
0.5
1
1.5
2
2.5


Downstream Lake mu = 0.00
c = 0.6
x(t+1) = x(t)
Total loading per period (control)
Accumulated phosphorus in the lake (state)
0.4
0.3
0.2
0.1
0
-0.1
0
0.5
1
1.5
2
2.5


Downstream Lake mu = 0.01
c = 0.6
x(t+1) = x(t)
Total loading per period (control)
Accumulated phosphorus in the lake (state)
0.4
0.3
0.2
0.1
0
-0.1
0
0.5
1
1.5
2
2.5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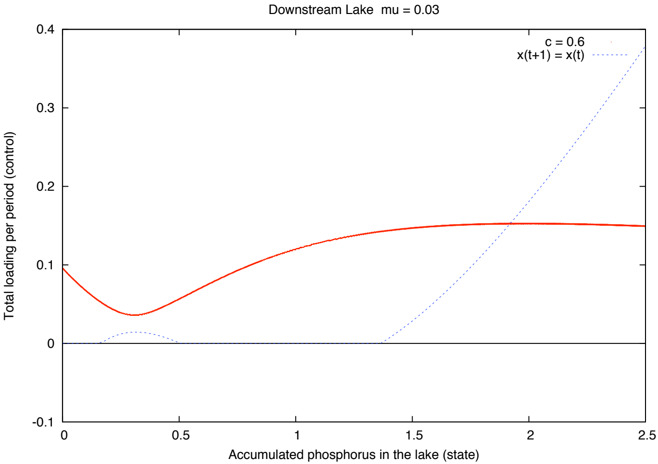
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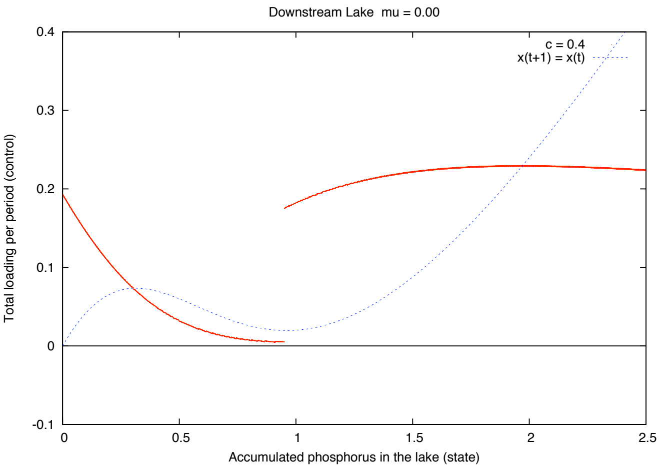
Downstream Lake mu = 0.00
c = 0.4
x(t+1) = x(t)
0.4
0.3
0.2
0.1
0
-0.1
0
0.5
1
1.5
2
2.5
Total loading per period (control)
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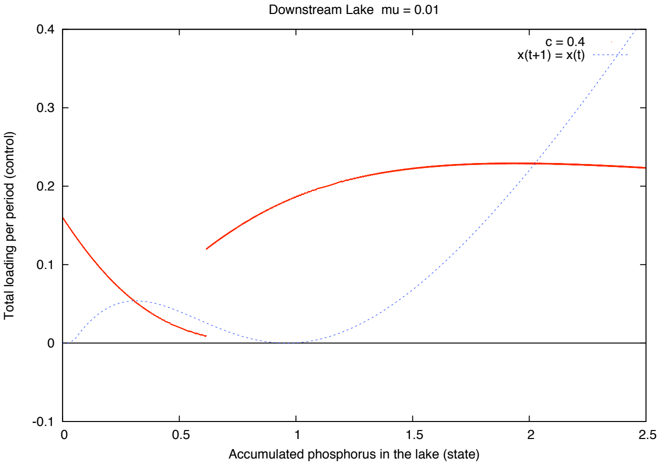
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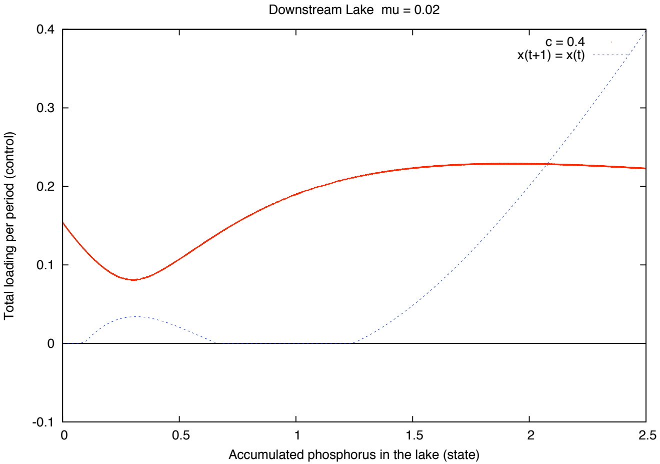
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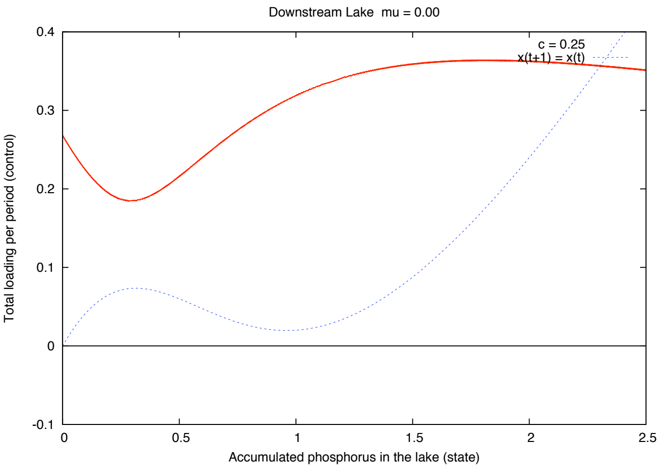
Downstream Lake mu = 0.00
c = 0.25
x(t+1) = x(t)
0.4
0.3
0.2
0.1
0
-0.1
0
0.5
1
1.5
2
2.5
Total loading per period (control)
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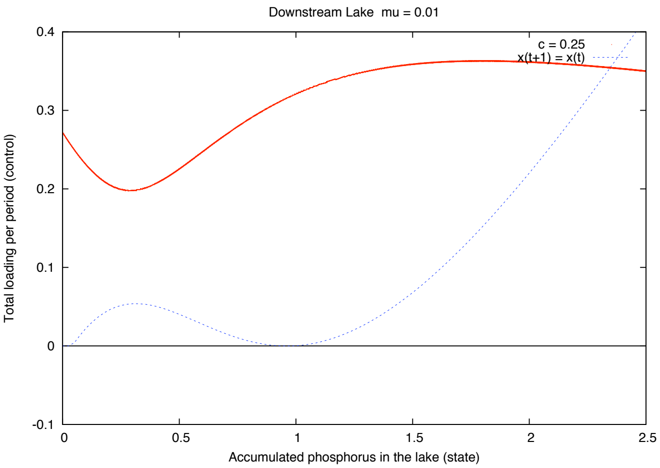
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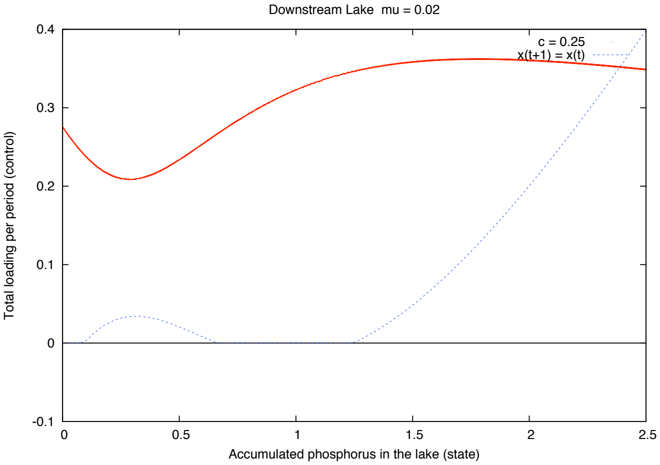
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