```
// -------------------------------------------------------------- -*- C++ -*-
// File: ilodiet.cpp
// Version 12.6.3
// --------------------------------------------------------------------------
// Licensed Materials - Property of IBM
// 5725-A06 5725-A29 5724-Y48 5724-Y49 5724-Y54 5724-Y55 5655-Y21
// Copyright IBM Corporation 2000, 2015. All Rights Reserved.
//
// US Government Users Restricted Rights - Use, duplication or
// disclosure restricted by GSA ADP Schedule Contract with
// IBM Corp.
// --------------------------------------------------------------------------
//
// A dietary model.
//
// Input data:
// foodMin[j]      minimum amount of food j to use
// foodMax[j]      maximum amount of food j to use
// foodCost[j]     cost for one unit of food j
// nutrMin[i]      minimum amount of nutrient i
// nutrMax[i]      maximum amount of nutrient i
// nutrPer[i][j]   nutrition amount of nutrient i in food j
//
// Modeling variables:
// Buy[j]          amount of food j to purchase
//
// Objective:
// minimize sum(j) Buy[j] * foodCost[j]
//
// Constraints:
// forall nutr i: nutrMin[i] <= sum(j) Buy[j] * nutrPer[i][j] <= nutrMax[i]
//

#include <ilcplex/ilocplex.h>           // entêtes obligatoires
ILOSTLBEGIN

void usage(const char* name) {
   cerr << endl;
   cerr << "usage:   " << name << " [options] <file>" << endl;
   cerr << "options: -c  build model by column" << endl;
   cerr << "         -i  use integer variables" << endl;
   cerr << endl;
}

void buildModelByRow(IloModel           mod,
                     IloNumVarArray     Buy,
                     const IloNumArray  foodMin,
                     const IloNumArray  foodMax,
                     const IloNumArray  foodCost,
                     const IloNumArray  nutrMin,
                     const IloNumArray  nutrMax,
                     const IloNumArray2 nutrPer,
                     IloNumVar::Type type) {

   IloEnv env = mod.getEnv();
   Buy.clear();
   IloNumVarArray tmp(env, foodMin, foodMax, type);
          // tableau temporaire de variables créé d'un coup avec bornes
   Buy.add(tmp);                    // copie dans tableau permanent
   tmp.end();             // détruire objet temporaire

   IloInt i, j;
   IloInt n = foodCost.getSize();
   IloInt m = nutrMin.getSize();

   mod.add(IloMinimize(env, IloScalProd(Buy,foodCost)));
                                       // création de l'objectif d'un coup
   for (i = 0; i < m; i++) {
      IloExpr expr(env);
      for (j = 0; j < n; j++) {
         expr += Buy[j] * nutrPer[i][j];
                           // remplissage de l'expression de la contrainte
      }
      mod.add(nutrMin[i] <= expr <= nutrMax[i]);
                           // création et ajout de la contrainte au modèle
      expr.end();      // détruire objet temporaire
   }
}

void buildModelByColumn(IloModel           mod,
                        IloNumVarArray     Buy,
                        const IloNumArray  foodMin,
                        const IloNumArray  foodMax,
                        const IloNumArray  foodCost,
                        const IloNumArray  nutrMin,
                        const IloNumArray  nutrMax,
                        const IloNumArray2 nutrPer,
                        IloNumVar::Type    type) {

   IloEnv env = mod.getEnv();
   IloInt i, j;
   IloInt n = foodCost.getSize();
   IloInt m = nutrMin.getSize();

   IloRangeArray range (env, nutrMin, nutrMax);
  // tableau temporaire de contraintes (pour créer intersections avec variables)
   mod.add(range);
   IloObjective cost = IloAdd(mod, IloMinimize(env));
                                          // objectif créé et ajouté au modèle

   for (j = 0; j < n; j++) {
      IloNumColumn col = cost(foodCost[j]);
                                       // coût de la colonne j dans l'objectif
      for (i = 0; i < m; i++) {
         col += range[i](nutrPer[i][j]);
                                 // coeff de la colonne j pour la contrainte i
      }
      Buy.add(IloNumVar(col, foodMin[j], foodMax[j], type));
                           // création de la variable j et ajout dans tableauu
      col.end();   // détruire objet temporaire
   }
   range.end();    // détruire objet temporaire
}
```

```
int main(int argc, char **argv)
{
   IloEnv env;  // création de l'environnement

   try {
      const char*     filename  = "../../../examples/data/diet.dat";
      IloBool         byColumn  = IloFalse;
      IloNumVar::Type varType   = ILOFLOAT;
      IloInt i;

      for (i = 1; i < argc; i++) {
         if (argv[i][0] == '-') {
            switch (argv[i][1]) {
            case 'c':
               byColumn = IloTrue;
               break;
            case 'i':
               varType = ILOINT;
               break;
            default:
               usage(argv[0]);
               throw (-1);
            }
         }
         else {
            filename = argv[i];
            break;
         }
      }

      ifstream file(filename);
      if ( !file ) {
         cerr << "ERROR: could not open file '" << filename
              << "' for reading" << endl;
         usage(argv[0]);
         throw (-1);
      }

      // model data

      IloNumArray  foodCost(env), foodMin(env), foodMax(env);
      IloNumArray  nutrMin(env), nutrMax(env);      // tableaux de nombres
      IloNumArray2 nutrPer(env);     // matrice 2D de nombres

//------------------------------------------------------------------
// Ne pas utiliser car suppose un format de fichier particulier
// Lire normalement le fichier et stocker dans des IloArray<>
// ou autres structures
      file >> foodCost >> foodMin >> foodMax;
      file >> nutrMin >> nutrMax;
      file >> nutrPer;
 //------------------------------------------------------------------

      IloInt nFoods = foodCost.getSize();
      IloInt nNutr  = nutrMin.getSize();

      if ( foodMin.getSize() != nFoods ||
           foodMax.getSize() != nFoods ||
           nutrPer.getSize() != nNutr  ||
           nutrMax.getSize() != nNutr    ) {
         cerr << "ERROR: Data file '" << filename
              << "' contains inconsistent data" << endl;
         throw (-1);
      }

      for (i = 0; i < nNutr; i++) {
         if (nutrPer[i].getSize() != nFoods) {
            cerr << "ERROR: Data file '" << argv[0]
                 << "' contains inconsistent data" << endl;
            throw (-1);
         }
      }

      // Build model

      IloModel       mod(env);   // création du modèle
      IloNumVarArray Buy(env);   // création du tableau de variables

      if ( byColumn ) {
         buildModelByColumn(mod, Buy, foodMin, foodMax, foodCost,
                            nutrMin, nutrMax, nutrPer, varType);
      }
      else {
         buildModelByRow(mod, Buy, foodMin, foodMax, foodCost,
                         nutrMin, nutrMax, nutrPer, varType);
      }

      // Solve model

      IloCplex cplex(mod);      // création de l'objet CPLEX à partir du modèle
      cplex.exportModel("diet.lp");  // écriture du modèle dans un fichier texte

      cplex.solve();               // résolution du modèle

        // récupération de la solution
      cplex.out() << "solution status = " << cplex.getCplexStatus() << endl;
      cplex.out() << endl;
      cplex.out() << "cost   = " << cplex.getObjValue() << endl;
      for (i = 0; i < foodCost.getSize(); i++) {
         cplex.out() << "  Buy" << i << " = " << cplex.getValue(Buy[i]) << endl;
      }
   }
   catch (IloException& ex) {              // gestion des exceptions
      cerr << "Error: " << ex << endl;
   }
   catch (...) {
      cerr << "Error" << endl;
   }

   env.end();
            // destruction de l'environnement ainsi que de tous les objets liés

   return 0;
}
```

```
// -------------------------------------------------------------- -*- C++ -*-
// File: facility.cpp
// Version 12.6.3
// --------------------------------------------------------------------------
// Licensed Materials - Property of IBM
// 5725-A06 5725-A29 5724-Y48 5724-Y49 5724-Y54 5724-Y55 5655-Y21
// Copyright IBM Corporation 2000, 2015. All Rights Reserved.
//
// US Government Users Restricted Rights - Use, duplication or
// disclosure restricted by GSA ADP Schedule Contract with
// IBM Corp.
// --------------------------------------------------------------------------


#include <ilcplex/ilocplex.h>
ILOSTLBEGIN

typedef IloArray<IloNumArray>    FloatMatrix;        // matrice 2D de nombres
typedef IloArray<IloNumVarArray> NumVarMatrix;      // matrice 2D de variables


//------------------------------------------------------------------------
// Input data:
//
// capacity[j] : capacité de chaque dépôt (en clients)
// fixedCost[j] : coût fixe de chaque dépôt
// cost[i][j] : coût de relier un client i au dépôt j
//
// Variables:
//
// open[j] : 1 si dépôt ouvert, 0 sinon
// supply[i][j] : i si client i est relié au dépôt j
//
// objective:
// minimize sum(j) fixedCost(j) * open(j) + sum(i,j) cost(i,j) * supply(i,j)
//
// constraints:
//
// sum(j) supply(i,j) == 1     // chaque client est relié à un dépôt
//
// sum(i) supply(i,j) <= capacity(j) * open(j)  // capacité des dépôts
//
//------------------------------------------------------------------------




int main(int argc, char **argv)
{
   IloEnv env;   // création de l'environnement
   try {
      IloInt i, j;
      IloNumArray capacity(env), fixedCost(env);
      FloatMatrix cost(env);
      IloInt      nbLocations;
      IloInt      nbClients;
```

```
   //------------------------------------------------------------------
   // Ne pas utiliser car suppose un format de fichier particulier
   // Lire normalement le fichier et stocker dans des IloArray<>
   // ou autres structures

      const char* filename  = "../../../examples/data/facility.dat";
      if (argc > 1)
         filename = argv[1];
      ifstream file(filename);
      if (!file) {
         cerr << "ERROR: could not open file '" << filename
              << "' for reading" << endl;
         cerr << "usage:   " << argv[0] << " <file>" << endl;
         throw(-1);
      }

      file >> capacity >> fixedCost >> cost;
   //------------------------------------------------------------------

      nbLocations = capacity.getSize();
      nbClients   = cost.getSize();

      IloBool consistentData = (fixedCost.getSize() == nbLocations);
      for(i = 0; consistentData && (i < nbClients); i++)
         consistentData = (cost[i].getSize() == nbLocations);
      if (!consistentData) {
         cerr << "ERROR: data file '"
              << filename << "' contains inconsistent data" << endl;
         throw(-1);
      }

      IloNumVarArray open(env, nbLocations, 0, 1, ILOINT);
                        // tableau de variables binaires de taille nbLocations
      NumVarMatrix supply(env, nbClients);
      for(i = 0; i < nbClients; i++)
         supply[i] = IloNumVarArray(env, nbLocations, 0, 1, ILOINT);
           // matrice 2D de variables binaires de taille nbClients * nbLocations

      IloModel model(env);
      for(i = 0; i < nbClients; i++)
         model.add(IloSum(supply[i]) == 1);  // ∑j supply[i][j] == 1
      for(j = 0; j < nbLocations; j++) {
         IloExpr v(env);
         for(i = 0; i < nbClients; i++)
            v += supply[i][j];                  // ∑i supply[i][j]
         model.add(v <= capacity[j] * open[j]);
                                               // ∑i supply[i][j] <= cap[j]*open[j]
         v.end();     // détruire objet temporaire
      }

      IloExpr obj = IloScalProd(fixedCost, open);
      for(i = 0; i < nbClients; i++) {
         obj += IloScalProd(cost[i], supply[i]); // ∑i,j supply[i][j]*cost[i][j]
      }
      model.add(IloMinimize(env, obj));
      obj.end();          // détruire objet temporaire
```

```
      IloCplex cplex(env);
      cplex.extract(model);  // équivaut à IloCplex cplex(model);
      cplex.solve();
      cplex.out() << "Solution status: " << cplex.getCplexStatus() << endl;

      IloNum tolerance =
                  cplex.getParam(IloCplex::Param::MIP::Tolerances::Integrality);
      cplex.out() << "Optimal value: " << cplex.getObjValue() << endl;
      for(j = 0; j < nbLocations; j++) {
         if (cplex.getValue(open[j]) >= 1 - tolerance) {
            cplex.out() << "Facility " << j << " is open, it serves clients ";
            for(i = 0; i < nbClients; i++) {
               if (cplex.getValue(supply[i][j]) >= 1 - tolerance)
                  cplex.out() << i << " ";
            }
            cplex.out() << endl;
         }
      }
   }
   catch(IloException& e) {             // gestion des exceptions
      cerr  << " ERROR: " << e << endl;
   }
   catch(...) {
      cerr  << " ERROR" << endl;
   }
   env.end();
   return 0;
}
```

```
// -------------------------------------------------------------- -*- C++ -*-
// File: cutstock.cpp
// Version 12.6.3
// --------------------------------------------------------------------------
// Licensed Materials - Property of IBM
// 5725-A06 5725-A29 5724-Y48 5724-Y49 5724-Y54 5724-Y55 5655-Y21
// Copyright IBM Corporation 2000, 2015. All Rights Reserved.
//
// US Government Users Restricted Rights - Use, duplication or
// disclosure restricted by GSA ADP Schedule Contract with
// IBM Corp.
// --------------------------------------------------------------------------
//
// Problème maître:
//
// données:
// roolWidth       longueur du rouleau
// amount[j]       nb. min. de chaque morceau
// size[i]         taille de chaque morceau
//
// variables:
// Cut[i]          patron de coupe i
// price[j]        variable duale de chaque contrainte de coupe i
// Use[j]          nb. de chaque patron choisi
//
// Objectif:
// minimize sum(i) Cut[i]
//
// Contraintes:
// forall morceau: sum(i) use[i][j] * Cut[i] >= amount[j]
//
//
// Sous-problème:
//
// Objectif:
// minimize sum(j) (-price[j]) + 1
//
// Contraintes:
// sum(j) size[j] * Use[j] <= rollwidth


#include <ilcplex/ilocplex.h>    // entêtes obligatoires
ILOSTLBEGIN

#define RC_EPS 1.0e-6

static void readData (const char* filename, IloNum& rollWidth,
                      IloNumArray& size, IloNumArray& amount);
static void report1 (IloCplex& cutSolver, IloNumVarArray Cut,
                     IloRangeArray Fill);
static void report2 (IloAlgorithm& patSolver,
                     IloNumVarArray Use,
                     IloObjective obj);
static void report3 (IloCplex& cutSolver, IloNumVarArray Cut);

/// MAIN PROGRAM ///

int
main(int argc, char **argv)
{
   IloEnv env;  // création de l'environnement
   try {
      IloInt  i, j;

      IloNum      rollWidth;
      IloNumArray amount(env);   // tableaux de nombres
      IloNumArray size(env);

      if ( argc > 1 )
         readData(argv[1], rollWidth, size, amount);
      else
         readData("../../../examples/data/cutstock.dat",
                  rollWidth, size, amount);

      /// CUTTING-OPTIMIZATION PROBLEM ///

      IloModel cutOpt (env);                        // modèle principal (PM)
      IloObjective   RollsUsed = IloAdd(cutOpt, IloMinimize(env));
                                          // objectif PM créé et ajouté au modèle
      IloRangeArray  Fill = IloAdd(cutOpt,
                                   IloRangeArray(env, amount, IloInfinity));
                     // tableau de contraintes de coupe créé et ajouté au modèle
      IloNumVarArray Cut(env);    // tableau de variables

      IloInt nWdth = size.getSize();
      for (j = 0; j < nWdth; j++) {
         Cut.add(IloNumVar(RollsUsed(1) + Fill[j](int(rollWidth / size[j]))));
                        // création des variables initiales et ajout au modèle
      }

      IloCplex cutSolver(cutOpt);    // problème maître

      /// PATTERN-GENERATION PROBLEM ///

      IloModel patGen (env);              // modèle secondaire (SP)

      IloObjective ReducedCost = IloAdd(patGen, IloMinimize(env, 1));
                                       // objectif SP créé et ajouté au modèle
      IloNumVarArray Use(env, nWdth, 0.0, IloInfinity, ILOINT);
                                                    // tableau de variables créé
      patGen.add(IloScalProd(size, Use) <= rollWidth);
                        // création des contraintes de patron et ajout au modèle

      IloCplex patSolver(patGen);    // sous-problème

      /// COLUMN-GENERATION PROCEDURE ///

      IloNumArray price(env, nWdth);
                                 // valeurs duales des contraintes de coupe (PM)
      IloNumArray newPatt(env, nWdth); // nouveaux patrons de coupe
```

```
      /// COLUMN-GENERATION PROCEDURE ///

      for (;;) {
         /// OPTIMIZE OVER CURRENT PATTERNS ///

         cutSolver.solve();            // résolution PM
         report1 (cutSolver, Cut, Fill);

         /// FIND AND ADD A NEW PATTERN ///

         for (i = 0; i < nWdth; i++) {
           price[i] = -cutSolver.getDual(Fill[i]);  // var. duales
         }
         ReducedCost.setLinearCoefs(Use, price);
                              // modification de l'objectif du SP

         patSolver.solve();    // résolution du SP
         report2 (patSolver, Use, ReducedCost);

         if (patSolver.getValue(ReducedCost) > -RC_EPS) break;
                 // critère d'arrêt de la génération de colonnes

         patSolver.getValues(newPatt, Use);
                         // récupération de la nouvelle colonne
         Cut.add( IloNumVar(RollsUsed(1) + Fill(newPatt)) );
           // création et ajout de la nouvelle variable au PM
      }

      cutOpt.add(IloConversion(env, Cut, ILOINT));
                                          // on passe au MILP (PM)

      cutSolver.solve();     // résolution en nombres entiers du PM
      cout << "Solution status: " << cutSolver.getCplexStatus() << endl;
      report3 (cutSolver, Cut);
   }
   catch (IloException& ex) {   // gestion des exceptions
      cerr << "Error: " << ex << endl;
   }
   catch (...) {
      cerr << "Error" << endl;
   }

   env.end();

   return 0;
}


static void readData (const char* filename, IloNum& rollWidth,
                      IloNumArray& size, IloNumArray& amount)
{
   ifstream in(filename);
   if (in) {
      in >> rollWidth;
      in >> size;
      in >> amount;
   }
   else {
      cerr << "No such file: " << filename << endl;
      throw(1);
   }
}

static void report1 (IloCplex& cutSolver, IloNumVarArray Cut,
                     IloRangeArray Fill)
{
   cout << endl;
   cout << "Using " << cutSolver.getObjValue() << " rolls" << endl;
   cout << endl;
   for (IloInt j = 0; j < Cut.getSize(); j++) {
      cout << "  Cut" << j << " = " << cutSolver.getValue(Cut[j]) << endl;
   }
   cout << endl;
   for (IloInt i = 0; i < Fill.getSize(); i++) {
      cout << "  Fill" << i << " = " << cutSolver.getDual(Fill[i]) << endl;
   }
   cout << endl;
}

static void report2 (IloAlgorithm& patSolver, IloNumVarArray Use,
                     IloObjective obj)
{
   cout << endl;
   cout << "Reduced cost is " << patSolver.getValue(obj) << endl;
   cout << endl;
   if (patSolver.getValue(obj) <= -RC_EPS) {
      for (IloInt i = 0; i < Use.getSize(); i++)  {
         cout << "  Use" << i << " = " << patSolver.getValue(Use[i]) << endl;
      }
      cout << endl;
   }
}

static void report3 (IloCplex& cutSolver, IloNumVarArray Cut)
{
   cout << endl;
   cout << "Best integer solution uses "
        << cutSolver.getObjValue() << " rolls" << endl;
   cout << endl;
   for (IloInt j = 0; j < Cut.getSize(); j++) {
      cout << "  Cut" << j << " = " << cutSolver.getValue(Cut[j]) << endl;
   }
}


/* Example Input file:
115
[25, 40, 50, 55, 70]
[50, 36, 24, 8, 30]
*/
```

```
// -------------------------------------------------------------- -*- C++ -*-
// File: ilobendersatsp.cpp
// Version 12.6.3
// --------------------------------------------------------------------------
// Licensed Materials - Property of IBM
// 5725-A06 5725-A29 5724-Y48 5724-Y49 5724-Y54 5724-Y55 5655-Y21
// Copyright IBM Corporation 2000, 2015. All Rights Reserved.
//
// US Government Users Restricted Rights - Use, duplication or
// disclosure restricted by GSA ADP Schedule Contract with
// IBM Corp.
// --------------------------------------------------------------------------
//
//
// Example ilobendersatsp.cpp solves a flow MILP model for an
// Asymmetric Traveling Salesman Problem (ATSP) instance
// through Benders decomposition.
//
// The arc costs of an ATSP instance are read from an input file.
// The flow MILP model is decomposed into a master ILP and a worker LP.
//
// The master ILP is then solved by adding Benders' cuts during
// the branch-and-cut process via the cut callback functions.
// The cut callback functions add to the master ILP violated Benders' cuts
// that are found by solving the worker LP.
//
// The example allows the user to decide if Benders' cuts have to be separated:
//
// a) Only to separate integer infeasible solutions.
// In this case, Benders' cuts are treated as lazy constraints through the
// class IloCplex::LazyConstraintCallbackI.
//
// b) Also to separate fractional infeasible solutions.
// In this case, Benders' cuts are treated as lazy constraints through the
// class IloCplex::LazyConstraintCallbackI.
// In addition, Benders' cuts are also treated as user cuts through the
// class IloCplex::UserCutCallbackI.
//
//
// To run this example, command line arguments are required:
//     ilobendersatsp.cpp {0|1} [filename]
// where
//     0         Indicates that Benders' cuts are only used as lazy constraints,
//               to separate integer infeasible solutions.
//     1         Indicates that Benders' cuts are also used as user cuts,
//               to separate fractional infeasible solutions.
//
//     filename  Is the name of the file containing the ATSP instance (arc
costs).
//               If filename is not specified, the instance
//               ../../../examples/data/atsp.dat is read
//
//
// ATSP instance defined on a directed graph G = (V, A)
// - V = {0, ..., n-1}, V0 = V \ {0}
// - A = {(i,j) : i in V, j in V, i != j }
// - forall i in V: delta+(i) = {(i,j) in A : j in V}
// - forall i in V: delta-(i) = {(j,i) in A : j in V}
// - c(i,j) = traveling cost associated with (i,j) in A
//
// Flow MILP model
//
// Modeling variables:
// forall (i,j) in A:
//    x(i,j) = 1, if arc (i,j) is selected
//           = 0, otherwise
// forall k in V0, forall (i,j) in A:
//    y(k,i,j) = flow of the commodity k through arc (i,j)
//
// Objective:
// minimize sum((i,j) in A) c(i,j) * x(i,j)
//
// Degree constraints:
// forall i in V: sum((i,j) in delta+(i)) x(i,j) = 1
// forall i in V: sum((j,i) in delta-(i)) x(j,i) = 1
//
// Binary constraints on arc variables:
// forall (i,j) in A: x(i,j) in {0, 1}
//
// Flow constraints:
// forall k in V0, forall i in V:
//    sum((i,j) in delta+(i)) y(k,i,j) - sum((j,i) in delta-(i)) y(k,j,i) =
q(k,i)
//    where q(k,i) =  1, if i = 0
//                 = -1, if k == i
//                 =  0, otherwise
//
// Capacity constraints:
// forall k in V0, for all (i,j) in A: y(k,i,j) <= x(i,j)
//
// Nonnegativity of flow variables:
// forall k in V0, for all (i,j) in A: y(k,i,j) >= 0
//

#include <string>

#include <ilcplex/ilocplex.h>       // entêtes obligatoires
ILOSTLBEGIN

typedef IloArray<IloIntVarArray> Arcs;  // matrice 2D de variables entières

// Declarations for functions in this program

void createMasterILP(IloModel mod, Arcs x, IloNumArray2 arcCost);

void createWorkerLP(IloCplex cplex, IloNumVarArray v, IloNumVarArray u,
                    IloObjective obj, IloInt numNodes);

IloBool separate(const Arcs x, const IloNumArray2 xSol, IloCplex cplex,
                 const IloNumVarArray v, const IloNumVarArray u,
                 IloObjective obj, IloExpr cutLhs, IloNum& cutRhs);

void usage(char *progname);
```

```
// Implementation class for the user-defined lazy constraint callback.
// The function BendersLazyCallback allows to add Benders' cuts as lazy
// constraints.
//
// S'applique aux noeuds entiers
//
ILOLAZYCONSTRAINTCALLBACK5(BendersLazyCallback, Arcs, x, IloCplex, workerCplex,
                           IloNumVarArray, v, IloNumVarArray, u,
                           IloObjective, workerObj)
{
   IloInt i;
   IloEnv masterEnv = getEnv();
   IloInt numNodes = x.getSize();

   // Get the current x solution

   IloNumArray2 xSol(masterEnv, numNodes);
   for (i = 0; i < numNodes; ++i) {
      xSol[i] = IloNumArray(masterEnv);
      getValues(xSol[i], x[i]);
   }

   // Benders' cut separation

   IloExpr cutLhs(masterEnv);   // expression de la coupe
   IloNum cutRhs;
   IloBool sepStat = separate(x, xSol, workerCplex, v, u, workerObj, cutLhs,
cutRhs);
   if ( sepStat ) {
      add(cutLhs >= cutRhs).end();
       // crée la coupe, l'ajoute au modèle et détruit l'objet temporaire créé
   }

   // Free memory

   cutLhs.end();
   for (i = 0; i < numNodes; ++i)
      xSol[i].end();
   xSol.end();

   return;

} // END BendersLazyCallback


void usage (char *progname)
{
   cerr << "Usage:     " << progname << " {0|1} [filename]"            << endl;
   cerr << " 0:        Benders' cuts only used as lazy constraints,"   << endl;
   cerr << "           to separate integer infeasible solutions."      << endl;
   cerr << " 1:        Benders' cuts also used as user cuts,"          << endl;
   cerr << "           to separate fractional infeasible solutions."   << endl;
   cerr << " filename: ATSP instance file name."                       << endl;
   cerr << "           File ../../../examples/data/atsp.dat "
        << "used if no name is provided."                              << endl;

} // END usage
```

```
// Implementation class for the user-defined user cut callback.
// The function BendersUserCallback allows to add Benders' cuts as user cuts.
//
// S'applique aux noeuds fractionnaires
//
ILOUSERCUTCALLBACK5(BendersUserCallback, Arcs, x, IloCplex, workerCplex,
                    IloNumVarArray, v, IloNumVarArray, u,
                    IloObjective, workerObj)
{
   // Skip the separation if not at the end of the cut loop

   if ( !isAfterCutLoop() )
      return;

   IloInt i;
   IloEnv masterEnv = getEnv();
   IloInt numNodes = x.getSize();

   // Get the current x solution

   IloNumArray2 xSol(masterEnv, numNodes);
   for (i = 0; i < numNodes; ++i) {
      xSol[i] = IloNumArray(masterEnv);
      getValues(xSol[i], x[i]);
   }

   // Benders' cut separation

   IloExpr cutLhs(masterEnv);         // expression de la coupe
   IloNum cutRhs;
   IloBool sepStat = separate(x, xSol, workerCplex, v, u, workerObj, cutLhs,
cutRhs);
   if ( sepStat ) {
      add(cutLhs >= cutRhs).end();
       // crée la coupe, l'ajoute au modèle et détruit l'objet temporaire créé
   }

   // Free memory

   cutLhs.end();
   for (i = 0; i < numNodes; ++i)
      xSol[i].end();
   xSol.end();

   return;

} // END BendersUserCallback
```

```
int main(int argc, char **argv)
{
   IloEnv masterEnv;   // 2 environnements distincts (1 seul nécessaire)
   IloEnv workerEnv;

   try {
      const char* fileName = "../../../examples/data/atsp.dat";

       // Check the command line arguments

      if ( argc != 2 && argc != 3) {
         usage (argv[0]);
         throw (-1);
      }

      if ( (argv[1][0] != '1' && argv[1][0] != '0') ||
           argv[1][1] != '\0' ) {
         usage (argv[0]);
         throw (-1);
      }

      IloBool separateFracSols = ( argv[1][0] == '0' ? IloFalse : IloTrue );

      masterEnv.out() << "Benders' cuts separated to cut off: ";
      if ( separateFracSols ) {
         masterEnv.out() << "Integer and fractional infeasible solutions." <<
endl;
      }
      else {
         masterEnv.out() << "Only integer infeasible solutions." << endl;
      }

      if ( argc == 3 )  fileName = argv[2];

      // Read arc_costs from data file (17 city problem)

      IloNumArray2 arcCost(masterEnv);    // matrice 2D de nombres (coûtsArc)
      ifstream data(fileName);
      if ( !data ) throw(-1);
      data >> arcCost;
      data.close();

      // create master ILP

      IloModel masterMod(masterEnv, "atsp_master");
      IloInt numNodes = arcCost.getSize();
      Arcs x(masterEnv, numNodes);
      createMasterILP(masterMod, x, arcCost);

      // Create worker IloCplex algorithm and worker LP for Benders' cuts
separation

      IloCplex workerCplex(workerEnv);
      IloNumVarArray v(workerEnv);     // tableaux de variables
      IloNumVarArray u(workerEnv);
      IloObjective workerObj(workerEnv);   // objectif du SP
      createWorkerLP(workerCplex, v, u, workerObj, numNodes);

      // Set up the cut callback to be used for separating Benders' cuts

      IloCplex masterCplex(masterMod);
      masterCplex.setParam(IloCplex::Param::Preprocessing::Presolve, IloFalse);
                           // met le presolve à OFF pour le problème maître


      // Set the maximum number of threads to 1.
      // This instruction is redundant: If MIP control callbacks are registered,
      // then by default CPLEX uses 1 (one) thread only.
      // Note that the current example may not work properly if more than 1
threads
      // are used, because the callback functions modify shared global data.
      // We refer the user to the documentation to see how to deal with
multi-thread
      // runs in presence of MIP control callbacks.

      masterCplex.setParam(IloCplex::Param::Threads, 1);

      // Turn on traditional search for use with control callbacks

      masterCplex.setParam(IloCplex::Param::MIP::Strategy::Search,
                           IloCplex::Traditional);

      masterCplex.use(BendersLazyCallback(masterEnv, x, workerCplex, v, u,
workerObj));
      if ( separateFracSols )
         masterCplex.use(BendersUserCallback(masterEnv, x, workerCplex, v, u,
workerObj));

      // Solve the model and write out the solution

      if ( masterCplex.solve() ) {

         IloCplex::CplexStatus solStatus= masterCplex.getCplexStatus();
         masterEnv.out() << endl << "Solution status: " << solStatus << endl;

         masterEnv.out() << "Objective value: "
                         << masterCplex.getObjValue() << endl;

         if ( solStatus == IloCplex::Optimal ) {

            // Write out the optimal tour

            IloInt i, j;
            IloNumArray2 sol(masterEnv, numNodes);  // matrice 2D de nombres
            IloIntArray succ(masterEnv, numNodes);
                                                 //matrice 2D de nombres entiers
            for (j = 0; j < numNodes; ++j)
               succ[j] = -1;

            for (i = 0; i < numNodes; i++) {
               sol[i] = IloNumArray(masterEnv);
               masterCplex.getValues(sol[i], x[i]);  // récupération solution
               for(j = 0; j < numNodes; j++) {
                  if ( sol[i][j] > 1e-03 ) succ[i] = j;
```

```
            }
         }

         masterEnv.out() << "Optimal tour:" << endl;
         i = 0;
         while ( succ[i] != 0 ) {
            masterEnv.out() << i << ", ";
            i = succ[i];
         }
         masterEnv.out() << i << endl;
      }
      else {
         masterEnv.out() << "Solution status is not Optimal" << endl;
      }
   }
   else {
      masterEnv.out() << "No solution available" << endl;
   }
}

catch (const IloException& e) {             // gestion exceptions
   cerr << "Exception caught: " << e << endl;
}
catch (...) {
   cerr << "Unknown exception caught!" << endl;
}

// Close the environments

masterEnv.end();
workerEnv.end();
return 0;

} // END main

// This routine creates the master ILP (arc variables x and degree constraints).
//
// Modeling variables:
// forall (i,j) in A:
//    x(i,j) = 1, if arc (i,j) is selected
//           = 0, otherwise
//
// Objective:
// minimize sum((i,j) in A) c(i,j) * x(i,j)
//
// Degree constraints:
// forall i in V: sum((i,j) in delta+(i)) x(i,j) = 1
// forall i in V: sum((j,i) in delta-(i)) x(j,i) = 1
//
// Binary constraints on arc variables:
// forall (i,j) in A: x(i,j) in {0, 1}
//
void createMasterILP(IloModel mod, Arcs x, IloNumArray2 arcCost)
{
   IloInt i, j;
   IloEnv env = mod.getEnv();
   IloInt numNodes = x.getSize();

   // Create variables x(i,j) for (i,j) in A
   // For simplicity, also dummy variables x(i,i) are created.
   // Those variables are fixed to 0 and do not participate to
   // the constraints.

   char varName[100];
   for (i = 0; i < numNodes; ++i) {
      x[i] = IloIntVarArray(env, numNodes, 0, 1);
      x[i][i].setBounds(0, 0);
      for (j = 0; j < numNodes; ++j) {
         sprintf(varName, "x.%d.%d", (int) i, (int) j);
         x[i][j].setName(varName);
      }
      mod.add(x[i]);
       // ajout au modèle de la matrice de variable x, vecteur par vecteur
   }

   // Create objective function: minimize sum((i,j) in A ) c(i,j) * x(i,j)

   IloExpr obj(env);
   for (i = 0; i < numNodes; ++i) {
      arcCost[i][i] = 0;
      obj += IloScalProd(x[i], arcCost[i]);
   }
   mod.add(IloMinimize(env, obj));
   obj.end();

   // Add the out degree constraints.
   // forall i in V: sum((i,j) in delta+(i)) x(i,j) = 1

   for (i = 0; i < numNodes; ++i) {
      IloExpr expr(env);
      for (j = 0;   j < i; ++j)
         expr += x[i][j];
      for (j = i+1; j < numNodes; ++j)
         expr += x[i][j];
      mod.add(expr == 1);
      expr.end();
   }

   // Add the in degree constraints.
   // forall i in V: sum((j,i) in delta-(i)) x(j,i) = 1

   for (i = 0; i < numNodes; i++) {
      IloExpr expr(env);
      for (j = 0;   j < i; j++)
         expr += x[j][i];
      for (j = i+1; j < numNodes; j++)
         expr += x[j][i];
      mod.add(expr == 1);
      expr.end();
   }

}// END createMasterILP

// This routine set up the IloCplex algorithm to solve the worker LP, and
```

```
// creates the worker LP (i.e., the dual of flow constraints and
// capacity constraints of the flow MILP)
//
// Modeling variables:
// forall k in V0, i in V:
//    u(k,i) = dual variable associated with flow constraint (k,i)
//
// forall k in V0, forall (i,j) in A:
//    v(k,i,j) = dual variable associated with capacity constraint (k,i,j)
//
// Objective:
// minimize sum(k in V0) sum((i,j) in A) x(i,j) * v(k,i,j)
//          - sum(k in V0) u(k,0) + sum(k in V0) u(k,k)
//
// Constraints:
// forall k in V0, forall (i,j) in A: u(k,i) - u(k,j) <= v(k,i,j)
//
// Nonnegativity on variables v(k,i,j)
// forall k in V0, forall (i,j) in A: v(k,i,j) >= 0
//
void createWorkerLP(IloCplex cplex, IloNumVarArray v, IloNumVarArray u,
                    IloObjective obj, IloInt numNodes)
{

   IloInt i, j, k;
   IloEnv env = cplex.getEnv();
   IloModel mod(env, "atsp_worker");

   // Set up IloCplex algorithm to solve the worker LP

   cplex.extract(mod);
   cplex.setOut(env.getNullStream());  // pas d'output de CPLEX

   // Turn off the presolve reductions and set the CPLEX optimizer
   // to solve the worker LP with primal simplex method.

   cplex.setParam(IloCplex::Param::Preprocessing::Reduce, 0);
   cplex.setParam(IloCplex::Param::RootAlgorithm, IloCplex::Primal);

   // Create variables v(k,i,j) forall k in V0, (i,j) in A
   // For simplicity, also dummy variables v(k,i,i) are created.
   // Those variables are fixed to 0 and do not partecipate to
   // the constraints.

   IloInt numArcs  = numNodes * numNodes;
   IloInt vNumVars = (numNodes-1) * numArcs;
   IloNumVarArray vTemp(env, vNumVars, 0, IloInfinity);
   for (k = 1; k < numNodes; ++k) {
      for (i = 0; i < numNodes; ++i) {
         vTemp[(k-1)*numArcs + i *numNodes + i].setBounds(0, 0);
      }
   }
   v.clear();
   v.add(vTemp);   // copie du vecteur de variables temporaires
                   // dans vecteur permanent
   vTemp.end();
   mod.add(v);

   // Set names for variables v(k,i,j)

   for (k = 1; k < numNodes; ++k) {
      for(i = 0; i < numNodes; ++i) {
         for(j = 0; j < numNodes; ++j) {
            char varName[100];
            sprintf(varName, "v.%d.%d.%d", (int) k, (int) i, (int) j);
            v[(k-1)*numArcs + i*numNodes + j].setName(varName);
         }
      }
   }

   // Associate indices to variables v(k,i,j)

   IloIntArray vIndex(env, vNumVars);
   for (j = 0; j < vNumVars; ++j)
   {
      vIndex[j] = j;
      v[j].setObject(&vIndex[j]);  // assigne un index (ptr) à la variable
   }

   // Create variables u(k,i) forall k in V0, i in V

   IloInt uNumVars = (numNodes-1) * numNodes;
   IloNumVarArray uTemp(env, uNumVars, -IloInfinity, IloInfinity);
   u.clear();
   u.add(uTemp);  // copie du vecteur de variables temporaires
                  // dans vecteur permanent
   uTemp.end();
   mod.add(u);

   // Set names for variables u(k,i)

   for (k = 1; k < numNodes; ++k) {
      for(i = 0; i < numNodes; ++i) {
         char varName[100];
         sprintf(varName, "u.%d.%d", (int) k, (int) i);
         u[(k-1)*numNodes + i].setName(varName);
      }
   }

   // Associate indices to variables u(k,i)

   IloIntArray uIndex(env, uNumVars);
   for (j = 0; j < uNumVars; ++j)
   {
      uIndex[j] = vNumVars + j;
      u[j].setObject(&uIndex[j]);  // assigne un index (ptr) à la variable
   }

   // Initial objective function is empty

   obj.setSense(IloObjective::Minimize);
   mod.add(obj);

   // Add constraints:
```

```
   // forall k in V0, forall (i,j) in A: u(k,i) - u(k,j) <= v(k,i,j)

   for (k = 1; k < numNodes; ++k) {
      for(i = 0; i < numNodes; ++i) {
         for(j = 0; j < numNodes; ++j) {
            if ( i != j ) {
               IloExpr expr(env);
               expr -= v[(k-1)*numArcs + i*(numNodes) + j];
               expr += u[(k-1)*numNodes + i];
               expr -= u[(k-1)*numNodes + j];
               mod.add(expr <= 0);
               expr.end();
            }
         }
      }
   }

}// END createWorkerLP


// This routine separates Benders' cuts violated by the current x solution.
// Violated cuts are found by solving the worker LP
//
IloBool separate(const Arcs x, const IloNumArray2 xSol, IloCplex cplex,
                 const IloNumVarArray v, const IloNumVarArray u,
                 IloObjective obj, IloExpr cutLhs, IloNum& cutRhs)
{
   IloBool violatedCutFound = IloFalse;

   IloEnv env = cplex.getEnv();
   IloModel mod = cplex.getModel();

   IloInt numNodes = xSol.getSize();
   IloInt numArcs  = numNodes * numNodes;
   IloInt i, j, k, h;

   // Update the objective function in the worker LP:
   // minimize sum(k in V0) sum((i,j) in A) x(i,j) * v(k,i,j)
   //          - sum(k in V0) u(k,0) + sum(k in V0) u(k,k)

   mod.remove(obj);               // enlève l'objectif précédent
   IloExpr objExpr = obj.getExpr();
   objExpr.clear();
   for (k = 1; k < numNodes; ++k) {
      for (i = 0; i < numNodes; ++i) {
         for (j = 0; j < numNodes; ++j) {
               objExpr +=  xSol[i][j] * v[(k-1)*numArcs + i*numNodes + j];
         }
      }
   }
   for (k = 1; k < numNodes; ++k) {
      objExpr += u[(k-1)*numNodes + k];
      objExpr -= u[(k-1)*numNodes];
   }
   obj.setExpr(objExpr);
   mod.add(obj);              //ajoute un nouvel objectif
   objExpr.end();


   // Solve the worker LP

   cplex.solve();

   // A violated cut is available iff the solution status is Unbounded

   if ( cplex.getCplexStatus() == IloCplex::Unbounded ) {

      IloInt vNumVars = (numNodes-1) * numArcs;
      IloNumVarArray var(env);
      IloNumArray val(env);

      // Get the violated cut as an unbounded ray of the worker LP

      cplex.getRay(val, var);

      // Compute the cut from the unbounded ray. The cut is:
      // sum((i,j) in A) (sum(k in V0) v(k,i,j)) * x(i,j) >=
      // sum(k in V0) u(k,0) - u(k,k)

      cutLhs.clear();
      cutRhs = 0.;

      for (h = 0; h < val.getSize(); ++h) {

         IloInt *index_p = (IloInt*) var[h].getObject();
            // récupère l'index (ptr) de la variable
         IloInt index = *index_p;

         if ( index >= vNumVars ) {
            index -= vNumVars;
            k = index / numNodes + 1;
            i = index - (k-1)*numNodes;
            if ( i == 0 )
               cutRhs += val[h];     // met à jour la valeur du RHS
            else if ( i == k )
               cutRhs -= val[h];
         }
         else {
            k = index / numArcs + 1;
            i = (index - (k-1)*numArcs) / numNodes;
            j = index - (k-1)*numArcs - i*numNodes;
            cutLhs += val[h] * x[i][j];
                           // met à jour l'expression de la contrainte
         }
      }

      var.end();
      val.end();

      violatedCutFound = IloTrue;
   }

   return violatedCutFound;

} // END separate
```