

The Mesh Adaptive Direct Search Algorithm for Discrete Blackbox Optimization

Sébastien Le Digabel
Charles Audet
Christophe Tribes

GERAD and École Polytechnique de Montréal

ICCOPT, Tokyo

2016-08-08

Presentation outline

Blackbox optimization

Motivating example

The MADS algorithm

Computational experiments

Discussion

Blackbox optimization

Motivating example

The MADS algorithm

Computational experiments

Discussion

Blackbox optimization (BBO) problems

- ▶ Optimization problem:

$$\min_{x \in \Omega} f(x)$$

- ▶ Evaluations of f (the **objective function**) and of the functions defining Ω are usually the result of a computer code (a **blackbox**).
- ▶ Variables are typically continuous, but in this work, some of them are discrete – **integers** or **granular variables**.

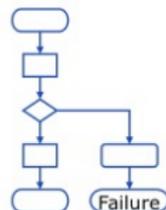
Blackboxes as illustrated by J. Simonis [ISMP 2009]



Long runtime



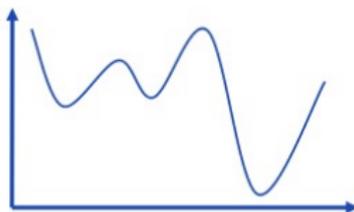
Large memory
requirement



Software
might fail



No derivatives
available



Local
optima



Non-smooth,
noisy

Blackbox optimization

Motivating example

The MADS algorithm

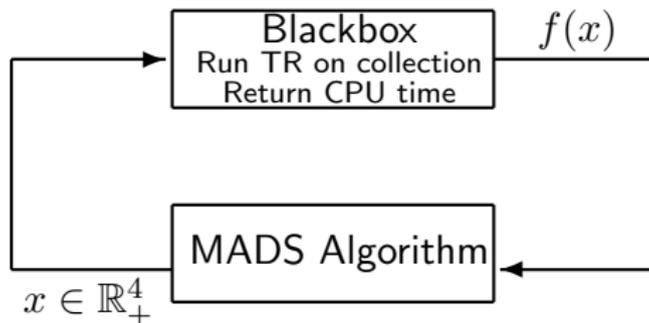
Computational experiments

Discussion

Motivating example: Parameters tuning (1/2)

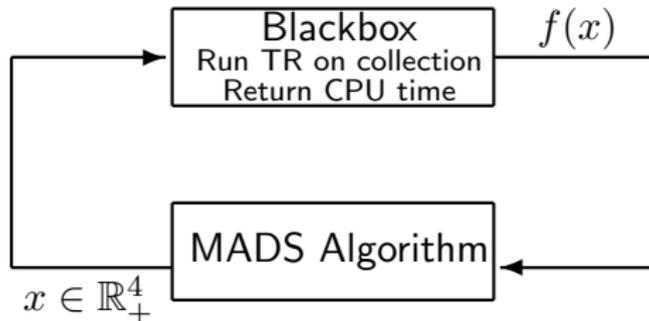
- ▶ [Audet and Orban, 2006].
- ▶ The classical Trust-Region algorithm depends on four parameters $x = (\eta_1, \eta_2, \alpha_1, \alpha_2) \in \mathbb{R}_+^4$.
- ▶ Consider a collection of 55 test problems from CUTEr.
- ▶ Let $f(x)$ be the CPU time required to solve the collection of problems by a Trust-Region algorithm with parameters x .
- ▶ $f(x_0) \simeq 3h45$ with the textbook values
 $x_0 = (1/4, 3/4, 1/2, 2)$.

Motivating example: Parameters tuning (2/2)



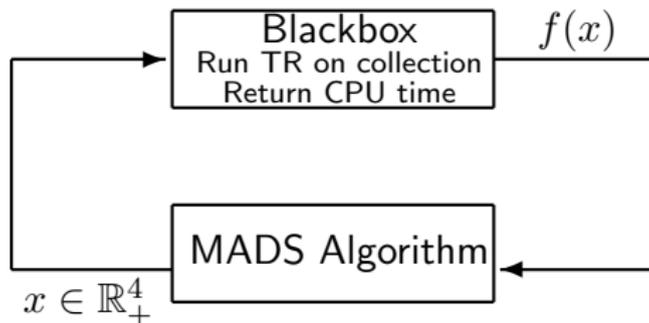
- ▶ This optimization produced \hat{x} with $f(\hat{x}) \simeq 2h50$.

Motivating example: Parameters tuning (2/2)



- ▶ This optimization produced \hat{x} with $f(\hat{x}) \simeq 2h50$. **Victory ?**

Motivating example: Parameters tuning (2/2)



- ▶ This optimization produced \hat{x} with $f(\hat{x}) \simeq 2h50$. **Victory ?**
No because
 $\hat{x} = (0.22125, 0.94457031, 0.37933594, 2.3042969)$.

Granular variables

The initial point $x_0 = (0.25, 0.75, 0.50, 2.00)$ is frequently used because each entry is a multiple of 0.25.

Its granularity is $\mathcal{G} = 0.25$

Granular variables

The initial point $x_0 = (0.25, 0.75, 0.50, 2.00)$ is frequently used because each entry is a multiple of 0.25.

Its granularity is $\mathcal{G} = 0.25$

- ▶ How can we devise a direct search algorithm so that it stops on a prescribed granularity?

With a granularity of $\mathcal{G} = 0.05$, the code might produce

$$\hat{x} = (0.20, 0.95, 0.40, 2.30).$$

Which is much nicer (for a human) than

$$\hat{x} = (0.22125, 0.94457031, 0.37933594, 2.3042969).$$

Granular variables

The initial point $x_0 = (0.25, 0.75, 0.50, 2.00)$ is frequently used because each entry is a multiple of 0.25.

Its granularity is $\mathcal{G} = 0.25$

- ▶ How can we devise a direct search algorithm so that it stops on a prescribed granularity?

With a granularity of $\mathcal{G} = 0.05$, the code might produce

$$\hat{x} = (0.20, 0.95, 0.40, 2.30).$$

Which is much nicer (for a human) than

$$\hat{x} = (0.22125, 0.94457031, 0.37933594, 2.3042969).$$

- ▶ This may be achieved using integer variables, together with a relative scaling, but there is a simpler way for mesh-based methods.

Blackbox optimization

Motivating example

The MADS algorithm

Computational experiments

Discussion

Mesh Adaptive Direct Search (MADS) in \mathbb{R}^n

- ▶ [Audet and Dennis, Jr., 2006].

Mesh Adaptive Direct Search (MADS) in \mathbb{R}^n

- ▶ [Audet and Dennis, Jr., 2006].
- ▶ Iterative algorithm that evaluates the blackbox at some **trial points** on a spatial discretization called the **mesh**.
- ▶ One iteration = **search** and **poll**.

Mesh Adaptive Direct Search (MADS) in \mathbb{R}^n

- ▶ [Audet and Dennis, Jr., 2006].
- ▶ Iterative algorithm that evaluates the blackbox at some **trial points** on a spatial discretization called the **mesh**.
- ▶ One iteration = **search** and **poll**.
- ▶ The search allows trial points generated anywhere on the mesh.
- ▶ The poll consists in generating a list of trial points constructed from **poll directions**. These directions grow dense.

Mesh Adaptive Direct Search (MADS) in \mathbb{R}^n

- ▶ [Audet and Dennis, Jr., 2006].
- ▶ Iterative algorithm that evaluates the blackbox at some **trial points** on a spatial discretization called the **mesh**.
- ▶ One iteration = **search** and **poll**.
- ▶ The search allows trial points generated anywhere on the mesh.
- ▶ The poll consists in generating a list of trial points constructed from **poll directions**. These directions grow dense.
- ▶ At the end of the iteration, the mesh size is reduced if no new success point is found.

[0] Initializations (x_0, Δ_0 : initial poll size)

[1] Iteration k

let $\delta^k \leq \Delta^k$ be the mesh size parameter

Search

test a finite number of mesh points

Poll (if the Search failed)

construct set of directions D_k

test poll set $P_k = \{x_k + \delta^k d : d \in D_k\}$

with $\|\delta^k d\| \simeq \Delta_k$

[2] Updates

if success

$x_{k+1} \leftarrow$ success point

increase Δ^k

else

$x_{k+1} \leftarrow x_k$

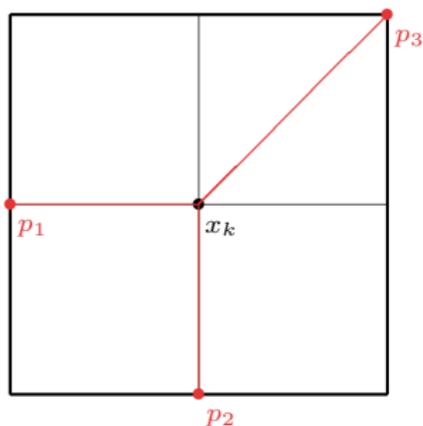
decrease Δ^k

$k \leftarrow k + 1$, stop if $\Delta^k \leq \Delta_{\min}$ or go to **[1]**

Poll illustration (successive fails and mesh shrinks)

$$\delta^k = 1$$

$$\Delta^k = 1$$

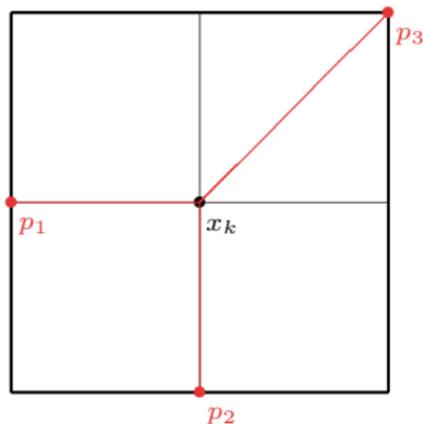


trial points = $\{p_1, p_2, p_3\}$

Poll illustration (successive fails and mesh shrinks)

$$\delta^k = 1$$

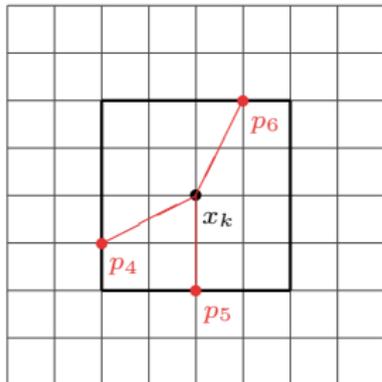
$$\Delta^k = 1$$



trial points = $\{p_1, p_2, p_3\}$

$$\delta^{k+1} = 1/4$$

$$\Delta^{k+1} = 1/2$$

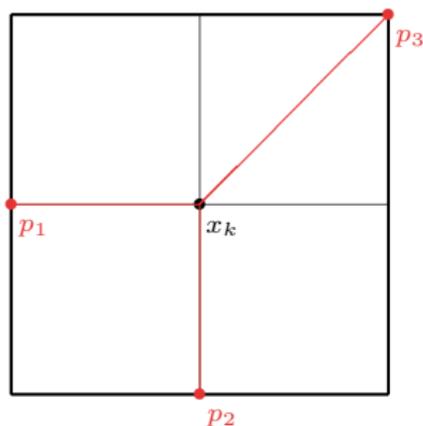


= $\{p_4, p_5, p_6\}$

Poll illustration (successive fails and mesh shrinks)

$$\delta^k = 1$$

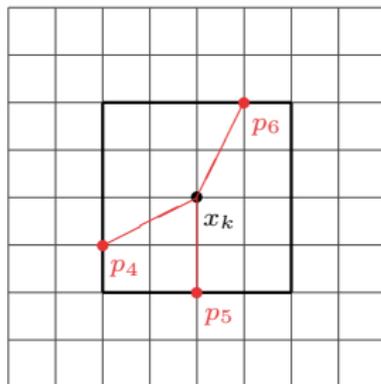
$$\Delta^k = 1$$



trial points = $\{p_1, p_2, p_3\}$

$$\delta^{k+1} = 1/4$$

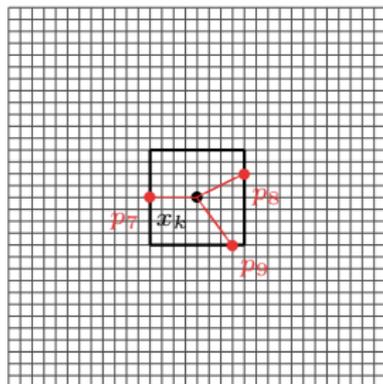
$$\Delta^{k+1} = 1/2$$



= $\{p_4, p_5, p_6\}$

$$\delta^{k+2} = 1/16$$

$$\Delta^{k+2} = 1/4$$



= $\{p_7, p_8, p_9\}$

Discrete variables in MADS – so far

- ▶ MADS has been designed for continuous variables.
- ▶ Some theory exists for **categorical variables** [Abramson, 2004].
- ▶ So far: Only a patch allows to handle integer variables:
Rounding + minimal mesh size of one.

In this work, we start from scratch and present direct search methods with a natural way of handling discrete variables.

Mesh refinement on $\min(x - 1/3)^2$

Δ^k	x^k
1	0
0.5	0.5
0.25	0.25
0.125	0.375
0.0625	0.3125
0.03125	0.34375
0.015625	0.328125
0.0078125	0.3359375
0.00390625	0.33203125
0.001953125	0.333984375

Mesh refinement on $\min(x - 1/3)^2$

Δ^k	x^k
1	0
0.5	0.5
0.25	0.25
0.125	0.375
0.0625	0.3125
0.03125	0.34375
0.015625	0.328125
0.0078125	0.3359375
0.00390625	0.33203125
0.001953125	0.333984375

alternately

Δ^k	x^k
1	0
0.5	0.5
0.2	0.4
0.1	0.3
0.05	0.35
0.02	0.34
0.01	0.33
0.005	0.335
0.002	0.332
0.001	0.333

Idea:

Instead of dividing Δ^k by 2, change it so that

10×10^b refines to 5×10^b

5×10^b refines to 2×10^b

2×10^b refines to 1×10^b

Mesh refinement on $\min(x - 1/3)^2$

Δ^k	x^k
1	0
0.5	0.5
0.25	0.25
0.125	0.375
0.0625	0.3125
0.03125	0.34375
0.015625	0.328125
0.0078125	0.3359375
0.00390625	0.33203125
0.001953125	0.333984375

alternately

Δ^k	x^k
1	0
0.5	0.5
0.2	0.4
0.1	0.3
0.05	0.35
0.02	0.34
0.01	0.33
0.005	0.335
0.002	0.332
0.001	0.333

Idea:

Instead of dividing Δ^k by 2, change it so that

- 10×10^b refines to 5×10^b
- 5×10^b refines to 2×10^b
- 2×10^b refines to 1×10^b

To get three decimals, one simply sets the granularity to 0.001. Integer variables are treated by setting the granularity to $\mathcal{G} = 1$.

Poll and mesh size parameter update

- ▶ The poll size parameter Δ^k is updated as
 $10 \times 10^b \longleftrightarrow 5 \times 10^b \longleftrightarrow 2 \times 10^b \longleftrightarrow 1 \times 10^b$

- ▶ The fine underlying mesh is defined with the mesh size parameter

$$\delta^k = \begin{cases} 1 & \text{if } \Delta^k \geq 1, \\ \max\{10^{2b}, \mathcal{G}\} & \text{otherwise, i.e. } \Delta^k \in \{1, 2, 5\} \times 10^b. \end{cases}$$

- ▶ Example: Granularity of $\mathcal{G} = 0.005$:

δ^k	Δ^k
1	5
1	2
1	1
0.01	0.5
0.01	0.2
0.01	0.1
0.005	0.05
0.005	0.02
0.005	0.01
0.005	0.005 ← stop

[0] Initializations ($x_0, \Delta_0 \in \{1, 2, 5\} \times 10^b, \mathcal{G}$ granularity)

[1] Iteration k

let $\delta^k \leq \Delta^k$ be the mesh size parameter

Search

test a finite number of mesh points

Poll (if the Search failed)

construct set of directions D_k

test poll set $P_k = \{x_k + \delta^k d : d \in D_k\}$

with $\|\delta^k d\| \simeq \Delta_k$

[2] Updates

if success

$x_{k+1} \leftarrow$ success point

increase Δ^k

else

$x_{k+1} \leftarrow x_k$

decrease Δ^k

$k \leftarrow k + 1$, stop if $\delta^k = \mathcal{G}$ or go to **[1]**

Theory

- ▶ MADS analysis relies on “*the sequence of trial points are located on some discretization of the space of variables called the mesh*”.
- ▶ By multiplying or dividing Δ^k by a rational number τ , [Torczon, 1997] showed that *all trial points from iteration 0 to ℓ were located on a fine underlying mesh*. The proof is not trivial and uses the fact that $\tau \in \mathbb{Q}$, and does not work for $\tau \in \mathbb{R}$ (paper won SIAM Outstanding Paper Prize).

Theory

- ▶ MADS analysis relies on “*the sequence of trial points are located on some discretization of the space of variables called the mesh*”.
- ▶ By multiplying or dividing Δ^k by a rational number τ , [Torczon, 1997] showed that *all trial points from iteration 0 to ℓ were located on a fine underlying mesh*.
- ▶ With the new mesh, that technical part of the proof becomes:

Consider any trial point t considered from iteration $k = 0$ to ℓ .

If a granularity of \mathcal{G}_i is requested on variable i ,

t_i lies on the mesh of granularity \mathcal{G}_i

if no granularity is requested on variable i ,

t_i lies on the mesh of granularity 10^{b_i}

with $b_i = \min\{b_i^k : k = 0, 1, \dots, \ell\}$.

Blackbox optimization

Motivating example

The MADS algorithm

Computational experiments

Discussion

Trust-Region parameters tuning

Find the values of the four parameters $x = (\eta_1, \eta_2, \alpha_1, \alpha_2) \in \mathbb{R}_+^4$ that minimize the overall CPU time to solve 55 CUTEr problems.

- ▶ A surrogate function s is defined as the time to solve a collection of small-sized problems.
- ▶ In 2005, $f(x) \simeq 3\text{h}-4\text{h}$ and $s(x) \simeq 1\text{m}$. The surrogate was 200 times faster (now $\simeq 140$ times).

Trust-Region parameters tuning

Find the values of the four parameters $x = (\eta_1, \eta_2, \alpha_1, \alpha_2) \in \mathbb{R}_+^4$ that minimize the overall CPU time to solve 55 CUTEr problems.

- ▶ A surrogate function s is defined as the time to solve a collection of small-sized problems.
- ▶ In 2005, $f(x) \simeq 3\text{h}-4\text{h}$ and $s(x) \simeq 1\text{m}$. The surrogate was 200 times faster (now $\simeq 140$ times).

Theorem (Moore's Law is valid)

Year	CPU $f(x)$	CPU $s(x)$	<i>Moore's Law: Speed doubles every 2 years:</i> 2016-2005 = 11 \Rightarrow 5.5 cycles, speedup of $2^{5.5} \simeq 45$.
2005	13800 s	69 s	
2016	330 s	2.3 s	
Ratio:	42	30	

Standard $\times 2 \div 2$

versus

New $\{1, 2, 5\} \times 10^b$

0.25	0.75	0.50	2	329.64729
0.25	0.80	0.50	1.55	254.32179
0.25	0.95	0.70	1	NaN
0.35	0.95	0.60	1	NaN
0.15	0.70	0.60	1	NaN
0.30	0.75	0.40	1	NaN
0.40	0.70	0.65	1.55	299.24209
0.85	0.90	0.55	4.70	304.38179
0.25	0.90	0.55	1.55	230.25339
0.25	1.	0.55	1.55	NaN
0.15	0.85	0.60	2.45	332.73279
0.25	1.	0.50	1.10	NaN
0.20	0.95	0.75	1.10	384.09890
0.85	0.90	0.50	1.10	315.53730
0.30	0.85	0.50	1	NaN
0.45	0.70	0.50	5.15	376.13209
0.30	0.90	0.55	1.55	234.08379
0.25	0.90	0.65	1.55	267.51490
0.15	0.90	0.55	2	307.73540
0.25	0.80	0.55	2	344.48180
0.30	0.95	0.55	2.45	284.48099
0.30	0.95	0.45	1	NaN
0.1875	0.9125	0.625	1.775	324.89690
0.2125	0.925	0.5375	1.55	237.63839
0.2625	0.90	0.55	2	297.25700
0.2375	0.8875	0.60	1.55	260.38650
0.225	0.9375	0.6625	1.6625	316.95100
0.3125	0.9375	0.525	1	NaN
0.271875	0.909375	0.59375	1.6625	312.84760
0.25	0.925	0.55	1.55	231.82699
0.26875	0.90	0.546875	1.409375	252.00210
0.24675	0.90	0.571875	1.4375	319.10470
0.234375	0.90	0.5375	1.409375	255.43649
0.25	0.875	0.54375	1.94375	303.55739
0.24375	0.90	0.54609375	1.59921875	238.37899
0.2409375	0.919375	0.553125	1.54296875	305.85789
0.2611875	0.903125	0.55234375	1.5640625	269.32650
0.24765625	0.89921875	0.5515625	1.6625	325.77249
0.24921875	0.8969375	0.56171875	1.52809625	252.59420
0.24609375	0.8990625	0.53125	1.4515625	313.61869
0.257125	0.898046875	0.552806875	1.562306875	259.61640
0.251171875	0.9068546875	0.5498046875	1.546484375	252.58099
0.2455078125	0.901171875	0.5515625	1.58515625	295.79539
0.25390625	0.89969375	0.5494140625	1.5939453125	244.50489
0.2515625	0.899046875	0.5560846875	1.5447265625	251.01300
0.248046875	0.89819375	0.5431540625	1.4796875	265.82049
0.25180640625	0.901220703125	0.55107421875	1.55703125	268.87639
0.248876953125	0.902685546875	0.551123046875	1.553076171875	265.05930
0.252783203125	0.901123046875	0.550439453125	1.54296875	306.20870
0.25830078125	0.900341796875	0.54921875	1.573927734375	315.93150
0.25044140625	0.89876953125	0.5502980625	1.557470703125	261.50459
0.247314453125	0.89697265625	0.546435546875	1.52055640625	262.96580
0.249853515625	0.8998291015625	0.5501986328125	1.5512084909375	239.99700
0.2500732421875	0.9014038089375	0.55042724609375	1.554612578125	268.07650
0.24975859375	0.9005126953125	0.5485595703125	1.5485717734375	238.33220
0.2497948046875	0.9004724609375	0.55035408390625	1.5370361328125	305.50679
0.2515258780625	0.9000732421875	0.54979248046875	1.547802734375	268.33620
0.2488647409375	0.8975830078125	0.55086669921875	1.56197509765625	264.80709
0.250225830078125	0.900227783203125	0.5490966796875	1.5502197265625	229.52039

0.25	0.75	0.5	2	330.1009
0.25	0.85	0.6	1	NaN
0.15	0.75	0.5	3	323.5899
0.05	0.75	0.5	4	340.3754
0.15	0.75	0.5	2	326.0686
0.05	0.75	0.4	4	330.5359
0.25	0.75	0.5	5	334.0560
0.15	0.85	0.5	3	316.2790
0.15	0.95	0.5	3	303.1888
0.15	1.	0.5	3	NaN
0.15	0.85	0.4	3	318.4759
0.05	1.	0.5	3	NaN
0.15	1.	0.6	3	NaN
0.35	1.	0.5	3	NaN
0.15	1.	0.5	5	NaN
0.05	0.05	0.4	1	NaN
0.15	0.85	0.475	3	313.9554
0.15	1.	0.476	2	NaN
0.15	1.	0.532	4	NaN
0.25	0.95	0.503	3	100286.7
0.15	0.95	0.55	2	303.1791
0.15	0.95	0.6	1	NaN
0.15	0.85	0.45	4	290.1130
0.15	0.75	0.35	6	338.5636
0.05	0.85	0.45	5	310.3020
0.15	0.75	0.45	1	NaN
0.15	1.	0.55	2	NaN
0.05	1.	0.25	3	NaN
0.05	0.55	0.55	3	400.4212
0.35	0.45	0.45	1	391.8666
0.2	0.65	0.45	4	351.8827
0.138	0.85	0.35	4	316.5801
0.151	0.85	0.45	6	291.6727
0.2	0.85	0.45	4	297.7219
0.149	1.	0.45	4	NaN
0.112	0.65	0.55	2	409.2380
0.141	0.85	0.4	5	316.5398
0.15	0.95	0.456	4	283.9470
0.15	1.	0.462	4	NaN
0.2	0.65	0.456	5	346.7783
0.147	0.95	0.556	4	311.6209
0.142	1.	0.456	3	NaN
0.174	1.	0.456	5	NaN
0.1	0.95	0.456	4	280.4177
0.05	0.95	0.456	4	283.5251
0.1	0.85	0.556	4	314.9984
0.1	0.95	0.456	5	281.0185
0.2	0.95	0.456	4	274.8974
0.3	0.95	0.456	4	282.7640
0.2	0.65	0.56	3	344.0217
0.3	0.95	0.456	5	282.3462
0.1	0.95	0.556	4	317.3498
0.2	0.75	0.456	4	329.5962
0.4	0.95	0.556	4	304.3251
0	1.	0.256	3	NaN
0.3	0.85	0.517	4	289.2744
0.3	0.95	0.414	3.5	237.7690
0.4	0.95	0.372	3	277.6089

Trust-Region parameter tuning: Results

	Standard $\times 2 \div 2$	New $\{1, 2, 5\} \times 10^b$	Granularity	
			0.01	0.05
CPU:	228 s	207 s	205 s	220 s
η_1	0.2508936274	0.5325498121	0.35	0.35
η_2	0.9008242693	0.9938378210	0.99	0.95
α_1	0.5464425868	0.4817164472	0.50	0.45
α_2	1.5505441560	3.5206901980	2.82	3.20

Observations:

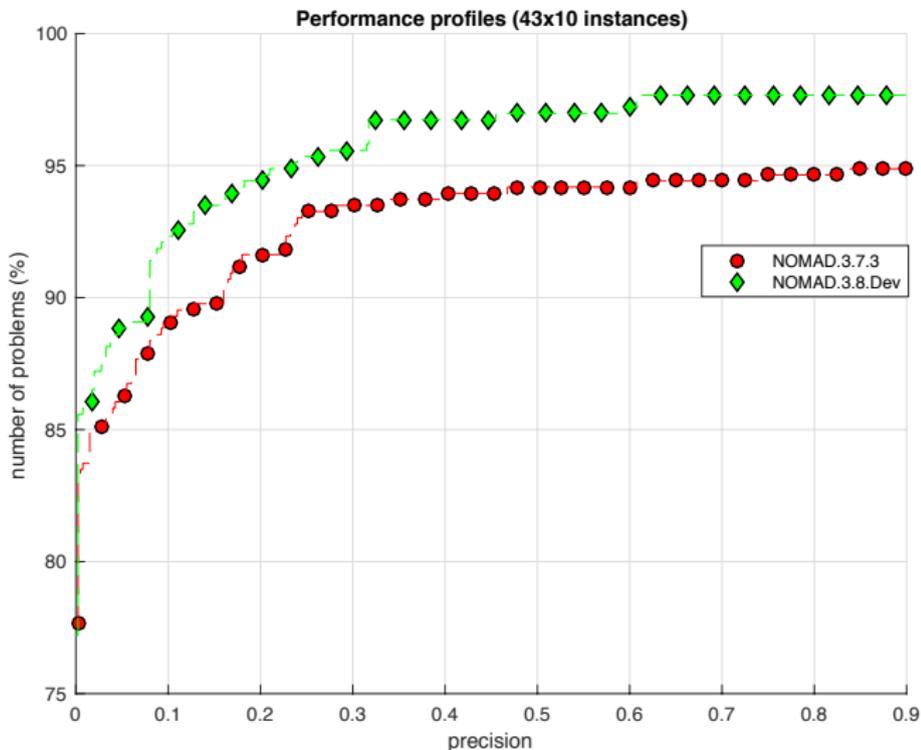
- ▶ On this example, the new strategies seem preferable.
- ▶ Trust-region recommendation for humans:

$$(\eta_1, \eta_2, \alpha_1, \alpha_2) = (0.35, 0.99, 0.5, 2.82).$$

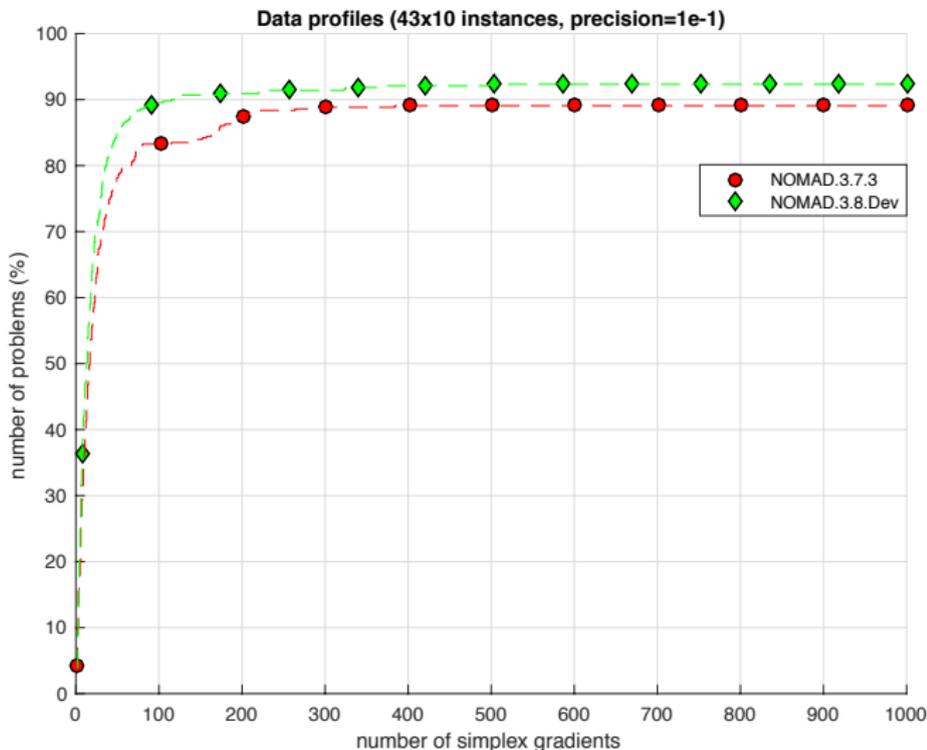
Results on a collection of mixed-integer problems

- ▶ 43 problems taken from 3 papers by J. Müller *et al.*:
 - ▶ SO-MI [Müller et al., 2013]: 19 problems, mixed, constrained.
 - ▶ SO-I [Müller et al., 2014]: 16 problems, discrete, constrained.
 - ▶ MISO [Müller, 2016]: 8 problems, mixed, unconstrained.
- ▶ 10 LHS starting points are considered for each problem, for a total of 430 instances.
- ▶ NOMAD 3.7.3 (current release, classic mesh)
vs
NOMAD 3.8.Dev (prototype, new mesh).
- ▶ Performance and data profiles [Moré and Wild, 2009].

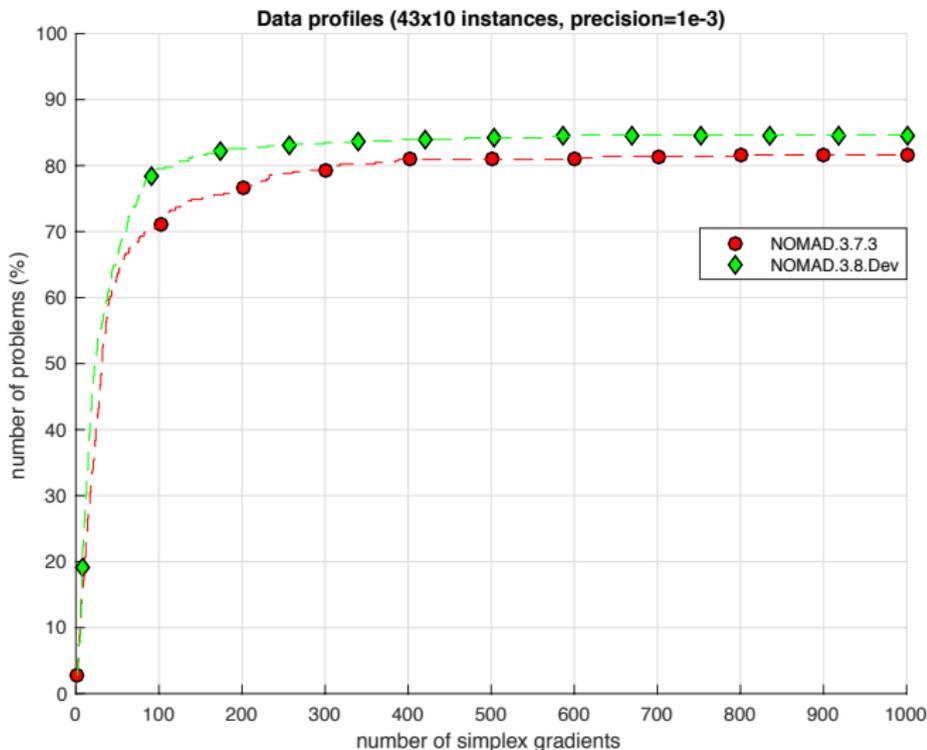
Performance profiles for the 430 instances



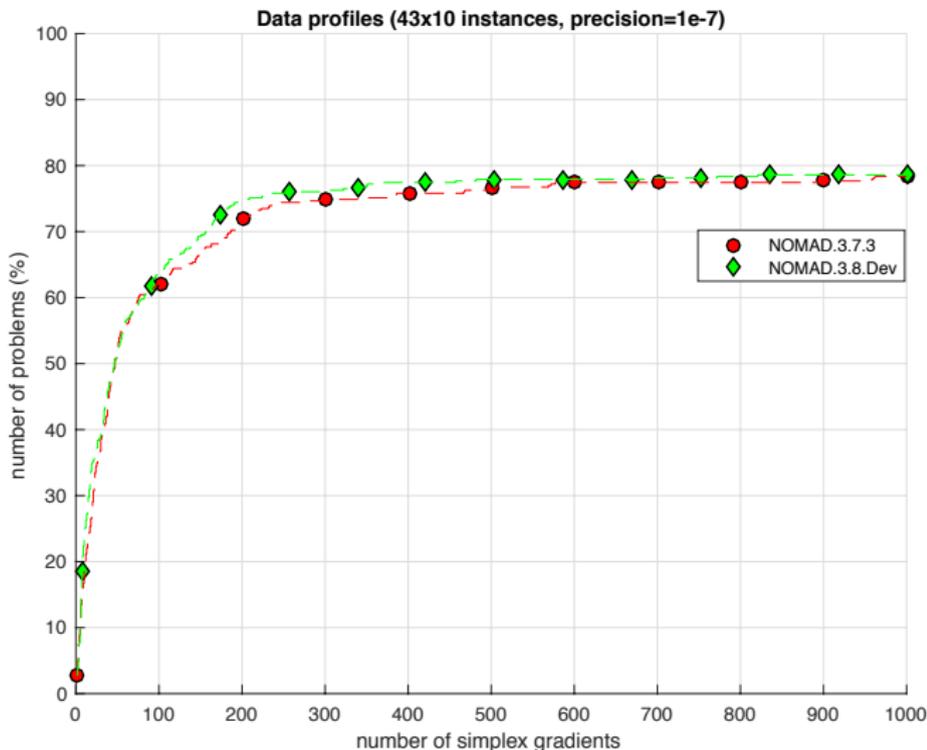
Data profiles for the 430 instances (precision 10^{-1})



Data profiles for the 430 instances (precision 10^{-3})



Data profiles for the 430 instances (precision 10^{-7})



Blackbox optimization

Motivating example

The MADS algorithm

Computational experiments

Discussion

Discussion (1/2)

- ▶ New mesh parameter update rules to control the number of decimals $\{1, 2, 5\} \times 10^b$:
 - ▶ A native way to handle granularity of variables \mathcal{G} .
 - ▶ Integer variables are handled by setting $\mathcal{G} = 1$.

Discussion (1/2)

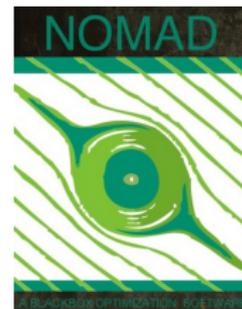
- ▶ New mesh parameter update rules to control the number of decimals $\{1, 2, 5\} \times 10^b$:
 - ▶ A native way to handle granularity of variables \mathcal{G} .
 - ▶ Integer variables are handled by setting $\mathcal{G} = 1$.
- ▶ Computational experiments on trust region parameters:
 - ▶ New parameters reduce CPU time by 35% (versus textbook).
 - ▶ New parameters have granularity 0.01 (readable by humans).
 - ▶ Experiments on other test problems in progress.

Discussion (1/2)

- ▶ New mesh parameter update rules to control the number of decimals $\{1, 2, 5\} \times 10^b$:
 - ▶ A native way to handle granularity of variables \mathcal{G} .
 - ▶ Integer variables are handled by setting $\mathcal{G} = 1$.
- ▶ Computational experiments on trust region parameters:
 - ▶ New parameters reduce CPU time by 35% (versus textbook).
 - ▶ New parameters have granularity 0.01 (readable by humans).
 - ▶ Experiments on other test problems in progress.
- ▶ Computational experiments on analytical problems:
 $\simeq 3\%$ performance improvement over the previous NOMAD version.

Discussion (2/2)

- ▶ This will be part of our NOMAD 3.8 software:
 - ▶ The only additional input from the user is \mathcal{G} .
 - ▶ ... and it is optional.
 - ▶ www.gerad.ca/nomad.



References I



Abramson, M. (2004).

Mixed variable optimization of a Load-Bearing thermal insulation system using a filter pattern search algorithm.

Optimization and Engineering, 5(2):157–177.



Audet, C. and Dennis, Jr., J. (2006).

Mesh adaptive direct search algorithms for constrained optimization.

SIAM Journal on Optimization, 17(1):188–217.



Audet, C. and Orban, D. (2006).

Finding optimal algorithmic parameters using derivative-free optimization.

SIAM Journal on Optimization, 17(3):642–664.



Le Digabel, S. (2011).

Algorithm 909: NOMAD: Nonlinear Optimization with the MADS algorithm.

ACM Transactions on Mathematical Software, 37(4):44:1–44:15.



Moré, J. and Wild, S. (2009).

Benchmarking derivative-free optimization algorithms.

SIAM Journal on Optimization, 20(1):172–191.

References II



Müller, J. (2016).

MISO: mixed-integer surrogate optimization framework.
Optimization and Engineering, 17(1):177–203.



Müller, J., Shoemaker, C., and Piché, R. (2013).

SO-MI: A surrogate model algorithm for computationally expensive nonlinear mixed-integer black-box global optimization problems.
Computers and Operations Research, 40(5):1383–1400.



Müller, J., Shoemaker, C., and Piché, R. (2014).

SO-I: a surrogate model algorithm for expensive nonlinear integer programming problems including global optimization applications.
Journal of Global Optimization, 59(4):865–889.



Torczon, V. (1997).

On the convergence of pattern search algorithms.
SIAM Journal on Optimization, 7(1):1–25.