Blackbox Optimization: Algorithm and Applications

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Presentation outline

Blackbox optimization

The MADS algorithm

The NOMAD software package

Snow Water Equivalent estimation

Characterization of objects from radiographs

Biobjective optimization of aircraft takeoff trajectories

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Blackbox optimization (BBO) problems

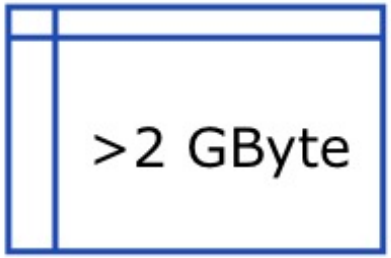
- Optimization problem:

$$\min_{x \in \Omega} \quad f(x)$$

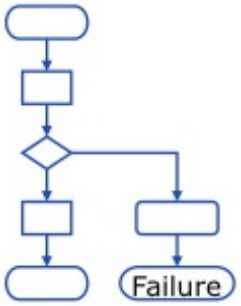
- Evaluations of f (the objective function) and of the functions defining Ω are usually the result of a computer code (a blackbox).
- n variables, m general constraints.

Blackboxes as illustrated by J. Simonis [ISMP 2009]

Long runtime

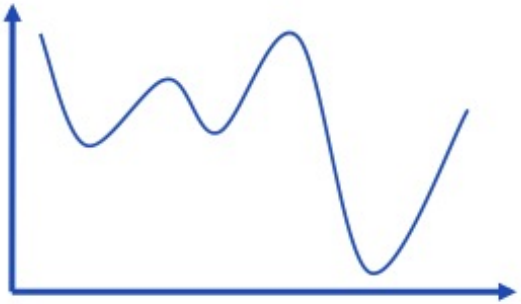


Large memory requirement



Software might fail

No derivatives available



Local optima

Non-smooth, noisy

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Mesh Adaptive Direct Search (MADS)

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- One iteration = search and poll.
- The search allows trial points generated anywhere on the mesh.
- The poll consists in generating a list of trial points constructed from poll directions. These directions grow dense.
- At the end of the iteration, the mesh size is reduced if no new success point is found.

[0] Initializations (x_0, Δ_0)

[1] Iteration k

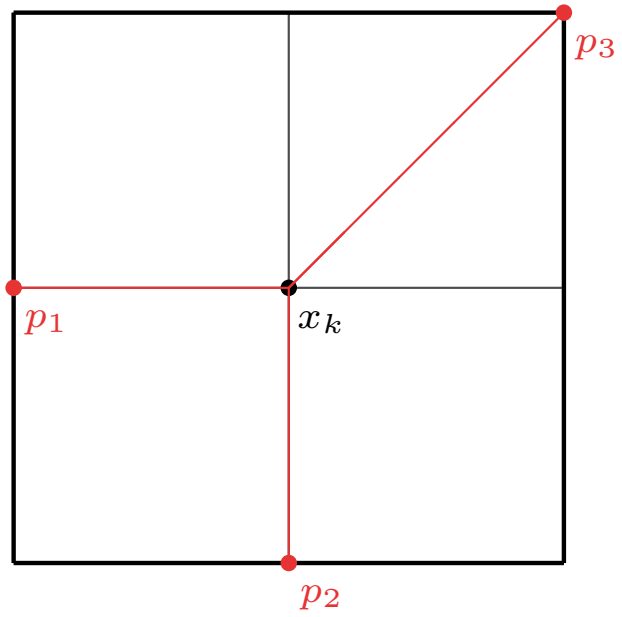
- **[1.1] Search**
  - select a finite number of mesh points
  - evaluate candidates opportunistically
- **[1.2] Poll** (if the Search failed)
  - construct poll set $P_k = \{x_k + \Delta_k d : d \in D_k\}$
  - sort(P_k)
  - evaluate candidates opportunistically

[2] Updates

- if success
  - $x_{k+1} \leftarrow$ success point
  - increase Δ_k
- else
  - $x_{k+1} \leftarrow x_k$
  - decrease Δ_k
- $k \leftarrow k + 1$, stop or go to **[1]**

Poll illustration (successive fails and mesh shrinks)

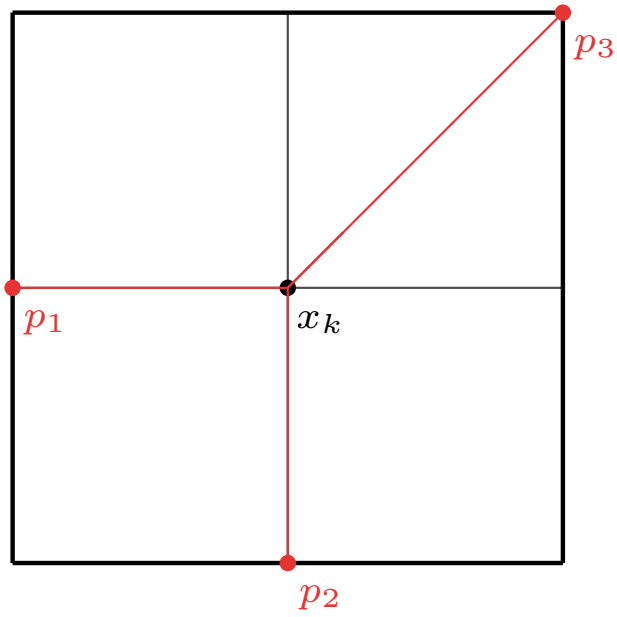
$$\Delta_k = 1$$

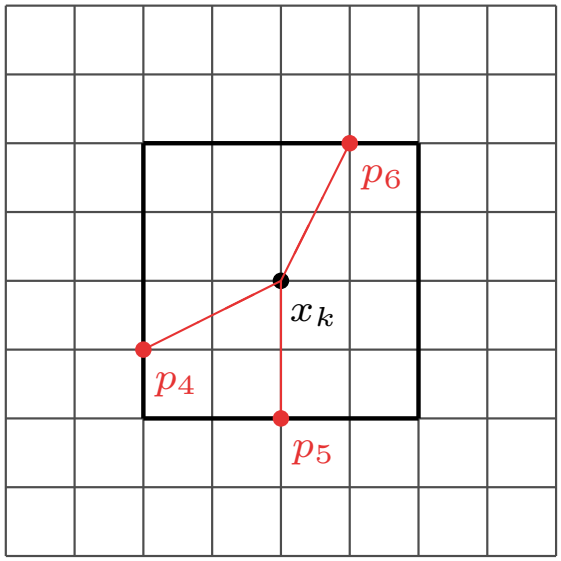


trial points$=\{p_1, p_2, p_3\}$

Poll illustration (successive fails and mesh shrinks)

$\Delta_k = 1$ $\Delta_{k+1} = 1/4$

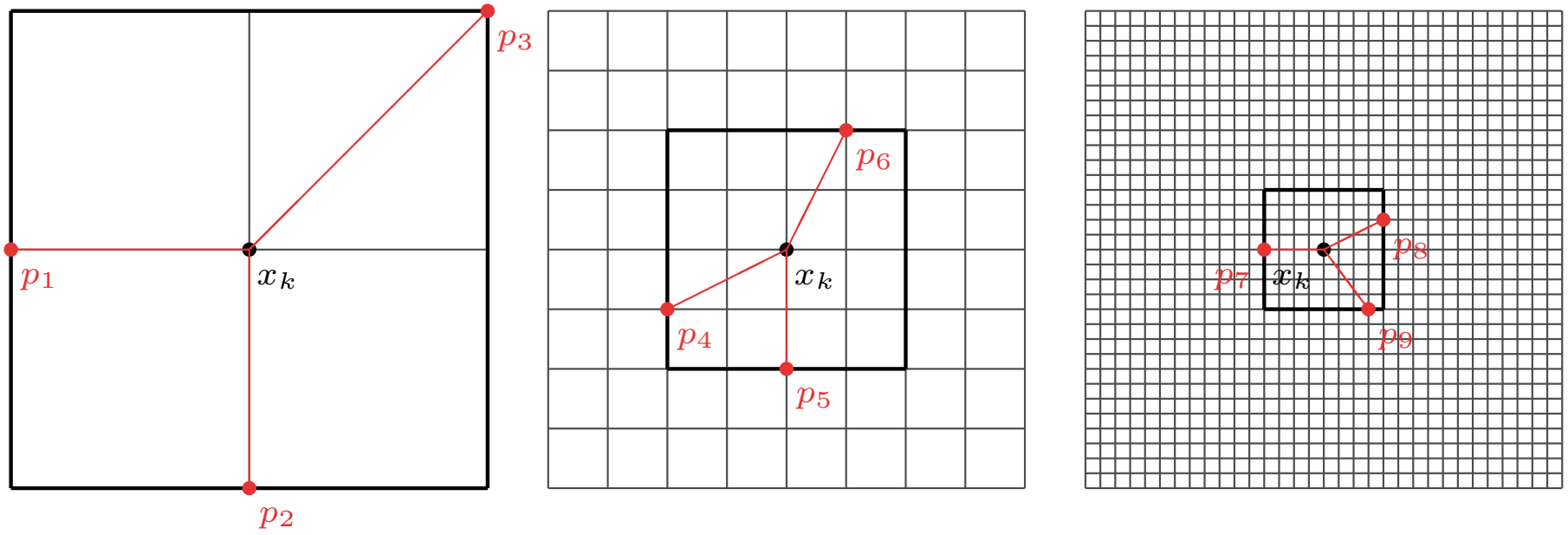




trial points$=\{p_1, p_2, p_3\}$ $= \{p_4, p_5, p_6\}$

Poll illustration (successive fails and mesh shrinks)

$\Delta_k = 1$ $\Delta_{k+1} = 1/4$ $\Delta_{k+2} = 1/16$



trial points$=\{p_1, p_2, p_3\}$ $= \{p_4, p_5, p_6\}$ $= \{p_7, p_8, p_9\}$

Convergence results

- MADS is backed by a convergence analysis based on the calculus for nonsmooth functions [Clarke, 1983].
- It produces solutions satisfying optimality conditions "proportional" to the smoothness of the problem.
- Summary of the results:

	Unconstrained	Constrained
Smooth	$\nabla f(x) = 0$	$f'(x;d) \geq 0$ for all $d \in T_\Omega(x)$
Nonsmooth	$0 \in \partial f(x)$	$f^\circ(x;d) \geq 0$ for all $d \in T_\Omega^H(x)$

MADS extensions

- Constraints handling with the Progressive Barrier technique [Audet and Dennis, Jr., 2009].
- Surrogates [Talgorn et al., 2015].
- Categorical variables [Abramson, 2004].
- Global optimization [Audet et al., 2008a].
- Parallelism [Le Digabel et al., 2010, Audet et al., 2008b].
- Multiobjective optimization [Audet et al., 2008c].
- Sensitivity analysis [Audet et al., 2012].
- Dynamic scaling [Audet et al., 2016].

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NOMAD (Nonlinear Optimization with MADS)

- C++ implementation of MADS.
- Standard C++, no other package needed.
- Parallel versions with MPI.
- Runs on Linux, Unix, Mac OS X and Windows.
- MATLAB versions.
- Command-line and library interfaces.
- Distributed under the LGPL license.
- Complete user guide available in the package.
- Doxygen documentation available online.
- Download at https://www.gerad.ca/nomad.

Run NOMAD

```
> nomad parameters.txt
```

```
delta:2 sebld$ nomad param.txt

NOMAD - version 3.5.1.TGP - www.gerad.ca/nomad

Copyright (C) 2001-2012 {
        Mark A. Abramson     - The Boeing Company
        Charles Audet        - Ecole Polytechnique de Montreal
        Gilles Couture       - Ecole Polytechnique de Montreal
        John E. Dennis, Jr.  - Rice University
        Sebastien Le Digabel - Ecole Polytechnique de Montreal
        Christophe Tribes    - Ecole Polytechnique de Montreal
}

Funded in part by AFOSR and Exxon Mobil.

License   : '$NOMAD_HOME/src/lgpl.txt'
User guide: '$NOMAD_HOME/doc/user_guide.pdf'
Examples  : '$NOMAD_HOME/examples'
Tools     : '$NOMAD_HOME/tools'

Please report bugs to nomad@gerad.ca

MADS run {

        BBE     SOL     OBJ

         1                1            1    0.589738091176242
         7               31            1    0.545072064762882
        10               31            1    0.545072064762882

} end of run (max number of blackbox evaluations)

blackbox evaluations    : 10
best feasible solution  : ( 31 1 ) h=0 f=0.5450720648
```

Other MADS distributions

- Available in the MATLAB Optimization Toolbox.
  Old version, not maintained.
- MATLAB version within the Opti Toolbox package.
  http://www.i2c2.aut.ac.nz/Wiki/OPTI.
- Excel with the OpenSolver tool.
  http://www.opensolver.org (GPLv3).

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Snow Water Equivalent (SWE) estimation

- Accurate estimate of water stored in snow is crucial to optimize hydroelectric plants management.
- Exact snow measurement is impossible.
- SWE is measured at specific sites and next interpolated over the territory.
- Territory is huge: Hydro-Québec (HQ) operates 565 dams, 75 reservoirs, and 56 hydroelectric power plants, located over 90 watersheds and covering more than 550,000 km^2.



source: Hydro-Québec.

Previous SWE estimation

- Done manually by weighing snow cores at specific sites.
- Each measurement campaign requires 2 weeks.
- Missing measurements due to adverse meteorological conditions.



source: Hydro-Québec.

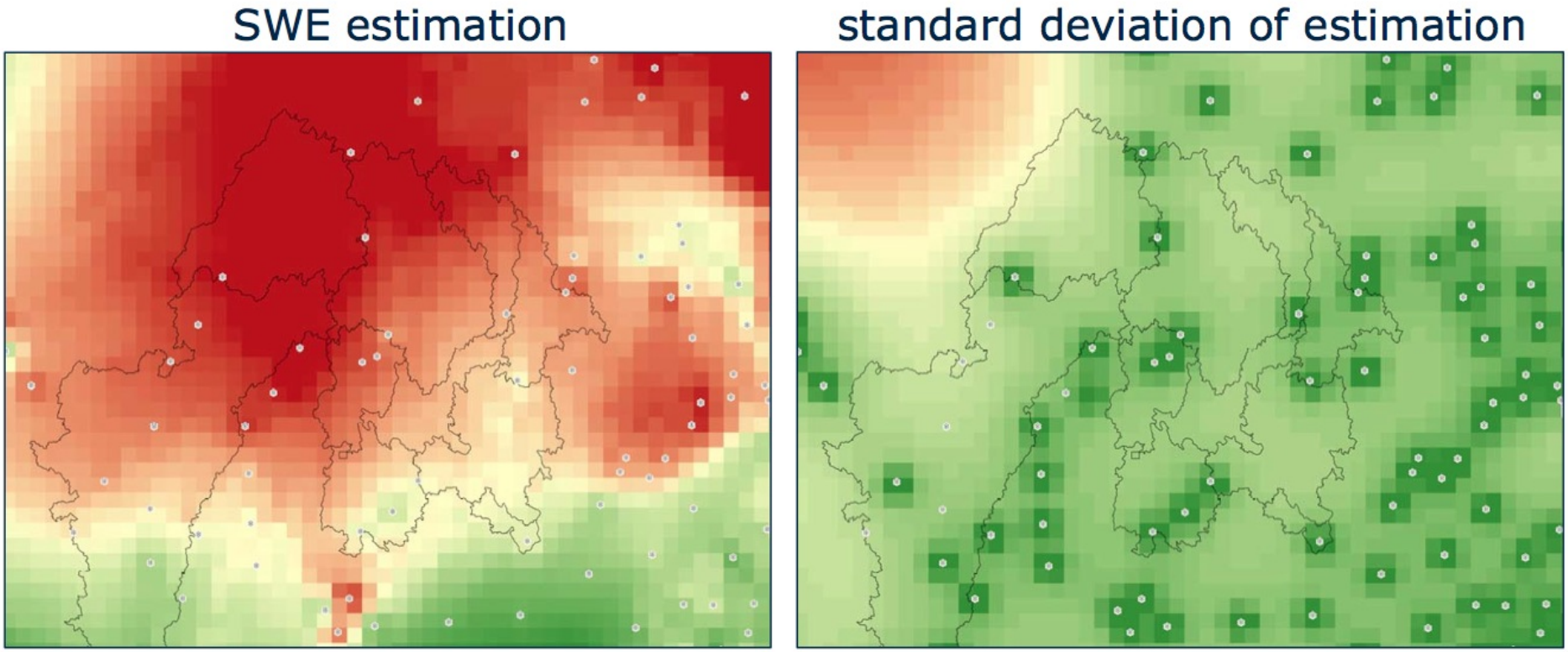
GMON device

- A new measuring instrument that provides daily automatic SWE.
- GMON for Gamma-MONitoring device: it measures the natural Gamma radiation emitted from the soil.
- Communicates via satellites.


source: Hydro-Québec.

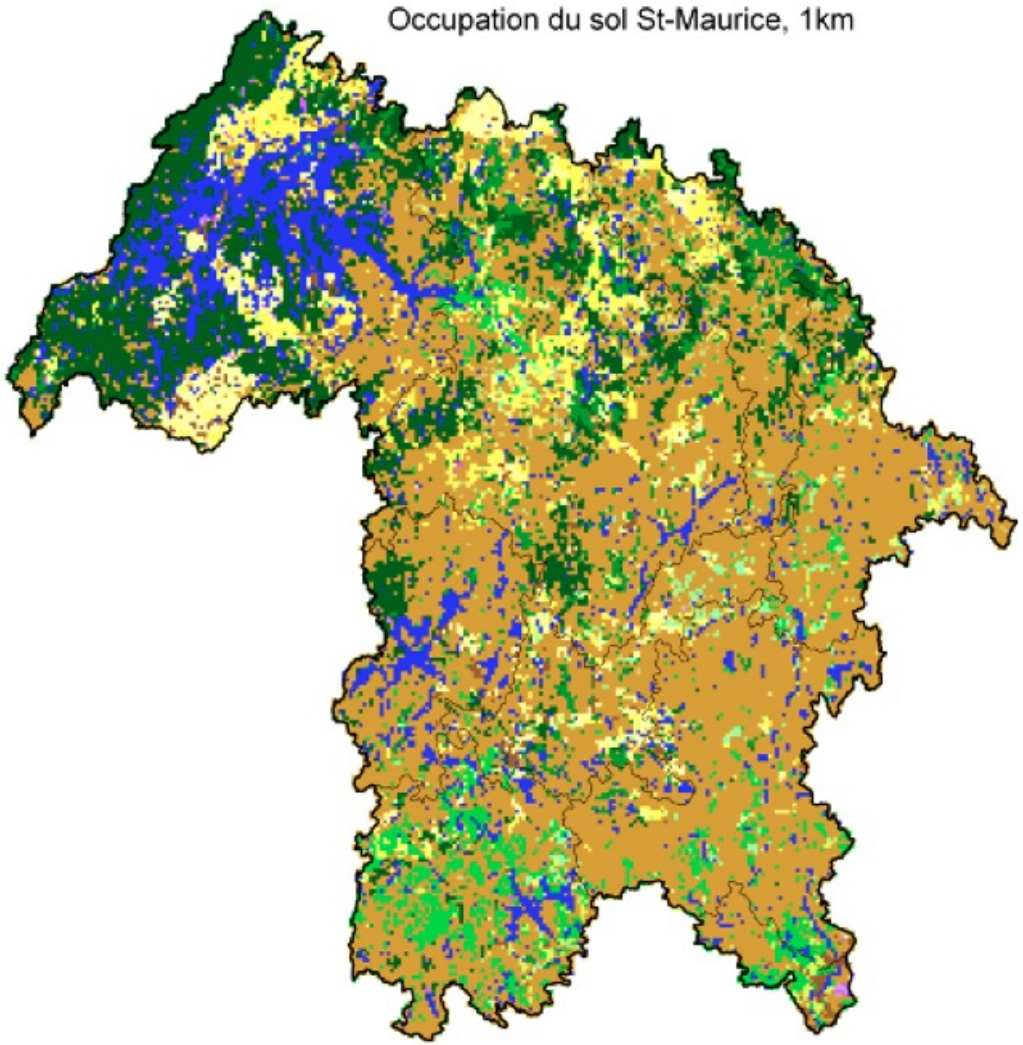
SWE estimation from GMON measures

- Kriging interpolation is used to obtain SWE estimation together with an error map.
- How to find the device locations that minimize the kriging interpolation error of the SWE?



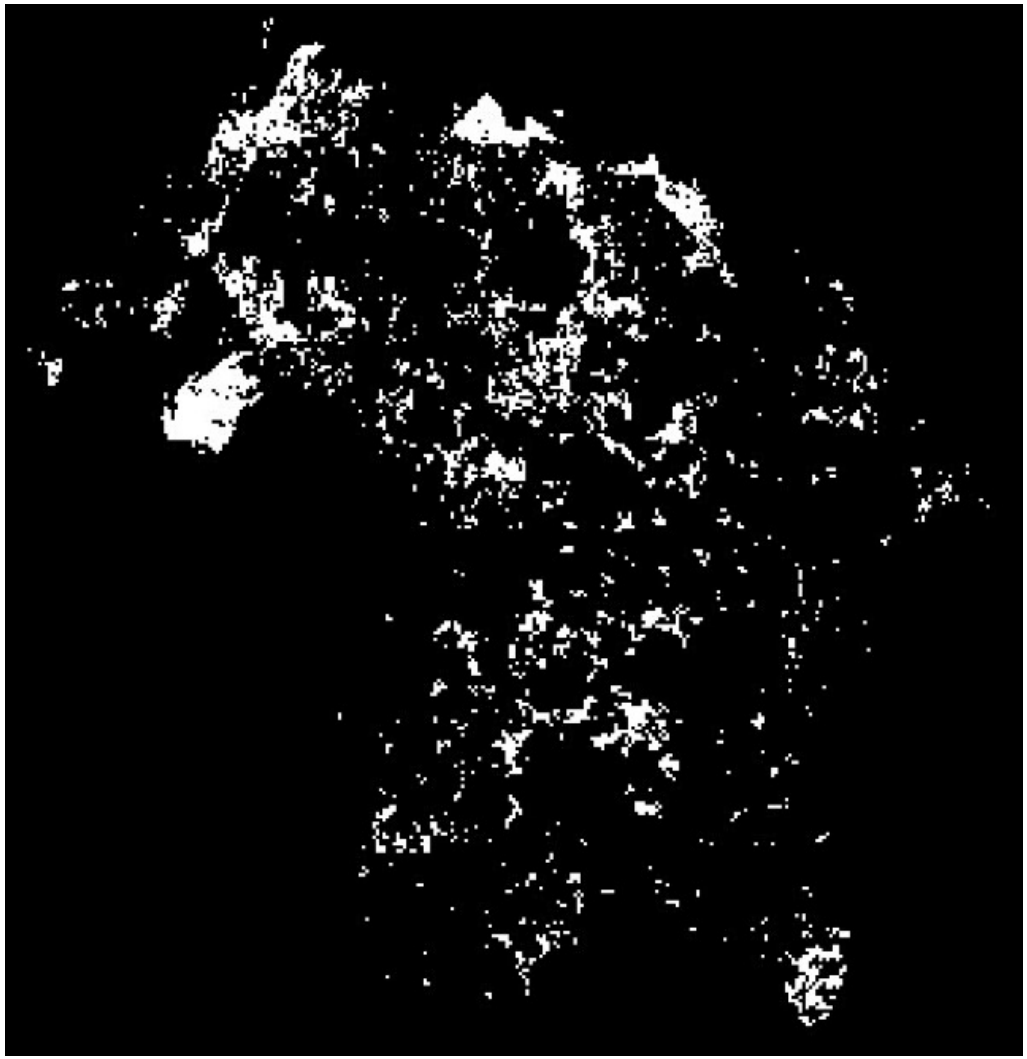
Constraints

- GMON stations cannot be located anywhere.
- Restrictions on:
  - subsoil properties,
  - slope,
  - vegetation,
  - exploitation,
  - etc.



Constraints

- GMON stations cannot be located anywhere.
- Restrictions on:
  - subsoil properties,
  - slope,
  - vegetation,
  - exploitation,
  - etc.
- Highly fragmented domain.



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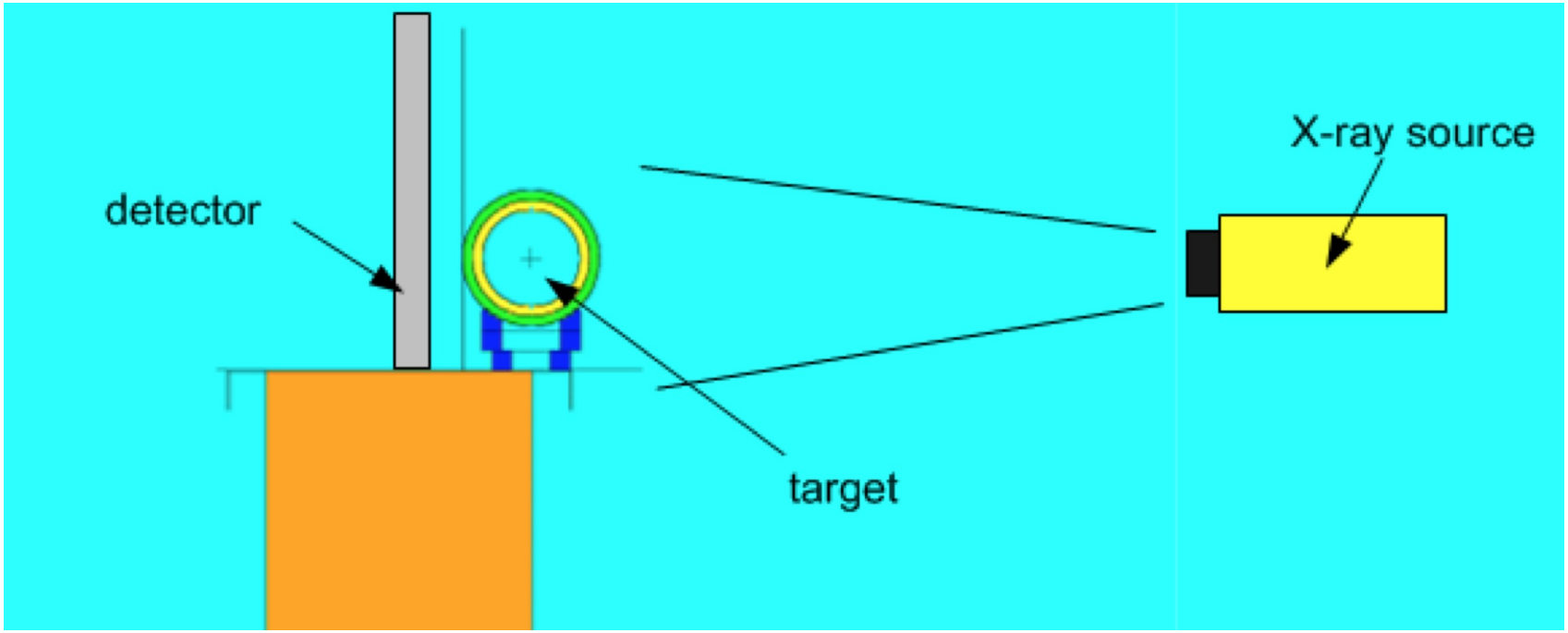
Setting

We want to identify an unknown object inside a box, using a x-ray source that gives an image on a detector.

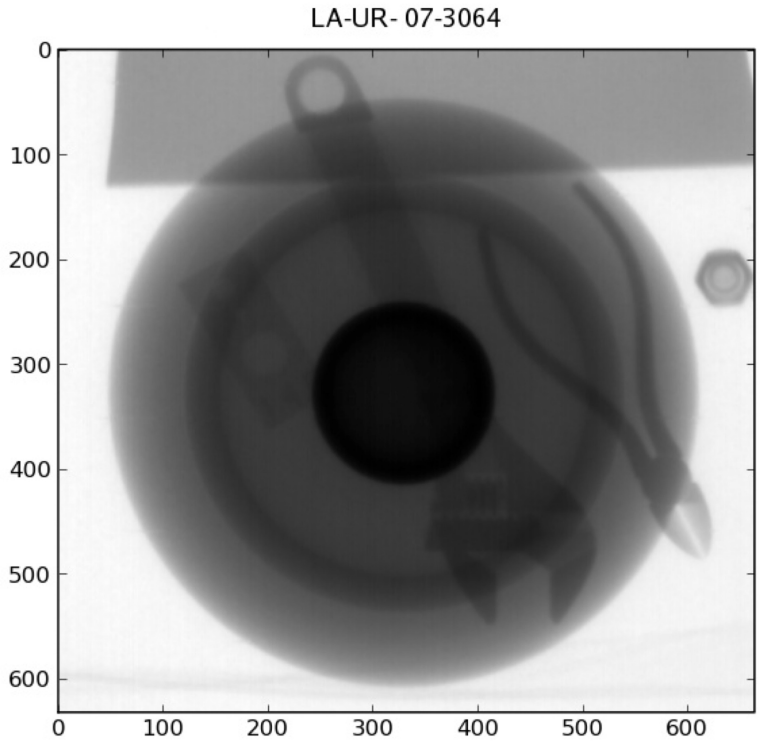


Source: LANL – LA-UR-11-0342

In this work, the unknown object is supposed to be spherical.

Radiograph

A radiograph is the observed image on the detector. For example:



source: LANL.

Description of the problem

- The problem consists to identify the unknown object with sufficient precision so that the object can be classified as dangerous or not.
- Must work rapidly.
- Must work for radiographs not created on a well-controlled experimental environment.
- Must not crash for unreasonable user inputs.

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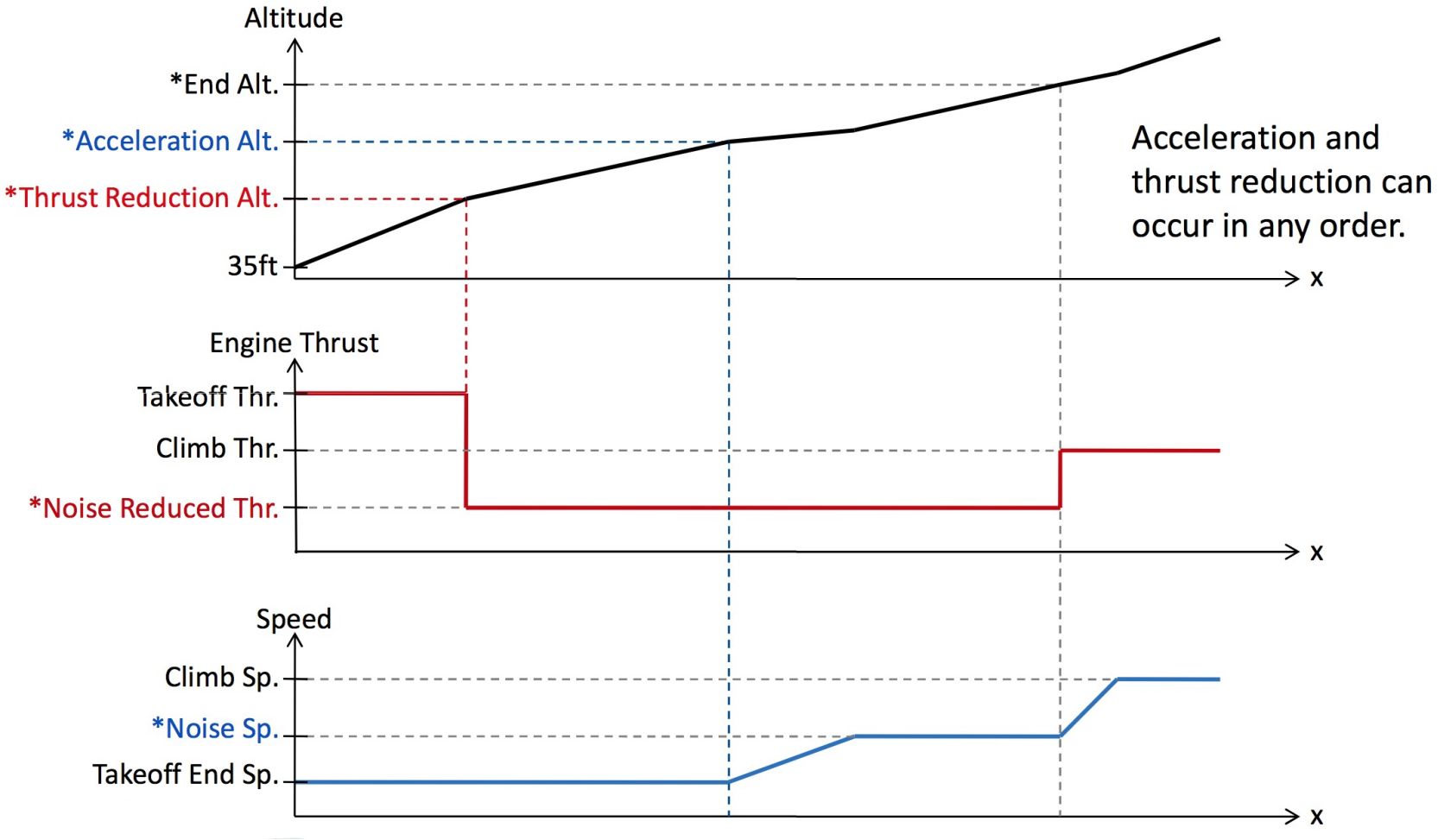
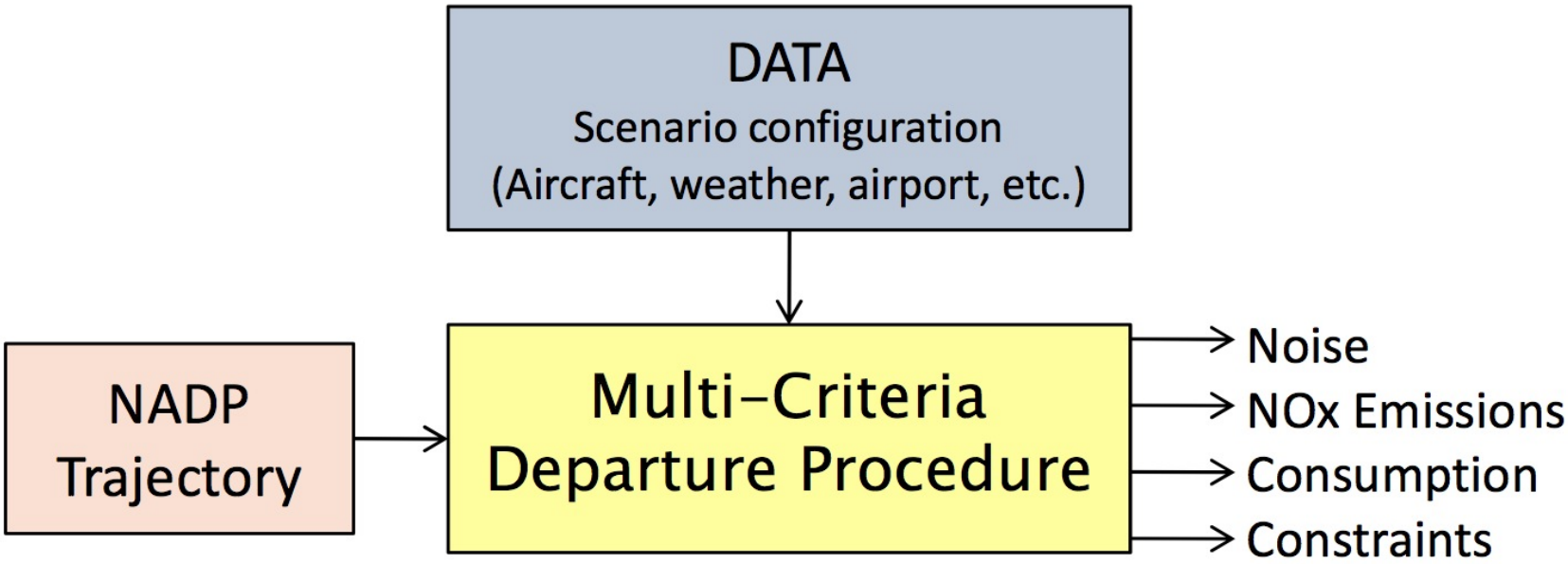
Aircraft takeoff trajectories

- Concept : Optimization of vertical flight path based on procedures designed to reduce noise emission at departure to protect airport vicinity.
- Minimization of environmental and economical impact: Noise and fuel consumption.
- NADP (Noise Abatement Departure Procedure) variables: During departure phase, the aircraft will target its climb configuration:
  - Increase the speed up to climb speed (acceleration phase).
  - Reduce the engine rate to climb thrust (reduction phase).
  - Gain altitude.

Parametric Trajectory: 5 optimization variables (*)



The blackbox: MCDP: Multi-Criteria Departure Procedure



One evaluation $\simeq$ 2 seconds.

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