CSE11

Snow water equivalent estimation using blackbox optimization

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Presentation outline

Problem description

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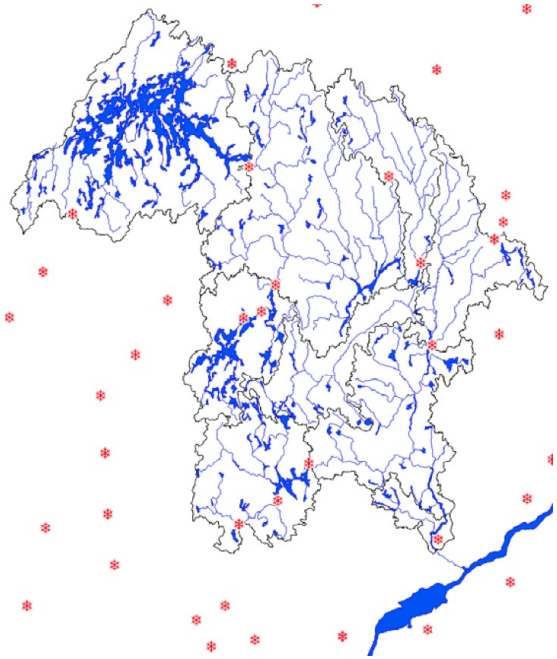
Importance of the Snow Water Equivalent (SWE)

- Most of the water reaching a reservoir is received during a 2-3 week spring flood period.
- Water inflows may exceed power plant capacities.
- Accurate estimate of water stored in snow is crucial to optimize hydroelectric plants management.
- Hydroelectric energy stabilizes market prices in North-Eastern U.S.



SWE estimation

- Territory is huge: Hydro-Québec (HQ) operates 565 dams, 75 reservoirs, and 56 hydroelectric power plants, located over 90 watersheds and covering more than 550,000 km^2.
- Exact snow measurement is impossible.
- SWE is measured at specific sites and next interpolated over the territory.



SWE estimation

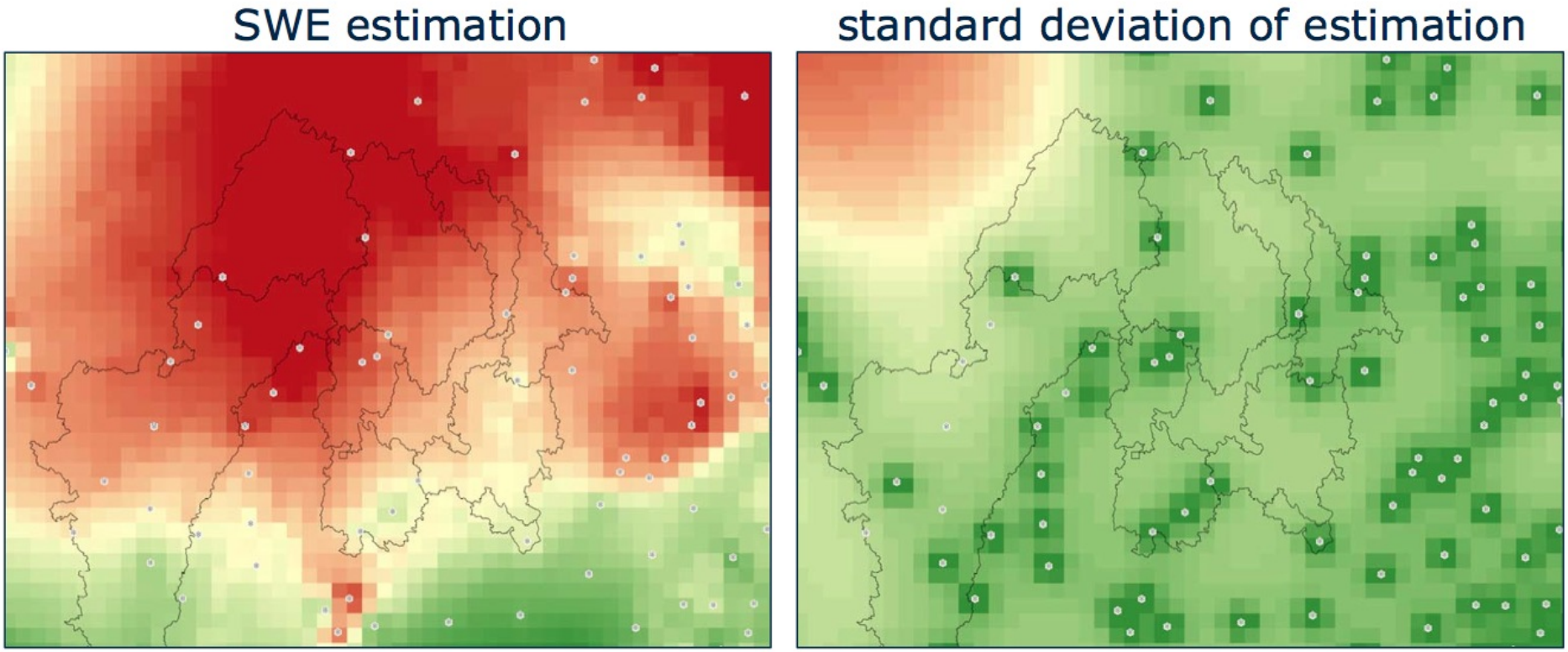
- Presently, done manually by weighing snow cores at specific sites.
- Each measurement campaign requires 2 weeks.
- Missing measurements due to adverse meteorological conditions.

GMON device

- A new measuring instrument that provides daily automatic SWE.
- GMON for Gamma-MONitoring device: it measures the natural Gamma radiation emitted from the soil.
- Communicates via satellites.

SWE estimation from GMON measures

- Kriging interpolation is used to obtain SWE estimation together with an error map.
- How to find the device locations that minimize the kriging interpolation error of the SWE?



Problem formulation

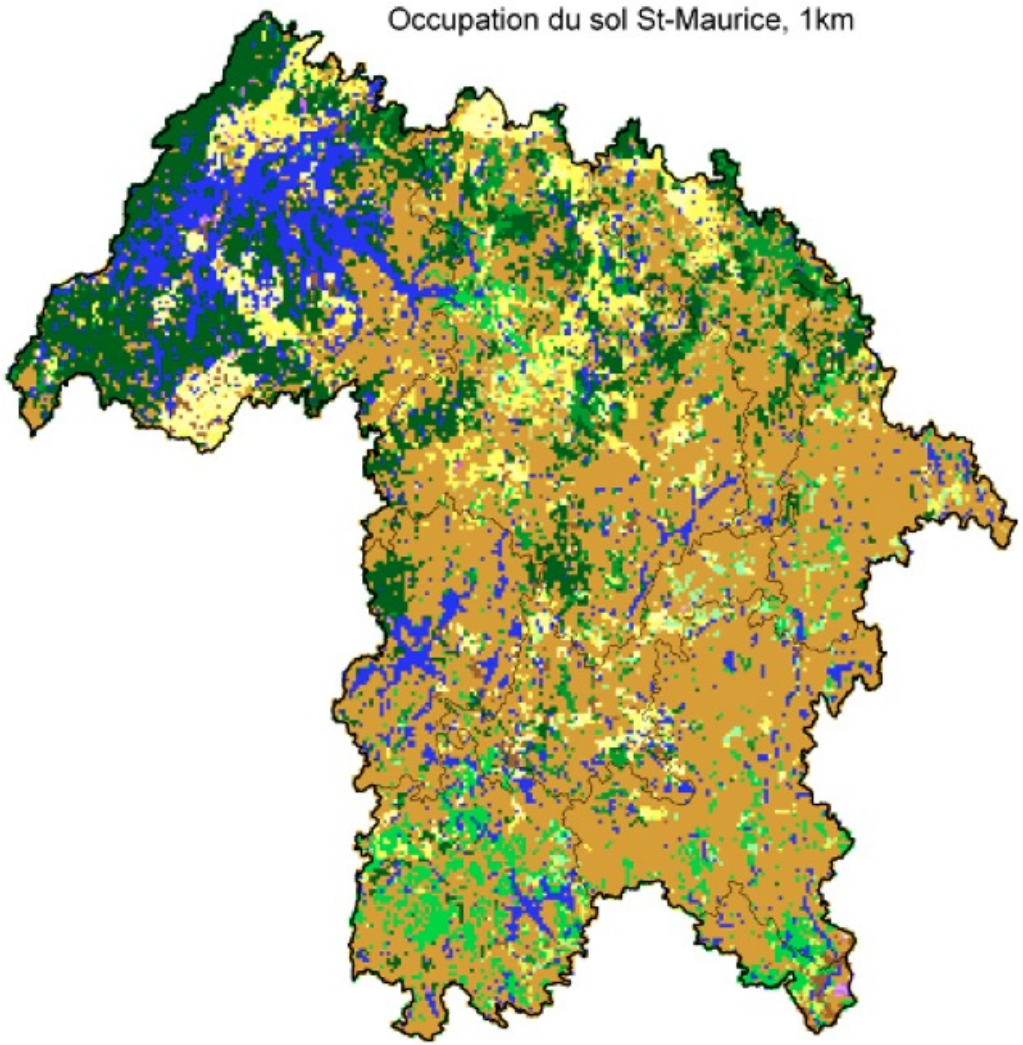
We consider the blackbox optimization problem:

$$\begin{aligned} \text{minimize} \quad & f(x) \\ \text{subject to} \quad & x \in \Omega^n \end{aligned}$$

- $x \in \mathbb{R}^{2n}$ are the locations of n stations.
- $\Omega \subseteq \mathbb{R}^2$ is the feasible domain where the stations can be located.
- $f(x)$ is a score based on the standard deviation map obtained by the kriging simulation and is considered as a blackbox.
- Each simulation requires $\simeq 2$ seconds, and can only be launched within the IREQ research center. Limited CPU resources are attributed to this project.

Constraints

- GMON stations cannot be located anywhere.
- Restrictions on:
  - subsoil properties,
  - slope,
  - vegetation,
  - exploitation,
  - etc.



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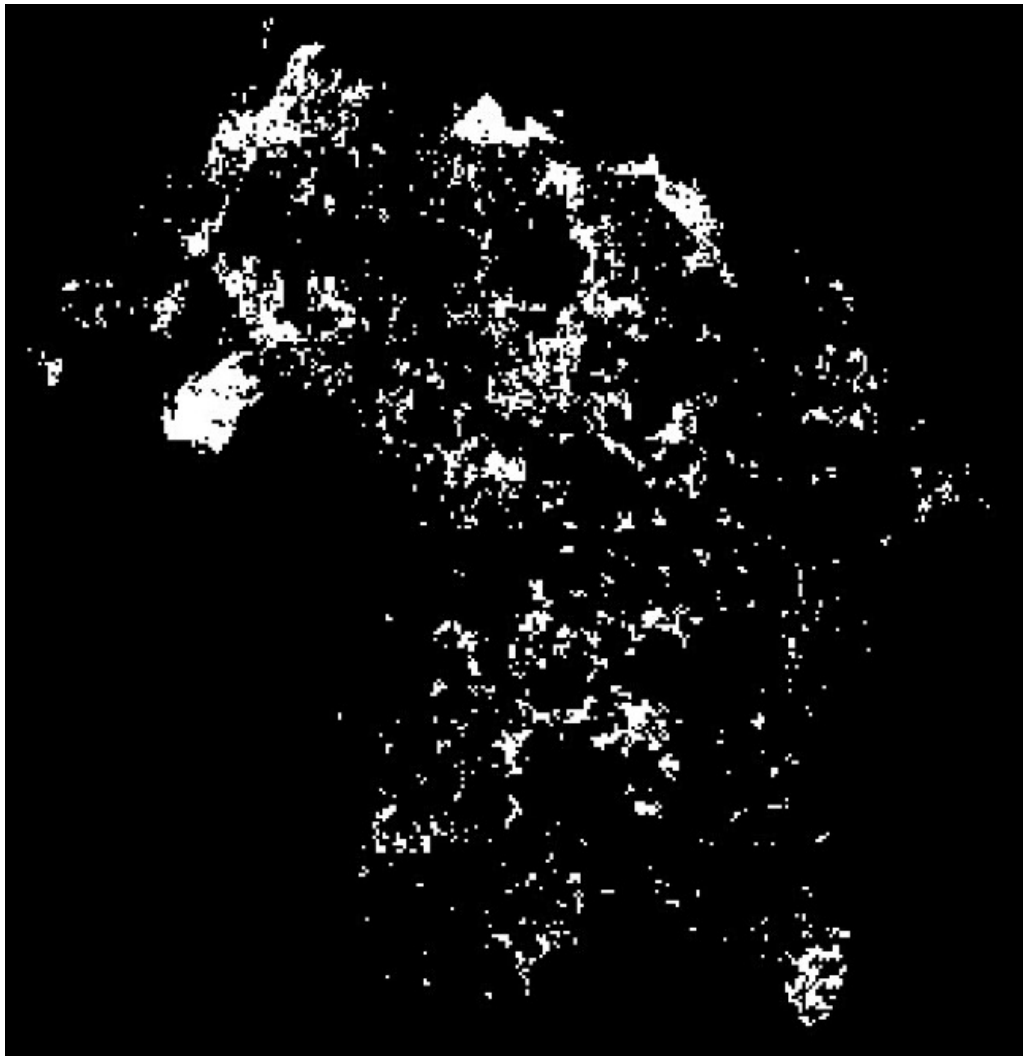

Constraints

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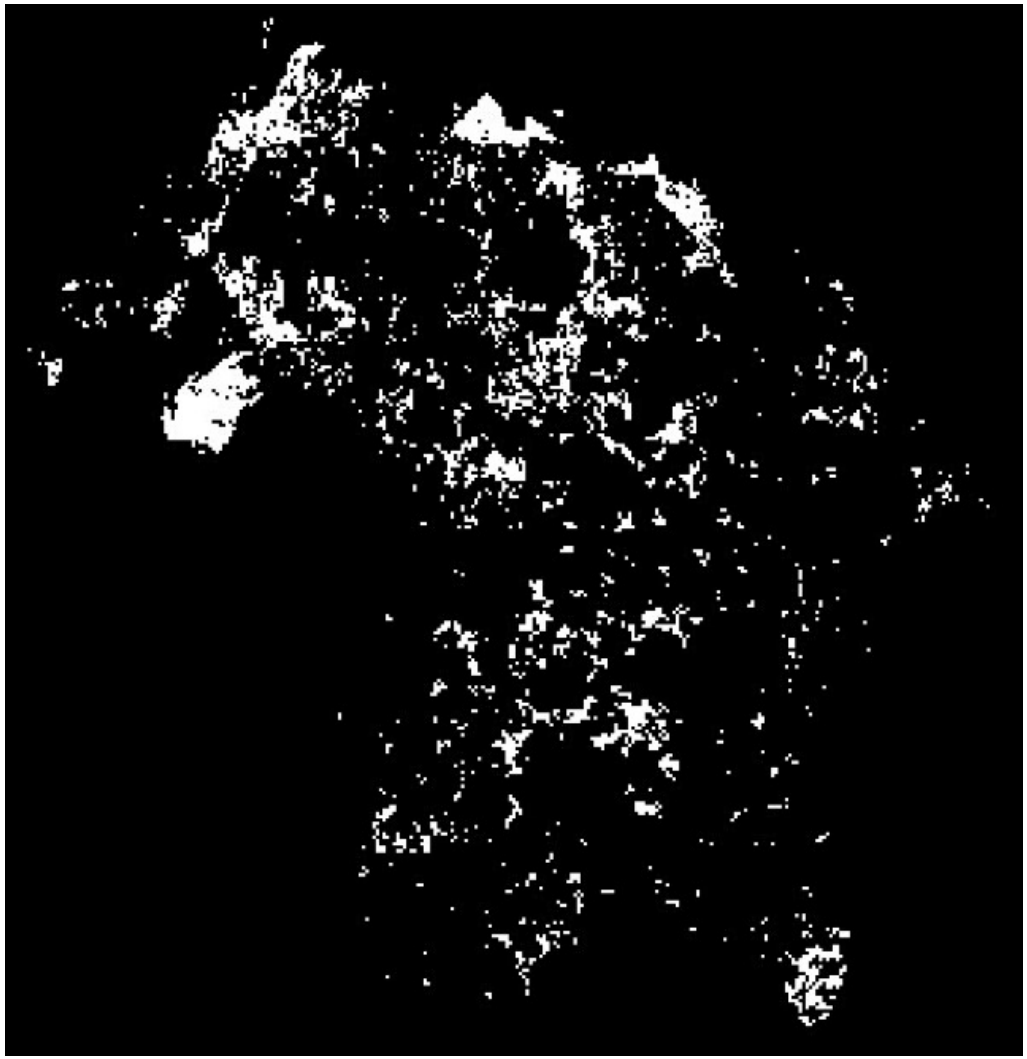
- Highly fragmented domain
- Solution: spiral walk integrated in the simulator to identify the closest feasible location.



Problem description

The MADS algorithm with groups of variables

Numerical results

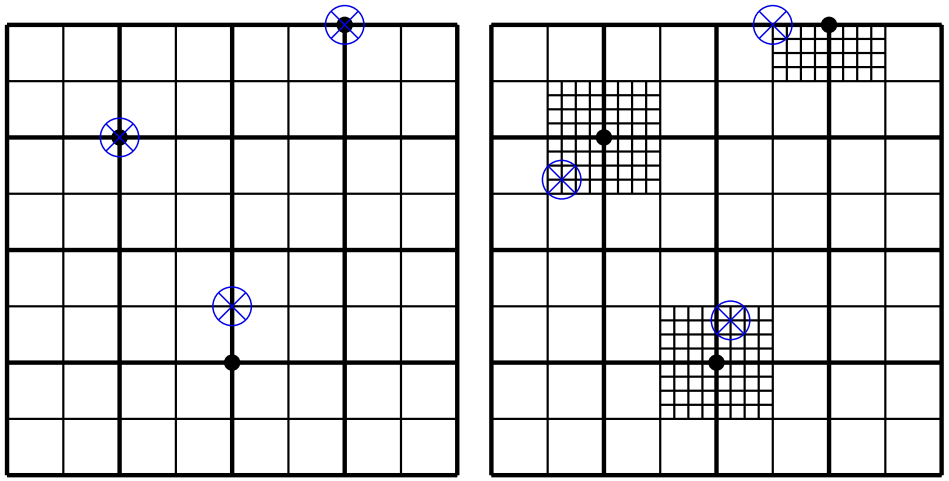
Discussion

Mesh Adaptive Direct Search (MADS)

- Audet and Dennis [SIOPT, 2006].
- Iterative algorithm that evaluates the blackbox functions at some trial points.
- Trial points generated on a spatial discretization: the mesh.
- One iteration consists in generating a list of trial points constructed from poll directions. These directions grow dense.
- At the end of the iteration, the mesh size is reduced if no new iterate is found.
- Algorithm is backed by a convergence analysis based on the Clarke Calculus for nonsmooth functions.

Groups of variables

- Blackbox problem with some knowledge on the structure: variables represent 2D locations.
- Makes sense to simultaneously move both GMON coordinates.
- Different grouping strategies are developed.
- Some are dynamic: groups are changed during the optimization.



CS MADS

Groups of variables

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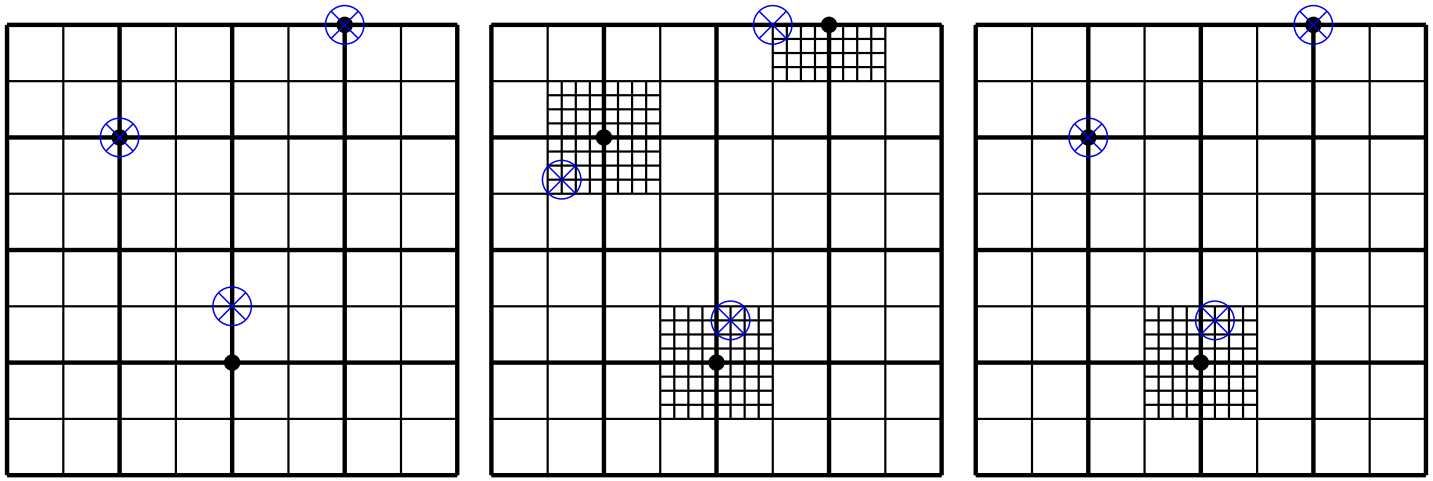


CS MADS New strategy

Grouping and regrouping strategies

- Initial grouping strategy:
  - INDIV: one group = one variable.
  - PAIRS: one group = one GMON (two variables).
  - ALL: all variables in a single group.
- Regrouping strategy:
  - STATIC: Keep the same groups (no regrouping).
  - DIST: Merge closest GMON devices.
  - CLUSTER: Merge closest devices using K-means method.
  - MVT: Group all idle GMON stations from previous run.
  - REGRES: Use linear regression to identify important variables.
- Stopping criteria for reconfiguration:
  1. $n_r = 1$.
  2. Launch MADS. Terminate after n_r failed MADS iterations.
  3. Set $n_r \leftarrow n_r + 1$ and go to step 2.

Convergence analysis

- As the algorithm is deployed, variables are progressively grouped together.
- After finitely many iterations, there is a group containing all variables.
- Consequently, the entire MADS convergence analysis holds for all of these regrouping strategies.
  - Hierarchical convergence analysis based on local smoothness.
  - Convergence to Clarke stationary points.

Non-adaptive surrogate

- Parameters defining the surrogate were chosen in collaboration with IREQ experts, by comparing corresponding error maps.

For m GMON stations on a $N_i \times N_j$ map:

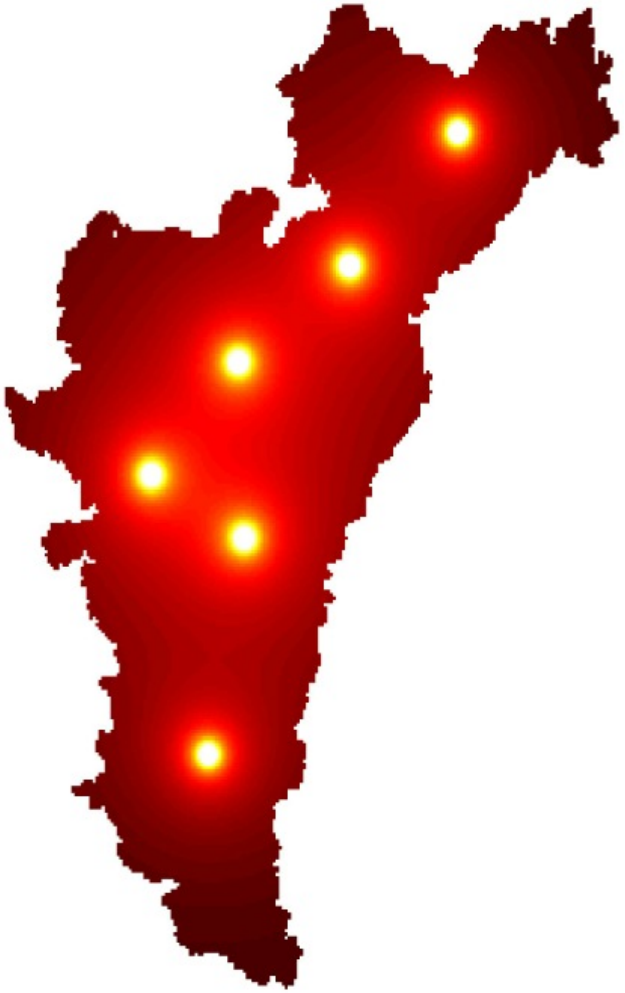
$$f(x) = \sum_{i=1}^{N_i} \sum_{j=1}^{N_j} \alpha_{ij} e_{ij}(x)$$

$\alpha_{ij} = 1$ if position (i, j) is inside the reservoir, 0 otherwise

$$e_{ij}(x) = \begin{cases} 1 & \text{if } d_{ij}(x) \geq 1, \\ (d_{ij}(x))^{-0.5} & \text{if } 0.030 \leq d_{ij}(x) < 1, \\ (d_{ij}(x))^{-0.6} & \text{if } 0.025 \leq d_{ij}(x) < 0.03, \\ (d_{ij}(x))^{-0.75} & \text{if } 0.020 \leq d_{ij}(x) < 0.025, \\ (d_{ij}(x))^{-1} & \text{if } d_{ij}(x) < 0.020, \end{cases}$$

$$d_{ij}(x) = \sum_{q=1}^{m} c_{ij}(x_{2q-1}, x_{2q})$$

$$c_{ij}(x_u, x_v) = \begin{cases} 1 & \text{if } x_u = i \text{ and } x_v = j, \\ \frac{0.8}{\sqrt{(x_u - i)^2 + (x_v - j)^2}} & \text{otherwise.} \end{cases}$$



Tests setup

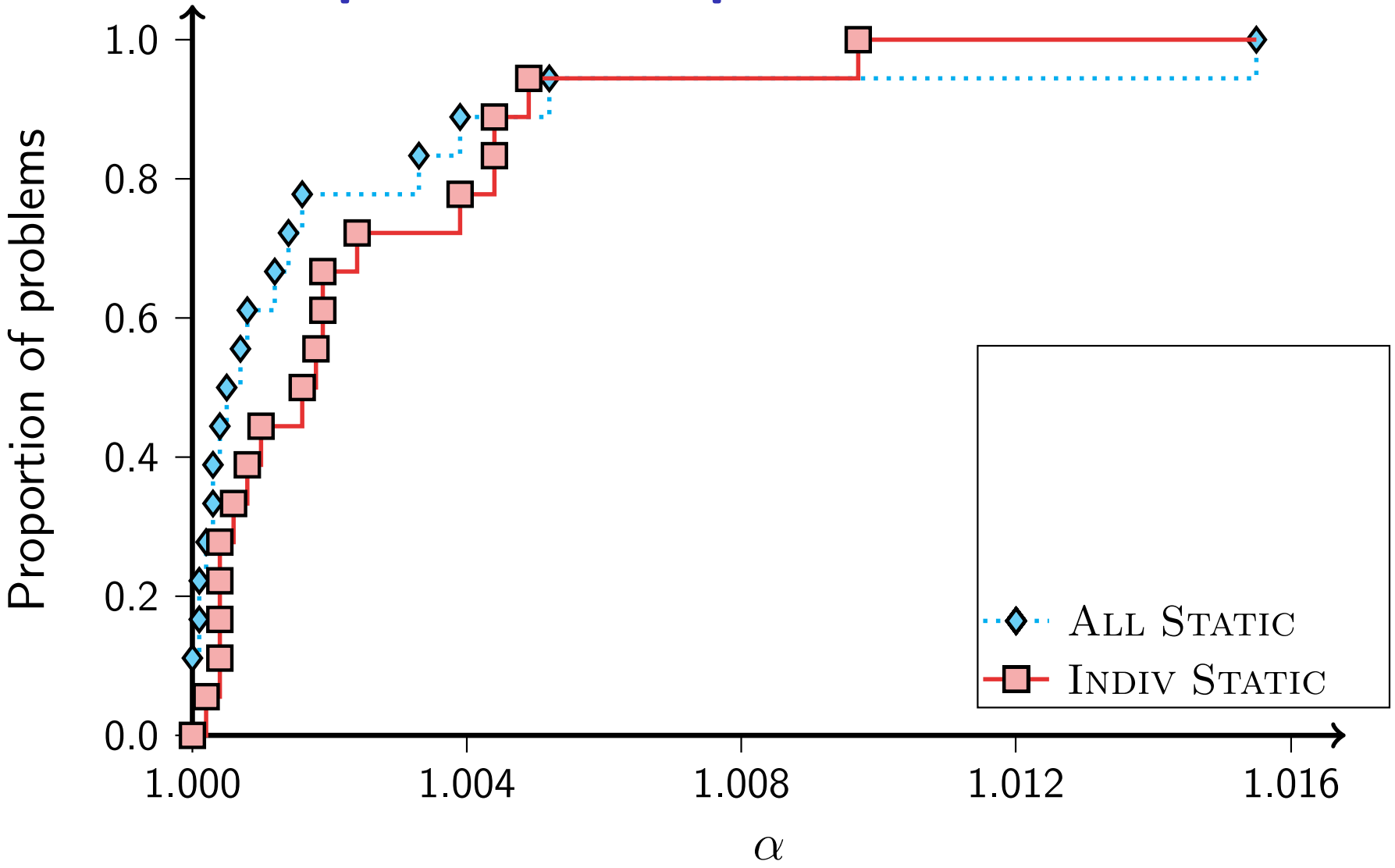
- Three maps are available: Gatineau, Saint-Maurice and La Grande.
- The number of GMON stations varies from $m = 5$ to 10, for a total of 18 test instances.
- A first series of intensive tests is performed on the surrogate.
- The six best strategies for the surrogate are then tested on the true function at HQ.
- Termination criteria based on a budget of TBE $= 1000$ evaluations for the true function, and various budgets for the surrogate.
- The surrogate function is used for the runs on the true function.
- The error map plays the role of a surrogate for the runs on the surrogate function.

The best six strategies obtained on the surrogate

Initial	Regroup	TEB = 250	TEB = 500	TEB = 1000	Mean
		Dynamic			
INDIV	DIST	1.38	1.59	1.72	1.56
PAIRS	REGRES	1.37	1.56	1.74	1.56
PAIRS	CLUSTER	1.34	1.60	1.70	1.55
INDIV	MVT	1.35	1.57	1.68	1.53
		Static			
INDIV	STATIC	1.40	1.65	1.74	1.60
ALL	STATIC	1.30	1.54	1.64	1.50

- Numbers correspond to the average relative improvement compared to the starting solution.
- Initial solution was chosen at hand.
- TEB: Total Evaluation Budget.

True function: performance profiles for 18 instances



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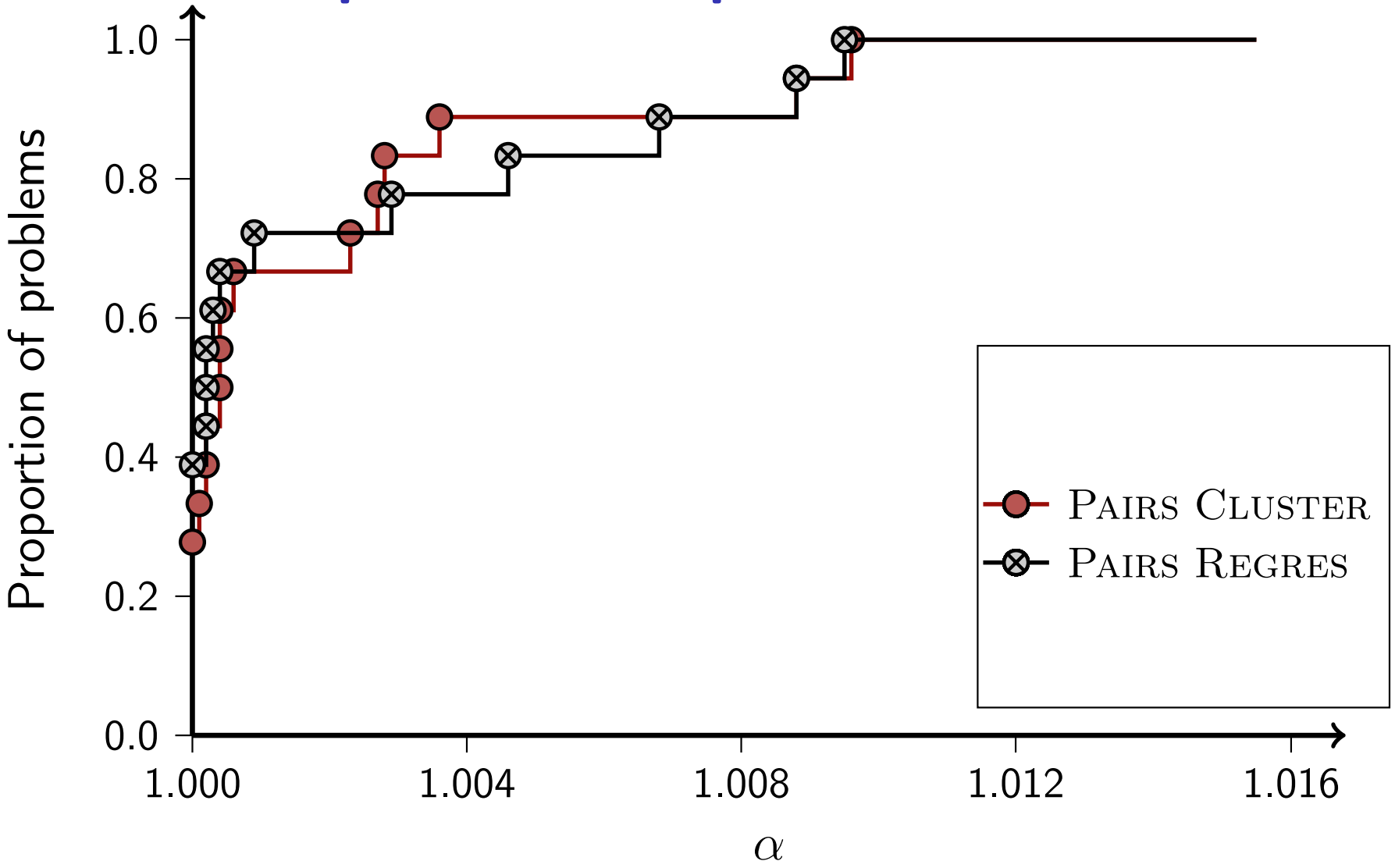

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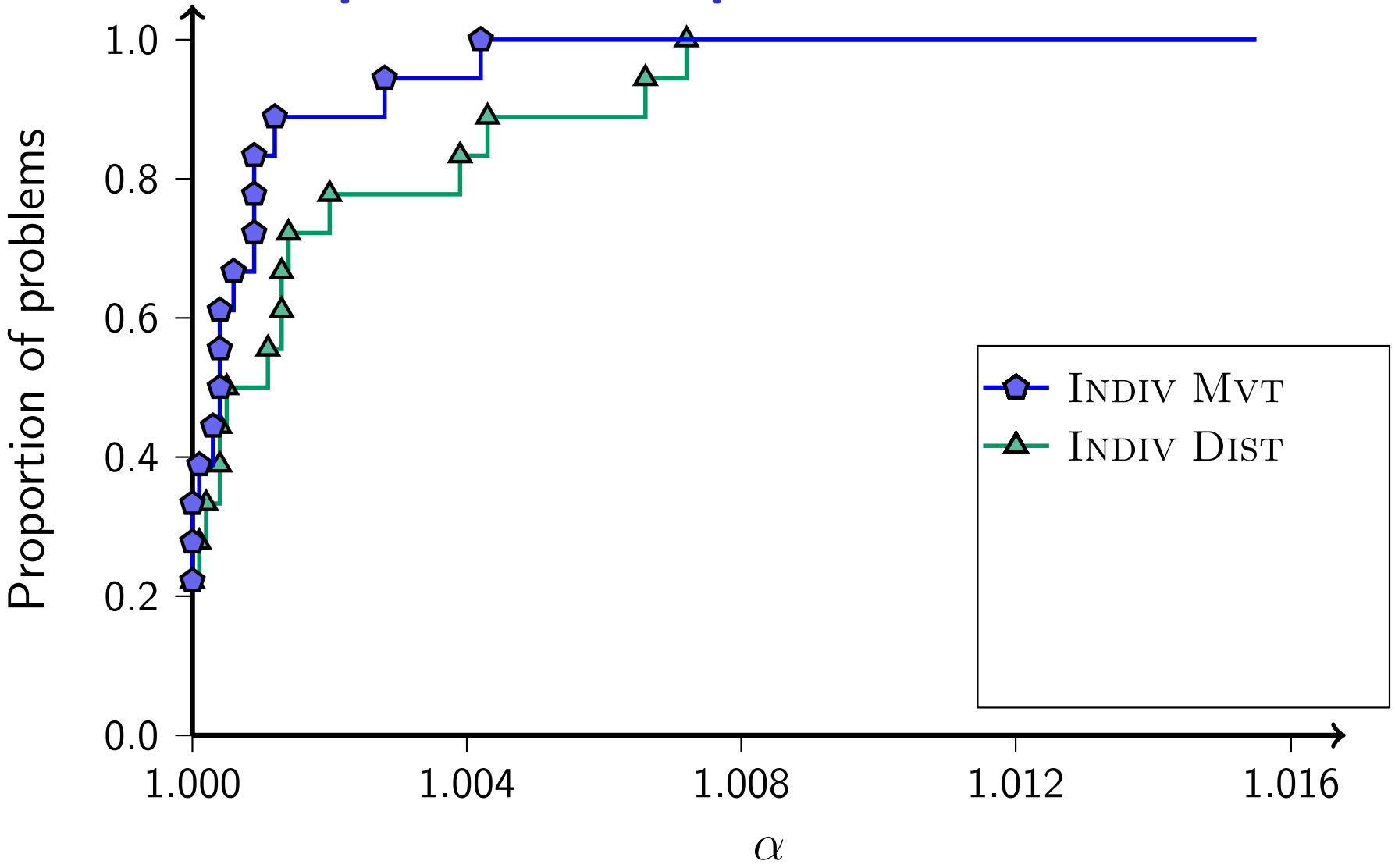

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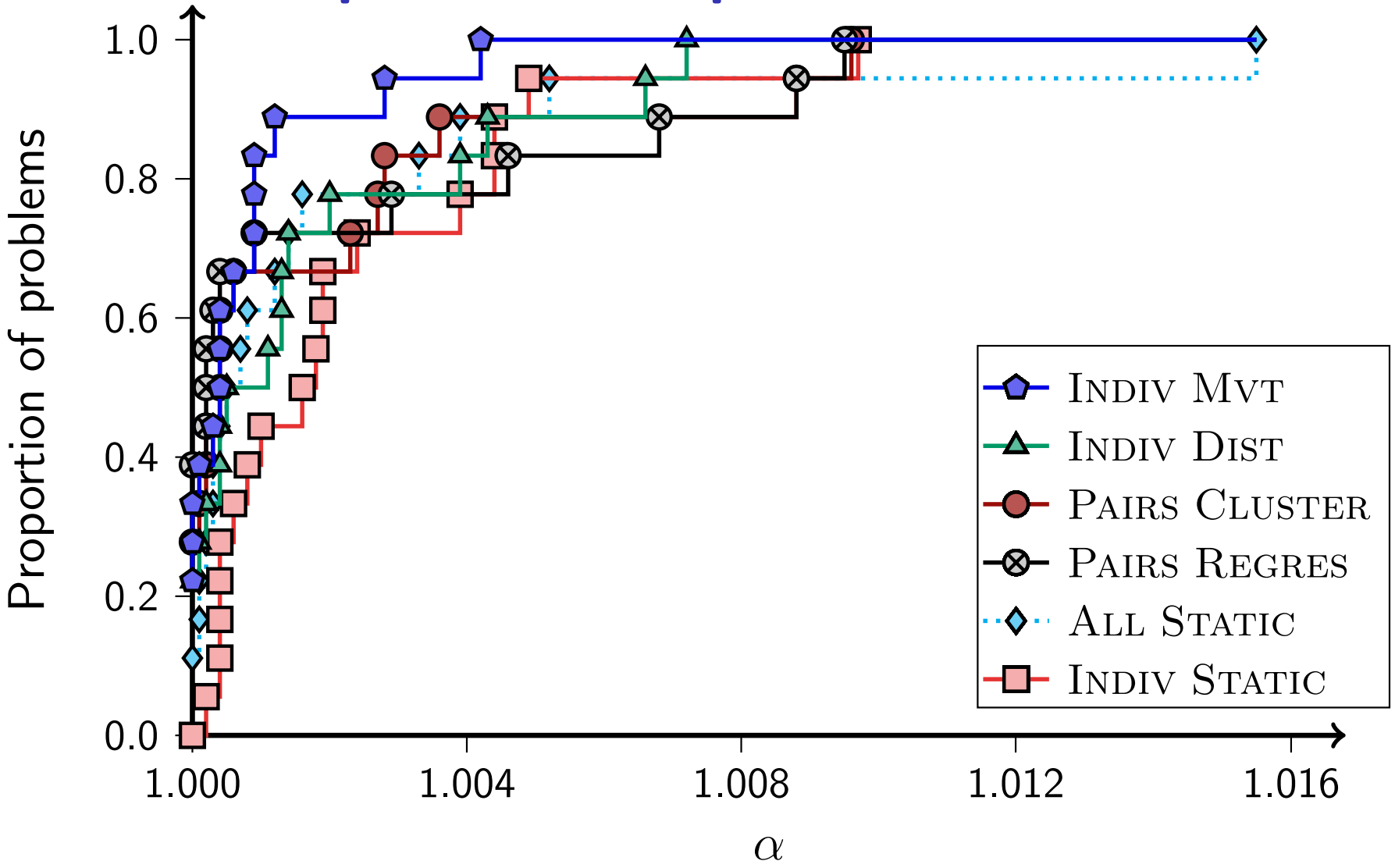

True function: summary of results

		STM	GAT	LG	Mean	
INDIV	MVT	6.28	4.77	4.34	5.13	
INDIV	DIST	6.23	4.63	4.25	5.04	
PAIRS	CLUSTER	6.10	4.70	4.28	5.03	
PAIRS	REGRES	6.15	4.58	4.32	5.02	
ALL	STATIC	6.26	4.50	4.27	5.01	= MADS
INDIV	STATIC	6.08	4.58	4.28	4.98	= CS

- The INDIV MVT strategy dominates all others.
- Dynamically regrouping the variables is preferable than either moving individual variables, or moving all variables simultaneously.
- Even if the variables represent locations in $\mathbb{R}^2$, pairing them initialy does not appear to be beneficial.

Discussion

- An industrial application: find GMON locations that minimize the SWE estimation error obtained by a blackbox simulator.
- A modification of the MADS algorithm using groups of variables was developed.
- Extensive testing on a surrogate function.
- Numerical experiments suggest that dynamically regrouping the variables improves the quality of the final solution compared to the standard MADS and CS methods.
- Some strategies developed in this work are specific to positioning problems, other are generic.
- This is the third application of MADS to location problems. Previous work on wells and tsunami buoy positioning.