Lower Bounding and Tabu Search Procedures for the Frequency Assignment Problem with Polarization Constraints

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Abstract

The problem retained for the ROADEF'2001 international challenge was a frequency assignment problem with polarization constraints (FAPP). This NP-hard problem was proposed by the *Centre d'Electronique de l'Armement de la Délégation Générale pour l'Armement* of the French Department of Defense, within the context of the CALMA project (Combinatorial ALgorithms for Military Applications). Twenty seven competitors took part to this contest, and we present in this paper the contribution of our team that allowed us to be selected as one of the six finalists qualified for the final round of the competition.
There is typically no solution satisfying all constraints of the FAPP. For this reason, some electromagnetic compatibility constraints can be progressively relaxed, and the objective is to find a feasible solution with the lowest possible level of relaxation. We have developed a procedure that computes a lower bound on the best possible level of relaxation, as well as two tabu search algorithms for the FAPP, one for the frequency assignment and one for the polarization assignment.
Keywords: assignment problems, tabu search, lower bounds

1. Introduction

The ever-increasing demand for communication, coupled with the limited spectra available, have made frequency assignment more and more difficult to accomplish effectively. Optimization of this process has therefore become a major issue for network administration and deployment, both civil and military. One role of the CELAR (Centre d'Electronique de l'ARmement) is to study new methods for optimizing the use of the available spectra for the French army. The ROADEF'2001 International Challenge (see [1] or the web site [2] for more details) was devoted to a frequency assignment problem in Hertzian telecommunication networks. This problem was first studied within the context of the CALMA project (Combinatorial ALgorithms for Military Applications) of the CELAR and further enriched so as to take polarizations into account as well as a controlled relaxation of certain electromagnetic compatibility constraints.
The Hertzian telecommunication network consists of a set of sites in which transmission equipments (antennae connected to emitters or receptors) are located. An

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Hertzian connection joins two geographic sites; it may consist of one or more paths. A *path* is a uni-directional radio-electric bond, established between antennae at distinct sites, with a given frequency and polarization. A frequency resource is defined as a pair (frequency, polarization) in which the components are respectively associated to the carrying frequency of the transmitted signal and the wave polarization. To simplify the model, the polarization is supposed to be a binary variable (e.g. only vertical or horizontal).

Let T denote the set of paths. A set of available resources is defined for each path $t \in T$: it is composed on the one hand of the frequency domain, referred to as $F_{\varphi(t)}$, and on the other hand of the polarization information $P_{\pi(t)}$. Two paths may have identical domains. The frequency assignment problem with polarization constraints (FAPP) consists in assigning a frequency $f_t \in F_{\varphi(t)}$ and a polarization $p_t \in P_{\pi(t)}$ to each path $t \in T$ while satisfying the following radio-electric compatibility constraints that are of three different types.

Let t and t' be two paths. The constraints of the first type impose that the absolute value of the difference between f_t and $f_{t'}$ must be equal to or different from a given value : $\mid f_t - f_{t'} \mid = \varepsilon_{tt'}$ or $\mid f_t - f_{t'} \mid \neq \varepsilon_{tt'}$ for some given $\varepsilon_{tt'}$. In particular, if $\varepsilon_{tt'} = 0$, then it is imposed that t and t' must have the same or a different frequency. The constraints of the second type impose that p_t must be equal to or different from $p_{t'}$: $p_t = p_{t'}$ or $p_t \neq p_{t'}$. Finally, there are electromagnetic compatibility constraints (called CEM constraints) imposing a minimum gap between f_t and $f_{t'}$. This gap depends on the polarizations as follows:

$$\mid f_t - f_{t'} \mid \geq \begin{cases} \gamma_{tt'} & \text{if} \quad p_t = p_{t'} \\ \delta_{tt'} & \text{if} \quad p_t \neq p_{t'} \end{cases}$$

where $\gamma_{tt'}$ and $\delta_{tt'}$ are two given numbers such that $\gamma_{tt'} \geq \delta_{tt'}$. The constraints of the first two types are considered as hard constraints and can therefore not be violated. There are typically no solutions satisfying all constraints of the FAPP. The CEM constraints are therefore progressively relaxed as follows. Eleven *relaxation levels* are defined, level zero corresponding to no relaxation, and level 11 to a complete relaxation of a CEM constraint. A CEM constraint is considered as satisfied at level k $(0 \leq k \leq 11)$ if

$$\mid f_t - f_{t'} \mid \geq \begin{cases} \gamma_{tt'}^{(k)} & \text{if} \quad p_t = p_{t'} \\ \delta_{tt'}^{(k)} & \text{if} \quad p_t \neq p_{t'} \end{cases}$$

where $\gamma_{tt'} = \gamma_{tt'}^{(0)} \geq \gamma_{tt'}^{(1)} \geq \ldots \geq \gamma_{tt'}^{(11)} = 0$, $\delta_{tt'} = \delta_{tt'}^{(0)} \geq \delta_{tt'}^{(1)} \geq \ldots \geq \delta_{tt'}^{(11)} = 0$, and $\gamma_{tt'}^{(k)} \geq \delta_{tt'}^{(k)}$ for all $k \in \{0, 1, \ldots, 11\}$.

A *k-feasible solution* is an assignment satisfying the constraints of the first two types as well as those of the third type at level k. The FAPP is said *k-feasible* if it has a k-feasible solution. The FAPP is a hierarchical optimization problem. Its objectives are, by order of priority :

1. to search for the lowest relaxation level k_{opt} for which a k_{opt}-feasible solution exists;
2. to minimize the number of CEM constraints not satisfied at level $k_{opt} - 1$;
3. to minimize the number of CEM constraints not satisfied at levels smaller than $k_{opt} - 1$.

The organizers of the challenge have generated three sets of benchmark problems. A first set A of 15 instances (problems fapp01 to fapp15) has been used to select 8 among the 27 competitors, for a second round. On the basis of the results obtained on the second set B of 15 instances (problems fapp16 to fapp30), six competitors were qualified for the final round that took place at FRANCORO III in Québec (CANADA) in May 2001. The algorithms developed by each of the six finalists were finally tested on a set X of 10 instances (problems fapp31 to fapp40). Our team was classified second in the "Junior" category, while the winner was Michel Vasquez whose contribution is described in [7].
This paper describes the contribution of our team to the ROADEF'2001 challenge. We first give a mathematical formulation of the FAPP in section 2. We then describe in section 3 a procedure that we have developed for the computation of a lower bound on the lowest possible level k_{opt} of relaxation. This lower bounding procedure will help us reducing the size of some frequency domains. We then describe in section 4 a constructive algorithm that builds an admissible solution for the FAPP. Section 5 starts with a description of a basic tabu search, and we then propose two adaptations of this method, one for the polarization assignment (thus fixing the CEM constraints), and one for the frequency assignment. Experimental results are given in section 6.

2. Mathematical formulation

In this section, we give a mathematical formulation of the FAPP. Remember first that there are only two possible values for the polarization of a path. We fix these two values to 1 and -1. This means that the polarization domain of a path $t \in T$ is either equal to $P_{-1} = \{-1\}$ (if polarization -1 is imposed), to $P_1 = \{1\}$ (if polarization 1 is imposed) or to $P_0 = \{-1, 1\}$. We denote $F_0, F_1, \ldots, F_N$ the different frequency domains (that contain positive integers) and we assume that we are given two functions φ and π which associate a frequency domain $F_{\varphi(t)}$ and a polarization domain $P_{\pi(t)}$ to each path $t \in T$. The constraints linking two paths t and t' are of one of the following types:

$$\left\{\begin{array}{rcll} \mid f_t - f_{t'} \mid & \geq & \frac{|p_t+p_{t'}|}{2}\gamma_{tt'}^{(k)} + \frac{|p_t-p_{t'}|}{2}\delta_{tt'}^{(k)} & \quad\textbf{CEM}(k) \\ \mid f_t - f_{t'} \mid & = & \varepsilon_{tt'} & \quad\textbf{C2} \\ \mid f_t - f_{t'} \mid & \neq & \varepsilon_{tt'} & \quad\textbf{C3} \\ f_t & = & f_{t'} & \quad\textbf{C4} \\ f_t & \neq & f_{t'} & \quad\textbf{C5} \\ p_t & = & p_{t'} & \quad\textbf{C6} \\ p_t & \neq & p_{t'} & \quad\textbf{C7} \end{array}\right.$$

where CEM(k) represents a CEM constraint at relaxation level k. We define a solution s of the FAPP as a set of $\mid T \mid$ couples $(f_t, p_t) \in F_{\varphi(t)} \times P_{\pi(t)}$. A solution is said *admissible* if it satisfies all constraints C2, C3, ..., C7, while it is said *k-feasible* if it is admissible and satisfies all CEM(k) constraints. To each admissible solution s, we associate a relaxation level $k(s)$ corresponding to the the smallest positive

integer such as s is $k(s)$-feasible. Remember that $\gamma_{tt'}^{(11)} = \delta_{tt'}^{(11)} = 0$, which means that $k(s) \leq 11$.
Now let $\Theta = \left|\left\{(i,j) : \gamma_{ij}^{(0)} + \delta_{ij}^{(0)} > 0\right\}\right|$ denote the total number of CEM constraints. For each positive integer $k \in \{0, 1, \ldots, k(s)-1\}$, let $V^{(k)}(s)$ denote the set of CEM(k) constraints that s violates. Notice that if a constraint belongs to a set $V^{(k)}(s)$, then it also belongs to the sets $V^{(0)}(s), \ldots, V^{(k-1)}(s)$, The objective of the FAPP is to minimize

$$f(s) = 10 \cdot k(s) \cdot \Theta^2 + 10 \cdot \left|V^{(k(s)-1)}(s)\right| \cdot \Theta + \sum_{i<k(s)-1} \mid V^{(i)}(s) \mid .$$

Let $f_a(s), f_b(s)$ and $f_c(s)$ denote the three terms of the above function f (i.e., $f(s) = f_a(s) + f_b(s) + f_c(s)$). The following property can easily be proved (see [8] for more details) and this shows that f respects the hierarchy of the objectives mentioned in the introduction.

Properties. Let s and s' be two admissible solutions. Then

(a) $f_a(s) < f_a(s') \Rightarrow f(s) < f(s')$
(b) $f_a(s) = f_a(s'), f_b(s) < f_b(s') \Rightarrow f(s) < f(s')$
(c) $f_a(s) = f_a(s'), f_b(s) = f_b(s'), f_c(s) < f_c(s') \Rightarrow f(s) < f(s')$

3. Preprocessing

We describe in this section a preprocessing procedure which reduces the size of the frequency domains of the paths, and computes a lower bound k_{inf} on the best possible relaxation level k_{opt}.
A frequency f for a path t is defined as *k-useless* if the FAPP has no k-feasible solution in which t has frequency f. It would be interesting to eliminate all k_{opt}-useless frequencies for each path t. However, k_{opt} is not known, and we therefore compute a lower bound k_{inf} on k_{opt}, and eliminate all k_{inf}-useless frequencies. The lower bound k_{inf} is initially set equal to 0, and is increased by one unit each time we can prove that the FAPP is not k_{inf}-feasible. In order to prove that the FAPP is not k_{inf}-feasible, we associate to each path t a frequency domain, denoted Dom_t, which is initially set equal to $F_{\varphi(t)}$, and which is then reduced by successively removing k_{inf}-useless frequencies. If a domain Dom_t of a path t becomes empty, this proves that no k_{inf}-feasible solution exists. Otherwise, the FAPP is possibly k_{inf}-feasible and the reduced sets Dom_t contain frequencies that are not k_{inf}-useless.
Observe that two paths t and t' with $F_{\varphi(t)} = F_{\varphi(t')}$ will probably not have the same reduced sets Dom_t and $Dom_{t'}$. More importantly, notice that it may happen that $k_{inf} < k_{opt}$. In such a case, we cannot guarantee that the FAPP has a k_{opt}-feasible solution if we restrict the frequency domains to the sets Dom_t instead of $F_{\varphi(t)}$. Indeed, k_{inf}-useless frequencies are not necessarily k_{opt}-useless. However, since the

number of admissible solutions is much smaller when considering the reduced sets Dom_t instead of $F_{\varphi(t)}$, we have decided to remove all k_{inf}-useless frequencies. Notice that according to our experiments, (see Table 1) k_{inf} is very often equal to k_{opt}. We now describe three techniques to determine k-useless frequencies.

3.1 Reductions based on C2 and C4 constraints

The first procedure called REDUCTION-C2C4 reduces the frequency domains by taking into account C2 and C4 constraints. The procedure is described in Fig.1 and illustrated on the following example.

Let t and t' be two paths with frequency domains $F_{\varphi(t)} = \{1,2,3,4\}$ and $F_{\varphi(t')} = \{3,4,5,6\}$, respectively. Assume that $\mid f_t - f_{t'} \mid$ must be equal to 1 (a C2 constraint). In that case, we easily see that these frequency domains can be reduced to $Dom_t = \{2,3,4\}$ and $Dom_{t'} = \{3,4,5\}$.

The input of procedure REDUCTION-C2C4 is a set of frequency domains for each path. It returns new frequency domains as well as an answer which can be "NO" or "MAY BE" : the answer is "NO" if and only if we have proved that the FAPP has no solution satisfying all C2 and C4 constraints.

Procedure REDUCTION-C2C4($Dom_1, \ldots, Dom_{|T|}$)

Set $continue = 1$ and REDUCTION-C2C4($Dom_1, \ldots, Dom_{|T|}$) = "MAY BE"

While $continue = 1$, do

1. set $continue = 0$
2. for all constraint $(t, t') \in$ C2 $\cup$ C4, do
   - $\forall f \in Dom_t$, remove f from Dom_t if $\nexists f' \in Dom_{t'}$ satisfying constraint (t, t')
   - $\forall f' \in Dom_{t'}$, remove f' from $Dom_{t'}$ if $\nexists f \in Dom_t$ satisfying constraint (t, t')
3. if a frequency has been removed from a domain in step 2, then set $continue = 1$

If $\exists$ a path t with $Dom_t = \emptyset$, then set REDUCTION-C2C4($Dom_1, \ldots, Dom_{|T|}$) = "NO"

Figure 1 : Reduction procedure based on C2 and C4 constraints

3.2 Reductions based on CEM constraints

In this section, we describe two reduction procedures based on CEM constraints. A CEM constraint concerning two paths t and t' is less restrictive when t and t' have different polarizations. In order to determine k-useless frequencies, we therefore assume that each CEM constraint is defined with $\delta_{tt'}^{(k)}$, unless it is imposed by C6 and C7 constraints, or by the polarization domains, that t and t' must have the same polarization.

Consider now a CEM constraint involving paths t and t'. If there is no frequency for t' which satisfies this constraint at relaxation level k with frequency f for t, then f is k-useless for t. The procedure, called REDUCTION1-CEM, performs this kind of reduction of the frequency domains. It is described in Fig.2 and illustrated on the following example. Consider two paths t and t' with frequency domains $\{5, \ldots, 16\}$

and $\{1,\ldots,12\}$, respectively. Assume that it is not imposed that t and t' should have the same polarization, and let $\delta_{tt'}^{(k)} = 9$. In that case, frequency 5 cannot be assigned to t for getting a k-feasible solution, since $\mid f - 5 \mid < 9$, $\forall f \in \{1,\ldots,12\}$. The same remark holds for frequencies 6, 7, 8 and 9 for t. In a similar way, frequencies 8 to 12 are k-useless for t'. Hence, the frequency domains for t and t' can be reduced to $\{10,\ldots,16\}$ and $\{1,\ldots,7\}$, respectively.
The input of procedure REDUCTION1-CEM is a set of frequency domains for each path as well as a relaxation level k. It returns new frequency domains and an answer which can be "NO" or "MAY BE" : the answer is "NO" if and only if we have proved that FAPP has no k-feasible solution.

Procedure REDUCTION1-CEM($k; Dom_1,\ldots,Dom_{|T|}$)

Set $continue = 1$ and REDUCTION1-CEM($k; Dom_1,\ldots,Dom_{|T|}$) = "MAY BE"

While $continue = 1$, do

1. set $continue = 0$
2. for all CEM constraints (t,t'), do
   - if C6 and C7 constraints or the polarization domains impose that t and t' must have the same polarization, then set $\alpha = \gamma_{tt'}^{(k)}$; else set $\alpha = \delta_{tt'}^{(k)}$
   - $\forall f \in Dom_t$, remove f from Dom_t if $\nexists f' \in Dom_{t'}$ such that $\mid f - f' \mid \geq \alpha$
   - $\forall f' \in Dom_{t'}$, remove f' from $Dom_{t'}$ if $\nexists f \in Dom_t$ such that $\mid f - f' \mid \geq \alpha$
3. if a frequency has been removed from a domain in step 2, then set $continue = 1$

If $\exists$ a path t with $Dom_t = \emptyset$, then set REDUCTION1-CEM($k; Dom_1,\ldots,Dom_{|T|}$) = "NO"

Figure 2 : First reduction procedure based on CEM constraints

The second reduction procedure based on CEM constraints, called REDUCTION2-CEM, considers triplets (t,t',t'') of paths involved pairwise in CEM constraints. At least two paths among t,t' and t'' must have the same polarization (since -1 and 1 are the unique possible polarization values). Hence, we consider four possible situations: $p_t = p_{t'} \neq p_{t''}$, $p_{t'} = p_{t''} \neq p_t$, $p_t = p_{t''} \neq p_{t'}$ and $p_t = p_{t'} = p_{t''}$. For each such situation, the CEM constraints involving t,t' and t'' are precisely defined (i.e., we know whether γ or δ should be used). Notice that some of the four above situations may be forbidden by C6 and C7 constraints or by polarization domains. For each possible situation, the frequencies f for t, f' for t' and f'' for t'' can be ordered in the six following ways : $f \leq f' \leq f''$, $f \leq f'' \leq f'$, $f' \leq f \leq f''$, $f' \leq f'' \leq f$, $f'' \leq f \leq f'$, $f'' \leq f' \leq f$. Hence, we have a total of $4 \cdot 6 = 24$ cases to examine. If given a frequency f for t there are no frequencies f' for t' and f'' for t'' which satisfy the CEM constraints at level k in at least one of the 24 above mentioned cases, then f is k-useless for t. This way of reducing the frequency domains is precisely described in Fig.3.

Procedure REDUCTION2-CEM($k; Dom_1, \ldots, Dom_{|T|}$)

Set $continue = 1$and REDUCTION2-CEM($k; Dom_1, \ldots, Dom_{|T|}$) = "MAY BE"

While $continue = 1$, do

1. set $continue = 0$
2. for each CEM constraint (t, t'), do
   - if $\min_{\hat{f} \in Dom_t} \hat{f} + \gamma^{(k)}_{tt'} > \max_{\hat{f} \in Dom_{t'}}$ and $\min_{\hat{f} \in Dom_{t'}} \hat{f} + \gamma^{(k)}_{tt'} > \max_{\hat{f} \in Dom_t}$, introduce constraint $p_t \neq p_{t'}$ in the constraint set C7
   - if the polarization domains of p_t and $p_{t'}$ impose that $p_t = p_{t'}$, but if $p_t \neq p_{t'}$ is a C7 constraint, then set REDUCTION2-CEM($k; Dom_1, \ldots, Dom_{|T|}$) = "NO" and STOP
3. for each path t such that $\exists t', t''$ where t, t' and t'' are pairwise involved in a CEM constraint, and for all f in Dom_t, do
   - set $useless = 1$
   - if C6 and C7 constraints or the polarization domains do not forbid $p_t = p_{t'} \neq p_{t''}$, then set $\Delta_{tt'} = \gamma^{(k)}_{tt'}, \Delta_{tt''} = \delta^{(k)}_{tt''}, \Delta_{t't''} = \delta^{(k)}_{t't''}$ and apply CHECK-USELESS
   - if C6 and C7 constraints or the polarization domains do not forbid $p_t = p_{t''} \neq p_{t'}$, then set $\Delta_{tt'} = \delta^{(k)}_{tt'}, \Delta_{tt''} = \gamma^{(k)}_{tt''}, \Delta_{t't''} = \delta^{(k)}_{t't''}$ and apply CHECK-USELESS
   - if C6 and C7 constraints or the polarization domains do not forbid $p_{t'} = p_{t''} \neq p_t$, then set $\Delta_{tt'} = \delta^{(k)}_{tt'}, \Delta_{tt''} = \delta^{(k)}_{tt''}, \Delta_{t't''} = \gamma^{(k)}_{t't''}$ and apply CHECK-USELESS
   - if C6 and C7 constraints or the polarization domains do not forbid $p_t = p_{t'} = p_{t''}$, then set $\Delta_{tt'} = \gamma^{(k)}_{tt'}, \Delta_{tt''} = \gamma^{(k)}_{tt''}, \Delta_{t't''} = \gamma^{(k)}_{t't''}$ and apply CHECK-USELESS
   - if $useless = 1$, then remove f from Dom_t and set $continue = 1$

If $\exists\, t \in T$ such that $Dom_t = \emptyset$, then set REDUCTION2-CEM($k; Dom_1, \ldots, Dom_{|T|}$) = "NO"

Procedure CHECK-USELESS : 6 cases to check

(a) : $f \leq f' \leq f''$: if $f + \Delta_{tt'} \leq \max_{\hat{f} \in Dom_{t'}} \hat{f}$ and $f + \max\{\Delta_{tt'} + \Delta_{t't''}, \Delta_{tt''}\} \leq \max_{\hat{f} \in Dom_{t''}} \hat{f}$, set $useless = 0$

(b) : $f \leq f'' \leq f'$: if $f + \Delta_{tt''} \leq \max_{\hat{f} \in Dom_{t''}} \hat{f}$ and $f + \max\{\Delta_{tt''} + \Delta_{t't''}, \Delta_{tt'}\} \leq \max_{\hat{f} \in Dom_{t'}} \hat{f}$, set $useless = 0$

(c) : $f' \leq f \leq f''$: if $\min_{\hat{f} \in Dom_{t'}} \hat{f} + \Delta_{tt'} \leq f$ and $f + \Delta_{tt''} \leq \max_{\hat{f} \in Dom_{t''}} \hat{f}$, set $useless = 0$

(d) : $f' \leq f'' \leq f$: if $\min_{\hat{f} \in Dom_{t'}} \hat{f} + \max\{\Delta_{t't''} + \Delta_{tt''}, \Delta_{tt'}\} \leq f$ and $\min_{\hat{f} \in Dom_{t''}} \hat{f} + \Delta_{tt''} \leq f$, set $useless = 0$

(e) : $f'' \leq f \leq f'$: if $\min_{\hat{f} \in Dom_{t''}} \hat{f} + \Delta_{tt''} \leq f$ and $f + \Delta_{tt'} \leq \max_{\hat{f} \in Dom_{t'}} \hat{f}$, set $useless = 0$

(f) : $f'' \leq f' \leq f$: if $\min_{\hat{f} \in Dom_{t''}} \hat{f} + \max\{\Delta_{t't''} + \Delta_{tt'}, \Delta_{tt''}\} \leq f$ and $\min_{\hat{f} \in Dom_{t'}} \hat{f} + \Delta_{tt'} \leq f$, set $useless = 0$

Figure 3 : Second reduction procedure based on CEM constraints

This procedure is now illustrated with the following example. Consider three paths t, t' and t'' with frequency domains $\{1, 2, 3\}$, $\{6, \ldots, 10\}$ and $\{9, \ldots, 13\}$, respectively. Assume that it is not imposed that t, t' or t'' should have the same polarization, and consider the values $\gamma^{(k)}_{tt'} = 16$, $\gamma^{(k)}_{tt''} = 12$, $\gamma^{(k)}_{t't''} = 18$, $\delta^{(k)}_{tt'} = 5$, $\delta^{(k)}_{tt''} = 4$ and $\delta^{(k)}_{t't''} = 3$. Notice first that procedure REDUCTION1-CEM is not able to reduce any of the three frequency domains. We now show how frequency $f = 2$ can be removed from the frequency domain of t. Frequencies f, f' and f'' can only be ordered in two different ways : $f \leq f' \leq f''$ and $f \leq f'' \leq f'$. Paths t' and t'' cannot have the same polarization since in such a case $| f' - f'' |$ should be larger or equal to $\gamma^{(k)}_{t't''} = 18$, which is impossible. Also, p_t and $p_{t'}$ must be different else $f' - f$ is too large. Hence the unique possible case is $p_t = p_{t''} \neq p_{t'}$. Now, frequency 2 is k-useless for t since $2 + \gamma^{(k)}_{tt''} = 14$, which is too large for f''. In a similar way, we can see that frequency 3 is k-useless for t, and frequencies $9, \ldots, 12$ are k-useless for t''. Hence the new reduced frequency domains for t, t' and t'' are $\{1\}, \{6, \ldots, 10\}$ and $\{13\}$, respectively.

3.3 Computation of the lower bound on k_{opt}

We can now describe the procedure that computes a lower bound k_{inf} on the best relaxation level k_{opt}. This procedure uses the three reduction procedures detailed in sections 3.1 and 3.2. In order to determine k_{inf}, we first apply successively each reduction procedure with $k = 0$. If the returned answer is "NO" at least one time, this means that there is no k-feasible solution, and we restart the process by setting $k = k + 1$, and so on until each procedure returns "MAY BE". The final value of k gives the value k_{inf} of the lower bound on k_{opt}. Moreover, the final frequency domains Dom_t produced as output do not contain any k_{inf}-useless frequency. This process is described in Fig.4.

Procedure LOWER-BOUND

Set $continue = 1$ and $k = -1$

While $continue = 1$, do

  set $continue = 0$, $k = k + 1$ and $stop = 0$

  set $Dom_t = F_{\varphi(t)}, \forall t \in T$

  while $stop = 0$, do

  - set $stop = 1$
  - apply REDUCTION-C2C4($Dom_1, \ldots, Dom_{|T|}$)
  - apply REDUCTION1-CEM($k; Dom_1, \ldots, Dom_{|T|}$)
  - apply REDUCTION2-CEM($k; Dom_1, \ldots, Dom_{|T|}$)
  - if at least one of the three procedures returns "NO", then set $continue = 1$ and $stop = 1$
  - else if at least one of the three procedures returns "MAY BE" and at least one of the three procedures has modified a frequency domain, then set $stop = 0$

Set $k_{inf} = k$.

Figure 4 : Computation of a lower bound on k_{opt}

The quality of the lower bound k_{inf}, as well as the size of the reduced frequency domains are analysed in Table 1. For each problem, we first give three values k_{opt}, k'_{inf} and k_{inf}, where k'_{inf} is the lower bound on k_{opt} obtained without using the third reduction procedure. The value of k_{opt} was given to us by the organizers of the challenge. We have also measured the average ratio $\frac{|Dom_t|}{|F_{\varphi(t)}|}$ over all paths t. To demonstrate the effectiveness of the three reduction procedures, we indicate this average ratio when using only the first procedure (line labelled C2C4), the two first procedures (line labelled C2C4,1-CEM), and all three procedures (line labelled C2C4,1-CEM,2-CEM). It can be observed that, in some cases, up to 90% of the frequencies are k_{inf}-useless. Notice also that when $k'_{inf} < k_{inf}$ (for example on fapp01 and fapp31), the above ratio is smaller when using the two first procedures instead of the three proposed ones. This is due to the fact that k'_{inf}-useless frequencies are not necessarily k_{inf}-useless. Notice finally that our reduction procedures are able to find the optimal level of feasibility (i.e., $k_{inf} = k_{opt}$) for all instances except fapp05 and fapp39.

Instance	fapp01	fapp02	fapp03	fapp04	fapp05	fapp06	fapp07	fapp08
$k_{opt}, k'_{inf}, k_{inf}$	4,3,4	2,2,2	7,7,7	1,1,1	11,8,8	5,5,5	9,9.9	5,5,5
C2C4	0.987	0.999	1	0.991	1	0.996	0.994	1
C2C4,1-CEM	0.566	0.448	0.628	0.374	0.707	0.555	0.713	0.663
C2C4,1-CEM,2-CEM	0.579	0.425	0.544	0.366	0.705	0.477	0.691	0.646
Instance	fapp09	fapp10	fapp11	fapp12	fapp13	fapp14	fapp15	fapp16
$k_{opt}, k'_{inf}, k_{inf}$	3,3,3	6,6,6	8,8,8	2,2,2	3,3,3	4,4,4	5,5,5	11,11,11
C2C4	0.994	1	0.999	1	0.999	0.999	0.998	1
C2C4,1-CEM	0.580	0.704	0.565	0.399	0.495	0.470	0.618	1
C2C4,1-CEM,2-CEM	0.559	0.659	0.552	0.374	0.485	0.454	0.612	1
Instance	fapp17	fapp18	fapp19	fapp20	fapp21	fapp22	fapp23	fapp24
$k_{opt}, k'_{inf}, k_{inf}$	4,4,4	8,8,8	6,6,6	10,10,10	4,4,4	7,7,7	9,9,9	7,7,7
C2C4	1	0.996	0.994	1	1	0.996	0.992	0.999
C2C4,1-CEM	0.017	0.03	0.014	0.024	0.067	0.015	0.011	0.016
C2C4,1-CEM,2-CEM	0.017	0.03	0.014	0.024	0.067	0.015	0.011	0.016
Instance	fapp25	fapp26	fapp27	fapp28	fapp29	fapp30	fapp31	fapp32
$k_{opt}, k'_{inf}, k_{inf}$	3,3,3	7,7,7	5,5,5	3,3,3	6,6,6	7,7,7	5,3,5	6,6,6
C2C4	0.997	0.999	0.999	0.999	0.998	0.999	1	0.999
C2C4,1-CEM	0.051	0.020	0.159	0.037	0.011	0.039	0.656	0.023
C2C4,1-CEM,2-CEM	0.051	0.020	0.159	0.037	0.011	0.026	0.649	0.023
Instance	fapp33	fapp34	fapp35	fapp36	fapp37	fapp38	fapp39	fapp40
$k_{opt}, k'_{inf}, k_{inf}$	5,5,5	4,4,4	6,6,6	7,7,7	5,5,5	3,3,3	3,2,2	4,4,4
C2C4	0.995	1	0.999	1	1	0.999	1	0.999
C2C4,1-CEM	0.04	0.069	0.035	0.043	0.032	0.017	0.656	0.023
C2C4,1-CEM,2-CEM	0.04	0.060	0.035	0.043	0.032	0.017	0.649	0.023

Table 1: Results of the lower bounding procedure

4. Construction of an admissible solution

In order to build an admissible solution, we first assign a polarization to each path, and we then choose a frequency for each path, without taking into account the CEM constraints. The algorithm given in Fig.5 is a constructive procedure that assigns a polarization to each path while satisfying C6 and C7 constraints. The paths are considered in a random order. Let A be the subset of paths for which the polarization has already been fixed, and let B be the set of paths t which do not have a polarization and such that $\mid P_{\pi(t)} \mid = 1$. At each iteration, if $B \neq \emptyset$, we randomly choose a path $t \in B$ and we fix its polarization p_t (there is only one possibility). Then, using the propagation of constraints C6 and C7, we define the set I_t of paths (including t) for which the polarization is fixed by p_t. For each path $t' \in I_t$, let $p_{t \to t'}$ be the polarization imposed by t on t' when p_t is assigned to t. If all paths $t' \in I_t$ are such that $p_{t \to t'} \in P_{\pi(t')}$, we fix the polarization of each $t' \in I_t$ to $p_{t \to t'}$, and we put every such path in A. Else, the procedure stops with a message indicating that the FAPP does not have any admissible solution. Notice that when we fix the polarization of a path $t \in B$, we possibly reduce the polarization domain of other paths $t' \in I_t$. For example, if paths t and t' respectively have $P_1 = \{1\}$ and $P_0 = \{-1, 1\}$ as polarization domains, and if a C6 constraint links t to t', then we associate the polarization domain P_1 to t', which means that function π is modified in order to reduce the size of the polarization domain of some other paths. When (or if) $B = \emptyset$, we randomly choose a path $t \notin A$, we assign a polarization p_t randomly chosen in $\{-1, 1\}$ to t, and we continue as described above by propagating C6 and C7 constraints in order to fix the polarization of the paths in I_t. This random choice is possible without loss of generality because the polarization domains have been reduced when B was not empty.

The input of function CHECK-POL described below is the current set A of paths to which a polarization has already been assigned as well as a path $t \notin A$ and a polarization $p_t \in P_{\pi(t)}$. This function returns value 0 if p_t cannot be assigned to t without violating a C6 or a C7 constraint. Otherwise, it returns value 1 and updates the set A by assigning polarization $p_{t \to t'}$ to all $t' \in I_t$. Moreover, if $t \in B$, the polarization domain of each $t' \in I_t$ is reduced to $P_{\pi(t')} = \{p_{t \to t'}\}$.

Procedure INIT-POL

1. Set $A = \emptyset$ and determine B
2. While $B \neq \emptyset$, do
   - (a) randomly choose $t \in B$
   - (b) assign polarization p_t to t (only one possibility)
   - (c) if CHECK-POL$(A, t, p_t) = 0$, then STOP : there is no admissible solution
   - (d) set $B = B - A$
3. While $A \neq T$, do
   - (a) randomly choose $t \in T - A$
   - (b) randomly choose a polarization $p_t \in P_{\pi(t)}$ for t
   - (c) apply CHECK-POL(A, t, p_t) (value 0 can not be returned, because $B = \emptyset$)

Function CHECK-POL(A, t, p_t)

1. Set CHECK-POL$(A, t, p_t) = 1$
2. Determine the set I_t of paths (including t) for which the polarizations are imposed by the one of t, and for each $t' \in I_t$, let $p_{t \to t'}$ be the polarization imposed by t to t' when p_t is assigned to t
3. If $p_{t \to t'} \notin P_{\pi(t')}$ for some $t' \in I_t$ then set CHECK-POL$(A, t, p_t) = 0$;
   else set $A = A \cup I_t$ and $p_{t'} = p_{t \to t'}$ $\forall\, t' \in I_t$; if $t \in B$ then set $P_{\pi(t')} = \{p_{t \to t'}\}$ for all $t' \in I_t$

Figure 5 : Admissible assignment of the polarizations

We now show how to determine a frequency assignment satisfying C2, C3, C4 and C5 constraints. Consider the graph $G_{2,4}$ containing a vertex for each path, and in which two vertices are linked by an edge if the corresponding paths are linked by a C2 or a C4 constraint. For a path t, let J_t denote the connected component of $G_{2,4}$ containing t. The algorithm described in Fig.6 is a constructive procedure that assigns a frequency to each path while satisfying all C2, C3, C4 and C5 constraints. Each path t is supposed to have a frequency domain Dom_t which can be a proper subset of $F_{\varphi(t)}$ (see section 3). The paths are considered in a random order. Let A be the subset of paths for which a frequency has already been fixed. For a path $t \notin A$, we randomly choose a frequency $f_t \in Dom_t$, and fix the frequency of each path in J_t using the propagation of constraints C2 and C4. More precisely, consider two adjacent vertices t and t' in the graph $G_{2,4}$ defined above. If t and t' are linked by a C4 constraint, then they must have the same frequency, which means that when the frequency has been chosen for t we have no other choice for t'. The situation is a little bit different if t and t' are linked by a C2 constraint. Indeed, in such a case, given a frequency f_t for t, there are two possibilities for the frequency $f_{t'}$ to be assigned to t', namely $f_t - \varepsilon_{tt'}$ and $f_t + \varepsilon_{tt'}$ (if both frequencies belong to Dom_t). When propagating f_t in J_t, we randomly choose one of these two frequencies for $f_{t'}$. If the choice of $f_t \in Dom_t$ and its propagation on J_t does not lead to a partial admissible solution, we repeat the process with another frequency for t, randomly

chosen in Dom_t. This process may cycle even if there are admissible solutions, but such a case never happened in our experiments.
The input of function CHECK-FREQ described below is the current set A of paths to which a frequency has already been assigned as well as a path $t \notin A$ and a frequency $f_t \in F_{\varphi(t)}$. This function first chooses a frequency $f_{t'} \in Dom_{t'}$ for each path $t' \in J_{t'}$, using the propagation of constraints C2 and C4. It then returns value 0 if this frequency assignment violates a C3 or a C5 constraint. Otherwise, it returns value 1 and updates the set A.

Procedure INIT-FREQ

1. Set $A = \emptyset$
2. While $A \neq T$, do
   (a) randomly choose $t \in T - A$
   (b) randomly choose a frequency $f_t \in Dom_t$ for t
   (c) if CHECK-FREQ$(A, t, f_t) = 0$, go to step 2b

Function CHECK-FREQ(A, t, f_t)

1. Determine the connected component J_t of $G_{2,4}$ containing path t
2. Choose a frequency $f_{t'} \in Dom_{t'}$ for each path t' of J_t using the propagation of constraints C2 and C4 (it is always possible, if not, procedure BORNE-INF would have detected this problem before)
3. If this frequency assignment does not violate constraints in C3 $\cup$ C5, then set $A = A \cup J_t$ and set CHECK-FREQ$(A, t, f_t) = 1$; else set CHECK-FREQ$(A, t, f_t) = 0$

Figure 6 : Admissible assignment of the frequencies

By combining the two procedures described in this section we typically obtain 11-feasible solutions that violate a lot of CEM(10) constraints (i.e., solutions s with a set $V^{(10)}(s)$ containing many constraints). In order to determine feasible solutions with a lower relaxation level, we have developed two tabu search algorithms that are described in the next section : the first one tries to improve a solution by modifying the frequency assignment, and the second one performs changes on the polarization assignment. Notice that these two procedures need an admissible solution as input and they will be used alternatively several times.

5. A tabu search algorithm for the FAPP

We start this section with a brief description of a basic tabu search. We then propose two adaptations of tabu search, one for the frequency assignment (called TABU-FREQ), and one for the polarization assignment (called TABU-POL). We end the section with the final proposed algorithm which includes the lower bounding procedure, the generation of an initial admissible solution, and the two adaptations of tabu search.

5.1 Basic tabu search

Tabu search is a local search algorithm which was originally proposed by Glover [3, 4] and Hansen [6]. Its basic version can be described as follows. Let S be the set of solutions to a combinatorial optimization problem, and let f be an objective function which has to be minimized on S. A set $N(s)$, called *neighbourhood* of s, is associated with each solution $s \in S$. The solutions in $N(s)$ (also called *neighbours* of s) are obtained from s by performing local changes called *moves*. Tabu search first constructs an initial solution s_0 in S. Then, it successively generates solutions $s_1, s_2, \ldots$ in S such that $s_{i+1} \in N(s_i)$. When a move is performed from s_i to s_{i+1}, the reverse of such a move is stored in a *tabu list* L, and it is forbidden (with some exceptions) to use such a move for a certain number of iterations. Solution s_{i+1} is set equal to $arg \min_{s \in N'(s_i)} f(s)$, where $N'(s)$ is a subset of $N(s)$ containing all solutions s' which can be obtained from s by performing a move that is not in L, or such that $f(s') < f(s^*)$, where s^* is the current best solution. The process is stopped after a fixed number of iterations without improvement of s^*, or when a time limit is reached. This basic version of tabu search is summarised in Fig.7. Many variants and extensions can be found in [5].

Initialisation

1. generate an initial solution s
2. set $s^* = s$, $f^* = f(s)$ and $L = \emptyset$

Repeat

1. determine $N'(s) = \{s' \in N(s)$ such that s' is obtained from s by performing a move that does not belong to L, or such that $f(s') < f(s^*)\}$
2. set $s' = arg \min_{s'' \in N'(s)} f(s'')$
3. if $f(s') < f^*$, then set $f^* = f(s')$ and $s^* = s'$
4. update L and set $s = s'$

Until a time limit is reached or s^* was not improved during the last p iterations (where p is a parameter)

Figure 7 : Basic tabu search

5.2 Tabu search for the frequency assignment

We describe in this section a tabu search procedure that tries to improve the frequency assignment. The function to be minimized is the objection function f of the FAPP described in section 2. A neighbour solution is obtained by first assigning a new frequency f_t to a path t, and then repairing the neighbour if it is not admissible. When modifying the frequency of a path t, we try to eliminate at least one violation of a CEM constraint involving t at a level $\leq k^* - 1$, where k^* denotes the best relaxation level encountered so far. More precisely, let $n_s(t, f)$ denote the number of CEM($k^* - 1$) constraints involving t that are violated if we modify the current solution s by assigning frequency f to t (while the frequencies of the other paths are not changed). Let $k_t < k^*$ be the largest integer such that there exists a CEM constraint involving t which is violated at level k_t. Finally, let $\widehat{Dom_t}$ be the subset of Dom_t containing all frequencies which, when assigned to t, eliminate a violation of a CEM(k_t) constraint. If $\widehat{Dom_t} \neq \emptyset$, we randomly choose a frequency $f_t \in \widehat{Dom_t}$ for t among those minimising $n_s(t, f)$. Otherwise ($\widehat{Dom_t} = \emptyset$) we randomly select a CEM constraint violated at level k_t. Let t' be the second path involved in this constraint. If there exists at least one pair $(f_t, f_{t'})$ in $Dom_t \times Dom_{t'}$ which satisfies the considered constraint at level k_t, we randomly choose such a pair $(f_t, f_{t'})$

and assign frequency f_t to t and $f_{t'}$ to t' to obtain a neighbour solution. It may happen that no such pair exists. If that situation occurs, two cases are possible. If $\mid P_{\pi(t)} \mid = \mid P_{\pi(t')} \mid = 1$, or if paths t and t' have different polarizations, this means that there exists no k_t-feasible solution. Otherwise, we change the polarization of t or t' and reapply the above procedure trying to eliminate a violation of a CEM(k_t) constraint.

It may happen that the assignment of frequency f_t to t violates a C2, C3, C4 or C5 constraint. In such a case we apply procedure CHECK-FREQ(A, t, f_t) with $A = T - J_t$ (see section 4) to check whether f_t can be propagated on J_t without violating C2, C3, C4 and C5 constraints. It may also happen that the assignment of $f_{t'}$ to t' (if $\widehat{Dom}_t = \emptyset$) creates a non-admissible solution. We then also apply procedure CHECK-FREQ$(A, t', f_{t'})$ with $A = T - J_{t'}$ to check whether these violations can be eliminated by constraint propagation. If all violations can be eliminated, then the output of CHECK-FREQ is considered as a neighbour solution. In summary, we move from a solution to a neighbour one by first assigning a new frequency f_t to a path t so that f_t eliminates at least one violation of a CEM(k_t) constraint and creates a minimum number of violations of CEM($k^* - 1$) constraints. We then repair such a neighbour if it is not admissible.

We put in the tabu list L all paths in J_t and we forbid to change the frequency of these paths during $\mid L \mid$ iterations. This may be considered as very restrictive. Indeed, when we change the frequency of a path $t' \in J_{t'}$ from f_{old} to f_{new}, one could think about only forbidding the assignment of frequency f_{old} to t' during $\mid L \mid$ iterations. However, in our experiments, the proposed tabu list turned to be more effective than less restrictive ones. Notice that a neighbour solution is obtained by modifying the frequency of all paths in J_t, and the move to such a neighbour is considered as tabu if at least one path in J_t belongs to L.

In order to keep control on the time needed to perform an iteration, we generate at most N_F neighbours per iteration, where N_F is a parameter. Let $T^{(k)}$ denote the set of paths involved in CEM(k) constraints that are violated in the current solution s. To generate these N_F neighbour solutions, we first select $\min\{N_F, \mid T^{(k^*-1)} \mid\}$ paths in $T^{(k^*-1)}$. If $\mid T^{(k^*-1)} \mid < N_F$, the remaining $N_F - \mid T^{(k^*-1)} \mid$ paths are chosen in $T^{(k^*-2)}$, then in $T^{(k^*-3)}$, and so on until N_F different paths have been found or all violated CEM constraints have been considered. We then apply the above mentioned neighbourhood process to each one of these N_F paths. In order to accelerate the algorithm, we stop the generation of neighbour solutions when we find a solution better than s^* (the best solution encountered so far).

5.3 Tabu search for the polarization assignment

We now briefly describe the second tabu search procedure which tries to improve an admissible solution by modifying the polarization assignment. If $\gamma_{tt'}^{(k)} > \delta_{tt'}^{(k)}$ for a CEM constraint involving paths t and t' at level k, then this CEM constraint is easier to satisfy at level k if t and t' have different polarizations. Let $D(k)$ be the number of pairs (t, t') of paths having different polarizations and such that $\gamma_{tt'}^{(k)} > \delta_{tt'}^{(k)}$. The proposed tabu search algorithm tries to maximize $D(k^* - 1)$ (i.e., this is the objective function which has to be maximized). A neighbour solution is obtained by assigning a new polarization p_t on a path t with $\mid P_{\pi(t)} \mid = 2$. Such a change may induce violations of C6 and C7 constraints. In such a case, we apply procedure CHECK-POL(A, t, p_t) with $A = T - I_t$ (see section 4) to check whether the violations can be eliminated by constraint propagation. If CHECK-POL(A, t, p_t) returns value 0, then no neighbour solution is generated with polarization p_t for t. Otherwise, all changes performed in CHECK-POL(A, t, p_t) define a neighbour solution. We always move to the best non tabu neighbour s' of the current solution s, and we put in the tabu list all paths which have a different polarization in s and s'.

5.4 A solution method for the FAPP

Our solution method for the FAPP is now summarized in Fig.8. The organizers have fixed a time limit of one hour CPU-time on a PC Pentium III (500 Mhz, RAM 128 Mo). The length of the tabu lists is set equal to 3, while the value of parameter N_F is an integer that is randomly chosen, at each iteration, in the interval $[5, 30]$.

1. Apply procedure LOWER-BOUND and let k_{inf} be the obtained lower bound on k_{opt}; let $Dom_1, \ldots, Dom_{|T|}$ be the reduced frequency domains at feasibility level k_{inf}
2. Apply procedures INIT-FREQ and INIT-POL to get an initial admissible solution s; set k^* equal to the relaxation level $k(s)$ of s.
3. Repeat the following process until one hour CPU is reached
   - Apply procedure TABU-FREQ and stop when $p = 5000$ iterations have been performed without improving the best solution (according to the objective function f), or when a $(k^* - p)$-feasible solution is reached with $p \geq 1$; let k denote the feasibility level of the output solution of TABU-FREQ
   - If $k < k^*$, then set $k^* = k$
   - Apply TABU-POL and stop when $p = 100$ iterations have been performed without improving the best solution (according to the objective function D), or when a $(k^* - p)$-feasible solution is reached with $p \geq 1$; let k be the feasibility level of the output solution of TABU-POL
   - If $k < k^*$, then set $k^* = k$

Figure 8 : The proposed solution method for the FAPP

6. Computational results

We have run the above algorithm on the 40 instances of the challenge. Remember that these instances are of three different types. The 15 problems of type A (fapp01 to fapp15) have between 200 and 3'000 paths, a maximum of 20'000 CEM constraints, a maximum of 2'200 other constraints, and a maximum of 250 frequencies per frequency domain. The 15 instances of type B (problems fapp16 to fapp30) are closer to real-life problems. The number of paths is approximately the same as for type A instances, while the frequency domains can have up to 500 frequencies, and the problems can have up to 31'000 CEM constraints and 13'000 other constraints. Type B instances are characterized by the fact that $\gamma_{tt'}^{(k)} = \delta_{tt'}^{(k)}$ for each CEM constraint, which means that the polarizations are not relevant for the satisfaction of a CEM constraint. The number of paths in the 10 instances of type X

(problems fapp31 to fapp40) ranges from 400 to 3'000. These instances have up to 30'000 CEM constraints, 2'500 other constraints, and a maximum of 500 frequencies per frequency domain. More information about these 40 instances can be found in [1] or on the web site [2].

The results are summarised in Table 2. For each instance, we give its type and its number $\mid T \mid$ of paths. We then compare the best possible level k_{opt} of feasibility (provided by the organizers) with the best level k^* of feasibility produced by our algorithm, and with the best level k_v^* of feasibility produced by the algorithm developed by Michel Vasquez (the winner of the competition). We also indicate the number $\mid V^{(k^*-1)} \mid$ of CEM constraints violated at level k^*-1 in our solutions, and the number $\mid V^{(k_v^*-1)} \mid$ of CEM constraints violated at level k_v^*-1 in Vasquez's solutions. Finally, we give the total number $\sum\limits_{k<k^*-1} \mid V^{(k)} \mid$ of CEM constraints violated in our solutions at a level strictly smaller than k^*-1, as well as the total number $\sum\limits_{k<k_v^*-1} \mid V^{(k)} \mid$ of CEM constraints violated in Vasquez's solutions at a level strictly smaller than k_v^*-1. The last column contains an asterisk when our algorithm has been able to find a better solution than the one produced by Vasquez's algorithm.

Instance	*type*	$\mid T \mid$	k_{opt}	k^*	k_v^*	$\mid V^{k^*-1} \mid$	$\mid V^{k_v^*-1} \mid$	$\sum_{k<k^*-1} \mid V^{(k)} \mid$	$\sum_{k<k_v^*-1} \mid V^{(k)} \mid$	best
fapp01	A	200	4	4	4	5	14	164	233	*
fapp02	A	250	2	4	2	2	20	273	195	*
fapp03	A	300	7	7	7	13	32	647	892	*
fapp04	A	300	1	4	1	1	184	190	0	
fapp05	A	350	11	11	11	1	364	715	5'694	*
fapp06	A	500	5	5	5	24	31	634	811	*
fapp07	A	600	9	9	9	26	106	1'339	3'375	*
fapp08	A	700	5	5	5	19	73	725	1'225	*
fapp09	A	800	3	5	3	1	104	1'071	846	
fapp10	A	900	6	7	6	23	103	2'411	2'003	
fapp11	A	1'000	8	10	8	1	119	3'052	4'191	
fapp12	A	1'500	2	9	2	64	62	7'588	1'310	
fapp13	A	2'000	4	10	5	13	132	9'651	3'645	
fapp14	A	2'500	5	10	5	43	217	9'566	5'045	
fapp15	A	3'000	5	10	5	30	192	14'031	4'727	
fapp16	B	260	11	11	11	5	514	57	5'189	*
fapp17	B	300	4	4	4	4	4	34	36	*
fapp18	B	350	8	8	8	4	4	55	59	*
fapp19	B	350	6	6	6	2	3	51	70	*
fapp20	B	420	10	10	10	5	7	97	142	*
fapp21	B	500	4	4	4	2	2	10	12	*
fapp22	B	1'750	7	7	7	15	25	187	503	*
fapp23	B	1'800	9	9	9	16	17	187	197	*
fapp24	B	2'000	7	7	7	6	9	71	91	*
fapp25	B	2'230	3	3	3	7	7	32	33	*
fapp26	B	2'300	7	7	7	9	10	74	86	*
fapp27	B	2'550	5	5	5	4	11	20	54	*
fapp28	B	2'800	3	3	3	13	42	32	142	*
fapp29	B	2'900	6	6	6	25	25	212	310	*
fapp30	B	3'000	7	7	7	13	48	148	1'045	*
fapp31	X	400	5	5	5	21	117	1'694	1'896	*
fapp32	X	550	6	11	6	1	10	81	235	
fapp33	X	650	5	5	5	7	10	69	235	*
fapp34	X	750	4	5	4	1	22	222	565	
fapp35	X	1'500	6	10	6	11	62	2'102	1'375	
fapp36	X	2'000	7	10	7	1	63	1'347	1'643	
fapp37	X	2'250	5	10	5	2	51	880	1'288	
fapp38	X	2'500	3	10	9	9	125	1'279	6'717	
fapp39	X	2'750	3	3	11	1'066	3'947	5'556	40'473	*
fapp40	X	3'000	4	10	4	5	64	3'484	1'252	

Table 2: Computational results

Our algorithm is better than Vasquez's algorithm on 24 instances. All these instances have $k^* = k_{opt}$ while we have not been able to reach the optimal level of feasibility k_{opt} for the 16 other instances. This indicates that when we reach the optimal level k_{opt}, we are very effective in the minimization of the violations of CEM(k) constraints with $k < k_{opt}$. However, our algorithm is not effective enough in the minimization of the feasibility level. The 24 successful instances all have $k^* = k_{inf}$ (see Table 1), except fapp05 and fapp39, which means that in these cases, our algorithm has proved that there is no feasible solution with a lower feasibility level.
We notice also that the 24 successful instances include all type B instances. There are several reasons which can explain the success of our algorithm on type B instances. Remember first that the reduction procedures proposed in section 3 assume that we are always in the most favourable situation, where the two paths involved in a CEM constraint have different polarizations (i.e., $\delta_{tt'}^{(k)}$ can be used instead of $\gamma_{tt'}^{(k)}$). We have mentioned that the polarizations are not relevant for type B instances since $\gamma_{tt'}^{(k)} = \delta_{tt'}^{(k)}$ for each CEM constraint. Hence, these instances can be considered as favourable cases for our lower bounding technique. The success of our algorithm on type B instances can perhaps also be explained by the big reduction of the frequency domains (up to 90%, see Table 1), which means that we have drastically reduced the number of admissible solutions. Notice however that for many instances of type X, our best feasibility level k^* is much larger than k_{opt}, while the frequency domains have also been reduced a lot. On the opposite, we have got some very good results for some instances of type A (e.g., fapp01, fapp03, fapp05, fapp06, fapp07 and fapp08) while we have not been able to get a large reduction of the frequency domains in these cases.
Most of the instances for which we have not been able to reach a k_{opt}-feasible solution are of large size, which indicates that the performance of our algorithm decreases with the increase of the problem size. We have however observed in section 3 that the quality of our lower bound k_{inf} does not seem to depend on the problem size. As mentioned above, the polarizations are not relevant for type B instances, while they are for those of type A and X. We therefore think that the medium performance of our algorithm on some instances of type A or X is probably due to a not enough adequate way of dealing with the polarizations. The performance of our algorithm can however be considered as reasonably good since we have always been able (except for fapp32) to find a k^*-feasible solution with $k^* < 11$ when such a solution exists, while this is not an easy task at all.
In conclusion, our second position in the final phase of the contest (to which 27 competitors have taken part), gives evidence that the combination of a tabu search with an effective reduction of the search space is an appropriate approach for the solution of the FAPP.

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