A price-based policy to coordinate investments in green infrastructure: the interplay between strategic behaviors and initial endowments in natural capital *

Denis Claude
CES – Univ. Panthéon-Sorbonne,
Paris, France.

Charles Figuières
INRA & Lameta,
Montpellier, France

Mabel Tidball
INRA & Lameta,
Montpellier, France

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Abstract

This paper explores the dynamic properties of price-based policies in a model of competition between two jurisdictions. Jurisdictions invest over time in infrastructure to increase the quality of the environment, a global public good. They are identical in all respects but one: initial stocks of infrastructure. This is a dynamic type of heterogeneity that disappears in the long run. Therefore, at the steady state, usual intuitions from static settings apply: identical jurisdictions inefficiently under-invest, calling for public subsidies. In the short run, however, counterintuitive properties are established: i) the evolution of capital stocks can be non-monotonic, ii) one jurisdiction can be temporarily taxed, even though it should increase its investment, whereas the other is subsidized. It is shown how these phenomena are related to initial conditions and the kind of interactions between infrastructure capitals, complementarity or substitutability.

1 Introduction

In recent years the theoretical analysis of price-based policies for the control of environmental externalities under imperfect competition has received a renewed attention. Previously, most of the literature focused on the design of optimal tax or subsidy policies in a static setting where several instantaneous effects of these instruments should be balanced; e.g., with respect to taxation, the gain in terms of social welfare arising from the reduction of pollution emissions against the loss from output restriction.

However, little is known about how intertemporal externalities affect the design and dynamic properties of price-based policies. Introducing the time dimension opens the possibility to raise the critical issue of credibility of public policies, namely how regulations should be framed to ensure that they remain optimal in their ability to achieve or increase social efficiency as circumstances change over time. Such an explicit consideration of credibility requirements may qualify

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substantially the intuitions about price-based regulations gained in a static setting and provide interesting and sometimes counter-intuitive policy advices.

An important contribution to this literature is Benchekroun and Long (1998). They consider efficiency inducing taxation[1] for the regulation of an oligopolistic industry which is responsible for releasing a stock pollutant – one for which pollution accumulation generates present as well as long-term environmental damages. They formulate a differential game of pollution control in which the environmental regulator imposes a taxation rule in a symmetric oligopolistic industry. In this game, the state variable is the pollution stock, the tax policy is the control of the environmental regulator and output decisions are the controls of the firms which are assumed to use either open-loop or markov strategies.

Benchekroun and Long (1998) analyze a Markovian tax policy whereby the output tax rate faced by a firm at any given time depends solely on the current pollution stock. By construction, such a linear markovian tax rule is credible, in the sense that it is time consistent and subgame perfect. The authors provide a characterization of the optimal tax rule that is shown to be increasing in the pollution stock and to 'decentralize' the socially optimal time-path of production. As for the dynamic properties of the tax, they obtain a surprising result. In an initial time interval where the stock of pollution is low, the tax rate may be negative implying a subsidy. Paradoxically, this subsidy induces firms to produce less than they would have if the industry had not been regulated. Upon reflection, the explanation for this result is simple. Since the tax rate at any given time depends solely on the pollution stock, firms anticipate that an increase in their production will lead to reduced subsidies in the future and eventually precipitate the turn of the subsidy into a tax. As noted by Benchekroun and Long (1998), this is an instance of 'carrot and stick' policy.

Motivated by a long standing concern for infrastructure competition, this paper elaborates on Benchekroun and Long (1998)'s seminal contribution by studying how differences in initial stocks of infrastructure alter the dynamic properties of the optimal and credible tax or subsidy policy. We consider a stylized dynamic extension of the model of interjurisdictional spillovers introduced by Wildasin (1991). The focus of our attention are two jurisdictions that are located in the same watershed or airshed[2]. Each jurisdiction invest in public (or green) infrastructure[3] in order to provide a public good to its own residents. However, the public good produced by one jurisdiction benefits also the residents in the other jurisdiction who cannot be excluded from 'its' consumption once it is provided. In the absence of any regulation initiative, the presence of positive interjurisdictional spillovers will result in the underprovision of public infrastructure. Indeed, local jurisdictions will not take into account the positive spillovers that benefit the non-residents when setting their investment policies. A first remedy is to elevate the decision making process to a higher level of jurisdiction so that external benefits of public infrastructure become internal to the jurisdiction which funds them. A drawback of this solution is that it alienates local residents from the control they have over issues that impact their local community and daily life. A second remedy is for the higher level of jurisdiction to implement an infrastructure capital subsidization policy that will help coordinate local investment decisions while preserving subsidiarity. This is the route we travel by in this paper.

The logic of regulation in our model is similar to that of Benchekroun and Long (1998). A benevolent authority sets the capital infrastructure subsidization scheme and local jurisdictions decides upon their expenditures in public infrastructure taking the subsidization rule as given. A key difference with Benchekroun and Long (1998) lies in the state of the system which is not scalar. Instead it is a two-dimensional vector describing the stocks of infrastructure of each jurisdiction at any given time.

Consistently with the purposes of our paper we expunge the model of any asymmetry across

[1] On efficiency inducing taxation, see also Bergstrom et al. (1981), Karp and Livernois (1992) and Karp and Livernois (1994).

[2] Here, we depart from the original meaning of the terms 'watershed' and 'airshed' and adopt the North-American usage in which they have come to describe the geographical boundary for water and air quality standards.

[3] Throughout this article, we shall use the terms 'public' and 'green', interchangeably.

jurisdictions except regarding their initial stock of infrastructure in order to highlight how this particular asymmetry affects the dynamic properties of the subsidy[4]. This is a dynamic type of heterogeneity that vanishes in the long-run. Consequently, at the steady state, usual intuitions from static symmetric settings apply. Due to the presence of positive interjurisdictional spillovers, both jurisdictions will inefficiently under-invest, which calls for the implementation of a green capital subsidization scheme. In the short run, however, a counterintuitive property appears: It is shown that the optimal scheme *may* require to simultaneously tax one jurisdiction and subsidize the other for an initial period of time. And which jurisdiction should be initially taxed depends on the degree to which stocks are substitutes or complements.

The remainder of this paper is organized as follows. Section 2 presents the basic model. In Section 3, the utilitarian social optimum is characterized. Then, in 4, we derive the optimal infrastructure capital subsidization scheme and we discuss its dynamic properties in 5. Section 6 provides examples and discusses intuitions. Finally, in 7, we conclude.

2 A dynamic framework for infrastructure competition

We consider a dynamic extension of the model of interjurisdictional spillovers introduced by Wildasin (1991). Two jurisdictions indexed by $i = 1,2$ are located in the same watershed or airshed. We shall assume that they are identical in all respects, except (possibly) their initial endowments in green infrastructure. Each jurisdiction is inhabited by identical households who are assumed to be immobile and infinitely lived. These households can be treated as a representative consumer whose preferences are defined over a composite private commodity, denoted by x_i, and an index of environmental quality denoted by s_i. Then, the representative consumer's preferences an be represented by the utility function

$$u_i(x_i,s_i) = x_i + s_i, \quad \forall i = 1,2. \tag{1}$$

Local stocks of infrastructure are the inputs in the production process of environmental quality. Letting $K_i(t)$ denote Jurisdiction i's stock of green infrastructure, this relationship can be expressed as:

$$s_i = P_i(K_i, K_j) = p_0 + p_1 K_i + p_2 K_j + \frac{p_3}{2} K_i^2 + p_4 K_i K_j + \frac{p_5}{2} K_j^2, \quad \forall i (\neq j) = 1,2, \tag{2}$$

with p_1, $p_2 > 0, p_3 < 0$ and $p_5 < 0$. Parameter p_4 is not restricted in sign and will play an important role in our investigation.

We assume that jurisdictions compete in public infrastructure over an infinite time period. Let $e_i(t)$ denote Jurisdiction i's expenditure on its public infrastructure at time $t \in [0,\infty[$. Each jurisdiction is endowed with an initial stock of green infrastructure equal to $K_i(0) = K_i^0$. In the remainder, we suppose that $K_1^0 \geq K_2^0$. Investment is a flow that allows jurisdictions to adjust their stocks of public infrastructure. Jurisdiction i's public expenditure $e_i(t)$ modifies its current stock of infrastructure according to the following law of motion

$$\dot{K}_i(t) = e_i(t) - \delta K_i(t), \quad \forall i = 1,2, \tag{3}$$

where δ is the constant rate of depreciation. We assume that investment is reversible and resale of infrastructure capital is impossible. In other words, $e_i(t)$ is restricted to be non-negative and δ is strictly positive.

Investment in infrastructure capital is costly. Let $C_i(e_i)$ denotes jurisdiction i's cost of infrastructure capital adjustment. We assume identical cost functions given by $C_i(e) = c_1\, e + \frac{c_2}{2}\, e^2$, $i =$

[4]We also assume away any informational obstacles to regulation. For a recent review on such issues, see for instance Lewis (1996).

1, 2, where $c_1 \geq 0$ and $c_2 > 0$ are cost parameters. Accordingly, jurisdiction i's cost of altering its infrastructure stock is an increasing and convex function of the rate of investment. In our model, where investment is reversible ($\delta > 0$), this assumption implies that instantaneous adjustments of capital stocks are ruled out.

We assume that each jurisdiction is endowed with an exogenous revenue y_i which can be used to finance public expenditures and consumption good expenses. Accordingly, jurisdiction i's budget constraint is $x_i + C_i(e_i) = y_i$. Plugging this constraint into the utility function (1) yields the reduced-form utility for each jurisdiction: $W_i(K_i, K_j, e_i) = y_i + P_i(K_i, K_j) - C_i(e_i)$.

Each local jurisdiction is assumed to select its investment strategy so as to maximize the integral of its discounted stream of net social benefits. Denoting by $r \in]0,1[$ the common discount factor of both jurisdictions, the objective of jurisdiction i is to maximize

$$J_i = \int_0^\infty \left[y_i + P_i\left(K_i(t), K_j(t)\right) - C_i\left(e_i(t)\right)\right] e^{-rt} dt, \quad \forall i = 1,2, \tag{4}$$

with respect to the state equations (3) and the non-negativity constraint $e_i(t) \geq 0$. To complete the description of each jurisdiction's problem, the information structure must be specified. In this paper, it is assumed that each jurisdiction is able to observe the state of the game at any given time and make investment decisions based on this information. Namely, we assume that both jurisdictions use markov strategies; i.e., decision rules of the form $e_i(t) = \phi_i(K_i(t), K_j(t))$.

The above elements define a game of competition in infrastructure which belongs to the extensively studied class of capital accumulation games [5]. In analyzing the outcome of interjurisdictional competition, the relevant solution concept is the markov perfect Nash equilibrium : a pair of markov perfect strategies that are mutual best-responses. Recall our assumption that both jurisdictions are identical in all respects, except (possibly) their initial endowments in green infrastructure. Under this assumption, the Nash equilibrium outcome predicts that both stocks of green infrastructure will converge to the same level K^∞ in the long-run. In other words, the only source of heterogeneity in the model disappears at the steady-state. We assume that inter-jurisdictional spillovers are positive in the long-run; i.e.,

$$\frac{\partial P_i(K_i, K_j)}{\partial K_j} \Big|_{K_i = K_j = K^\infty} > 0. \tag{5}$$

However, we allow for the existence of negative externalities in the short-run. Indeed, the possibility of a change in the sign of externalities over time as the state variable evolves is a distinguishing feature of dynamic games.

Observe that

$$\frac{\partial^2 P_i(K_i, K_j)}{\partial K_i \partial K_j} = p_4 \tag{6}$$

We introduce the following terminology due to Figuières (2009). Given $p_4 > 0$, an increase in the stock of capital accumulated by one jurisdiction enhances the marginal productivity of its rival's own stock of capital. Conversely, given $p_4 < 0$, an increase in the capital stock of one jurisdiction lowers the marginal productivity of its rival's own stock of capital. In the former case we shall say that the stocks are strategic complements, and in the latter strategic substitutes.

In our model where green infrastructures generate spillovers across the boundaries of jurisdictions, the Nash equilibrium outcome predicts that infrastructure provision will be suboptimal. This conclusion has been a major argument in favour of transferring decision making about public infrastructure to a higher level of jurisdiction that encompasses all the spillovers. In this paper, we consider a different remedy. We assume that the higher level jurisdiction wishes to coordinate local jurisdictions investment decisions and 'decentralize' the social optimum by means of a capital investment subsidization scheme. Specifically, we assume that the social regulator implements a

[5] See Dockner et al. (2000, chapt. 9) for an introduction. Also, see Driskill and McCafferty (1989); Fershtman and Muller (1984, 1986); Figuières (2002, 2009); Figuières, Gardères and Rychen (2002); Reynolds (1991).

linear markovian subsidization policy to support local expenditures in green capital. Under this tax scheme, each jurisdiction i is granted an amount $\tau_i(K_i, K_j)$ per unit of investment in public infrastructure capital e_i. It is important to note here that the unit rate of subsidization depends exclusively on the two jurisdictions stocks of infrastructure at any given time t.

Recall that parameter p4 is not restricted in sign. Given $p_4 > 0$ (resp. $p_4 < 0$), an increase in jurisdiction j's stock of green capital enhances (resp. reduces) the marginal productivity of jurisdiction i's own stock of capital. When $p_4 = 0$, the marginal productivity of firm i's stock of green infrastructure is independent of jurisdiction j's stock of infrastructure. More importantly, the sign of p_4 is known to determine the strategic features of the capital accumulation game. Let us assume that both jurisdictions use linear feedback strategies; i.e., strategies of the form $e_i(t) = \Phi(S(t)) = \phi_1 + \phi_2 S(t) + \phi_3 S(t)$. Proposition 3 in Figuières (2009) establishes that $\text{sign}(p_4) = \text{sign}(\phi_3)$. Accordingly, jurisdiction i's investment strategy will be increasing, decreasing or constant depending on whether p_4 is positive, negative or zero.

Definition 1 (Feedback Complementarity (Figuières 2002))**.** *When the strategies are decreasing functions of the rival stock, i.e.,* $\phi_3 < 0$ *then the players are said to display feedback substitutability. When the strategies are increasing functions of the rival's stock, i.e.* $\phi_3 > 0$*, then the players are said to display feedback complementarity.*

Feedback substitutability (resp. complementarity) corresponds to situations where an increase of the rival's stock reduces (resp. increases) the incentive to invest for greater future benefits.

3 The utilitarian social optimum

Let us assume that the responsibility for infrastructure financing has been transferred to a higher level of government that encompasses both local jurisdictions; e.g., an intercommunal or interregional association. As a consequence of this delegation, interjurisdictional spillovers are now internalized into the decision making of a single economic agent. Then, the problem faced by the social planner is to find the time-paths of investment $(e_1(.), e_2(.))$ that solve

$$\max_{(e_1(.),e_2(.))} J^1 + J^2 \tag{7}$$

subject to (3) and $e_i(t) \geq 0, \forall t \in]0, \infty[$. This amounts to solving a two-state variable optimal control problem. We will refer to the solution to this problem as the *utilitarian social optimum* and use it as a benchmark for the remainder of the analysis.

In this section we show that there exists a unique optimal path of investment in public infrastructure. To solve for the social optimum we make use of Pontryagin's maximum principle. The current value Hamiltonian of the centralized problem (7) is defined as[6]

$$H(e_1, e_2, K_1, K_2, \lambda_1, \lambda_2) = \sum_{i=1}^{2} \left(P_i(K_i, K_j) - C(e_i)\right) + \sum_{i=1}^{2} \lambda_i \left(e_i - \delta K_i\right), \tag{8}$$

where λ_1 and λ_2 are the co-state variables associated with $\dot{K}_1$ and $\dot{K}_2$, respectively.

Assuming interior solutions, Pontryagin's maximum principle implies the following necessary conditions for optimality:

$$\frac{\partial H}{\partial e_i} = 0, \quad C'(e_i) = \lambda_i, \quad \forall i = 1, 2, \tag{9}$$

$$\dot{\lambda}_i = r\lambda_i - \frac{\partial H}{\partial K_i}, \quad \dot{\lambda}_i = (r+\delta)\lambda_i - \frac{\partial P_i(K_i, K_j)}{\partial K_i} - \frac{\partial P_j(K_j, K_i)}{\partial K_i}, \quad i(\neq j) = 1, 2, \tag{10}$$

[6] We have not incorporated explicitly the constraints $e_i(t), e_j(t) \geq 0$ at the formulation stage of the problem. We preferred to solve it and check afterward that those constraints are verified. The same remark applies to the study of decentralized behaviors in the next section.

or

$$\lambda_i = c_1 + c_2\, e_i, \quad \forall i = 1, 2, \tag{11}$$

$$\dot{\lambda}_i = (r+\delta)\,\lambda_i - (p_1+p_2) - (p_3+p_5)\,K_1 - 2\,p_4\,K_2, \quad \forall i (i \neq j) = 1, 2, \tag{12}$$

along with the dynamic process of capital accumulation (3); the transversality condition at infinity is

$$\lim_{t\to\infty} \left[\lambda_1(t)\,(K_1(t) - K_1^c(t)) + \lambda_2(t)\,(K_2(t) - K_2^c(t))\right] e^{-rt} = 0, \tag{13}$$

where $K_i^c(.)$ denotes a candidate for optimization and $K_i(.)$ is any other path.

Using Equation (11) to eliminate e_i from (3), optimality conditions can be summarized as

$$\begin{pmatrix} \dot{\lambda}_1 \\ \dot{\lambda}_2 \\ \dot{K}_1 \\ \dot{K}_2 \end{pmatrix} = \begin{pmatrix} (r+\delta) & 0 & -(p_3+p_5) & -2p_4 \\ 0 & (r+\delta) & -2p_4 & -(p_3+p_5) \\ 1/c_2 & 0 & -\delta & 0 \\ 0 & 1/c_2 & 0 & -\delta \end{pmatrix} \begin{pmatrix} \lambda_1 \\ \lambda_2 \\ K_1 \\ K_2 \end{pmatrix} - \begin{pmatrix} (p_1+p_2) \\ (p_1+p_2) \\ c_1/c_2 \\ c_1/c_2 \end{pmatrix} \tag{14}$$

along with the initial conditions and the transversality condition 13. The above system of differential equations (14) can be rewritten more compactly as $\dot{x}{=}A\,x - b$. Setting $\dot{x}{=}0$, we obtain a system of algebraic equations which can be solved for $(K_1^\infty,\, K_2^\infty,\, \lambda_1^\infty,\, \lambda_2^\infty)$ to yield:

$$K_i^\infty{=}K^\infty{=}\frac{(r+\delta)\,c_1 - (p_1+p_2)}{(p_3+p_5) + 2\,p_4 - \delta\,(r+\delta)\,c_2}, \quad \text{and} \quad e_i^\infty{=}e^\infty{=}\delta\,K^\infty, \quad i{=}1,\,2. \tag{15}$$

The stability properties of the steady-state are determined by the eigenvalues of the coefficient matrix A. Solving the characteristic equation

$$\det(\rho I - A){=}\left[((p_3+p_5) - (r+\delta-\rho)(\delta+\rho)\,c_2)^2 - 4\,p_4^2\right]/c_2^2{=}0, \tag{16}$$

where I is the identity matrix, yields four real and distinct eigenvalues, two of which are positive and two of which are negative, confirming a saddle-point solution. The following proposition provides a characterization of the social optimum.

Proposition 2. *The socially optimal time-paths of investment in public infrastructure are*

$$e_i(t){=}(1/2)\,(\delta{+}\rho_1)(K_i^0{-}K_j^0)e^{\rho_1 t}{+}(1/2)\,(\delta{+}\rho_2)(K_i^0{+}K_j^0{-}2K^\infty)e^{\rho_2 t}{+}\delta\,K^\infty, i({\neq}j){=}1,\,2, \tag{17}$$

where $(\rho_1,\,\rho_2)$ are the negative roots of the coefficient matrix A:

$$\rho_1 = (1/2)\left[r - \sqrt{(r+2\,\delta)^2 - 4\,(p_3+p_5-2\,p_4)\,/c_2}\right], \tag{18}$$

$$\rho_2 = (1/2)\left[r - \sqrt{(r+2\,\delta)^2 - 4\,(p_3+p_5+2\,p_4)\,/c_2}\right]. \tag{19}$$

The stocks of green infrastructure $(K_1(t),\,K_2(t))$ evolve along the following trajectories

$$K_i(t){=}(1/2)\,(K_i^0 - K_j^0)e^{\rho_1 t} + (1/2)\,(K_i^0 + K_j^0 - 2K^\infty)e^{\rho_2 t} + K^\infty, i({\neq}j){=}1,\,2, \tag{20}$$

which converge to the unique steady state K^∞.

Proof. See Appendix A. □

Finally, note that the two jurisdictions' optimal rates of investment at any time t can be written as functions of the state vector $(K_1(t),\,K_2(t))$. The so-called feedback representations of the optimal controls then read as

$$\hat{e}_i(K_i,K_j){=}(1/2)\,(2\delta+\rho_1+\rho_2)\,K_i + (1/2)(\rho_2-\rho_1)K_j - \rho_2 K^\infty, \tag{21}$$

Having characterized the utilitarian optimal solution, we now turn to the analysis of the decentralized scenario.

4 A Pigovian remedy to infrastructure competition

Because of the externalities involved in our "duopolity" game, unregulated competition leads to a suboptimal provision of green infrastructure. In order to restore efficiency, the regulator needs to adopt a certain environmental policy. In this paper, we consider a situation in which the role of the higher level jurisdiction is limited to setting investment subsidies. In order for the subsidization scheme to be optimal it should allow for the decentralization of the social optimum. In other words, it should induce both jurisdictions to follow an investment path that coincides with the socially optimal investment path. We shall consider two different scenarios which correspond to alternative assumptions regarding the information available to the players and/or their commitment power. In subsection 4.1 we examine the case of open-loop subsidization. Because of its strong analogy with a static regulation problem, this case provides an useful benchmark against which to compare the optimal markov-perfect subsidization scheme characterized in 4.2.

4.1 Open-loop price policy

In this subsection, we assume that the social regulator as well as local jurisdictions use open-loop strategies; i.e., strategies that are conditional on calendar time. The higher-level jurisdiction seeks to decentralize the social optimum by implementing an appropriate fiscal policy. Suppose that this jurisdiction is committed to a couple of time-paths of investment subsidies $\{\tau_i(t),\ \tau_j(t)\}$ where $\tau_i(t)$ is not restricted in sign (it may be negative implying a tax). Let $e_i(t)$ denote Jurisdiction i's open-loop investment strategy. Then, Jurisdiction i has to solve the following dynamic optimization problem:

$$\max_{e_i \geq 0} \qquad J_i = \int_0^{+\infty} e^{-rt} \left[P_i(K_i,\ K_j) - C(e_i) + \tau_i e_i\right] dt \tag{22}$$

$$s.t. \qquad \dot{K}_i = e_i - \delta\, K_i, \quad K_i(0) = K_i^0. \tag{23}$$

The current value Hamiltonian for this problem is defined as $H_i = P_i(K_i\, K_j) - C(e_i) + \tau_i\, e_i + \lambda_i(e_i - \delta\, K_i)$ where λ_i is the costate variable associated with $\dot{K}_i$. The first-order conditions for optimality are given by

$$C'(e_i) - \tau_i = \lambda_i, \quad \text{and} \quad \dot{\lambda}_i = (r+\delta)\,\lambda_i - \frac{\partial P_i(K_i,\ K_j)}{\partial K_i}, \quad i(\neq j) = 1,\ 2. \tag{24}$$

along with the usual transversality conditions and the dynamics of capital accumulation (3). Conditions (24) can be summarized as:

$$\dot{\lambda}_i = (r+\delta)\ \left(C'(e_i) - \tau_i\right) - \frac{\partial P_i(K_i,\ K_j)}{\partial K_i}. \tag{25}$$

From equations (9) and (10), we obtain

$$\dot{\lambda}_i = (r+\delta)\, C'(e_i) - \frac{\partial P_i(K_i,\ K_j)}{\partial K_i} - \frac{\partial P_j(K_j,\ K_i)}{\partial K_i}. \tag{26}$$

By definition, the optimal price instrument $\tau_i^o(t)$ ensures that equations (25) and (26) coincide. By direct comparison, we get

$$\tau_i^o = \frac{1}{(r+\delta)} \left(\frac{\partial P_j(K_j,\ K_i)}{\partial K_i}\right) \quad i(\neq j) = 1,\ 2. \tag{27}$$

We obtain the familiar result that, along the optimal path, the Pigouvian instrument is equal to the present value of the interjurisdictional spillover. Hence, $\tau_i^o(t)$ is proportional to, and of the same sign as, $\partial P_j(K_j,\ K_i)/\partial K_i,\ i(\neq j) = 1,\ 2$.

4.2 Markovian price policy

The analysis of the preceding subsection has confined itself to strategic situations in which all players use strategies that are functions of calendar time only. Consequently, it ignores the strategic interactions of the players through the evolution of the state variable over time. Such an exclusion is legitimate only if they are able to commit themselves to the chosen time-path of investment at the start of the game. In actual practice however, the players usually lack such a commitment power or may use the information acquired over time to deviate from the announced strategy path. In such cases, the optimal open-loop price policy fails to satisfy time-consistency requirements (subgame perfection) and the social optimal will not be reached.

In the remainder of this paper, to ensure that the social regulator's fiscal policy as well as the local jurisdictions' investment strategies are credible, we assume that they are of the markovian type. Namely, we assume that local jurisdictions use decision rules of the form $e_i(t)= e_i(K_i(t), K_j(t)),\ i(\neq j)=1,\ 2$, such that they condition their current capital outlay on current stock levels $(K_i(t),\ K_j(t))$. Similarly, we suppose that the social regulator seeks to decentralize the social optimum by implementing a fiscal policy of the form $T=\{\tau_i(K_i(t), K_j(t)), \tau_j(K_j(t), K_i(t))\}$. Then, each jurisdiction i is granted a subsidy (or tax) $\tau_i(K_i(t), K_j(t))$ per unit of expenditure in public infrastructure at time t, where the unit subsidy (or tax) rate $\tau_i(.)$ is a function of the current value of the two stocks $(K_i(t), K_j(t))$.

Under the above assumptions, the duopoly subgame is solved for a Markov-Perfect Nash Equilibrium and the environmental regulation game for a Non-degenerate Markov-Perfect Stackelberg Equilibrium. By definition, these solution concepts yield time-consistent equilibrium strategies. However, without further restrictions on the set of admissible strategies, unicity of the equilibrium is not warranted. Here, we restrict our attention to symmetric equilibria in linear strategies. There are at least three selection arguments in favor of such a restriction restriction. First, the symmetric equilibrium of a symmetric game constitutes a focal point solution. Second, in a symmetric game the MPE converges to the symmetric and linear MPE as t tends to infinity (if the limit MPE exists). Finally, non-linear MPE exist only when the state-variable takes values in a specific interval.

Let us consider how the fiscal policy implemented by the social planner alters the incentives of local jurisdictions to invest in green infrastructure. Suppose that the higher-level jurisdiction announces the markovian policy $T=(\tau_i(K_i, K_j), \tau_j(K_j, K_i))$. Then, at a markov perfect Nash equilibrium, jurisdiction i solves the dynamic optimization problem

$$\max_{e_i} \qquad J_i=\int_0^{+\infty} e^{-rt}\left[P_i(K_i,\ K_j) - C(e_i) + \tau_i(K_i,\ K_j)e_i\right] dt \tag{28}$$

$$s.t. \qquad \dot{K}_i=e_i - \delta\, K_i, \quad K_i(0)=K_i^0, \tag{29}$$

$$\dot{K}_j=e_j(K_j,\ K_i) - \delta K_j, \quad K_j(0)=K_j^0. \tag{30}$$

taking $\tau_i(K_i, K_j)$ as given. The current value Hamiltonian for this problem is defined as

$$H_i=P_i(K_i, K_j) - C(e_i) + \tau_i(K_i, K_j)e_i + \mu_i(e_i - \delta K_i) + \sigma_i(e_j(K_j, K_i) - \delta K_j), \tag{31}$$

where μ_i and σ_i are the costate variables associated with $\dot{K}_i$ and $\dot{K}_j$, respectively. Let us recall that the optimal strategies of jurisdiction i's opponent are necessarily of form $e_j(K_j, K_i)=\phi_1+\phi_2 K_j+\phi_3 K_i$, given the linear-quadratic structure of the game. Assuming interior solutions, Pontryagin's maximum principle then implies that the following conditions

$$\mu_i = c_1 + c_2\, e_i - m - n\, K_i - q\, K_j, \tag{32}$$

$$\dot{\mu}_i = (r+\delta)\,\mu_i - (p_1 + p_3\, K_i + p_4\, K_j) - n\, e_i - \sigma_i\phi_3, \tag{33}$$

$$\dot{\sigma}_i = (r+\delta-\phi_2)\sigma_i - (p_2 + p_4\, K_i + p_5\, K_j) - q\, e_i. \tag{34}$$

hold along jurisdiction i's optimal trajectory of investment (where the transversality condition has been omitted for sake of brevity).

The optimality conditions provide the higher level jurisdiction with the information needed to foresee how the proposed policy alters local jurisdictions' incentives to invest in public infrastructure. On the basis of this information, the regulator selects the subsidization scheme T^* so as to decentralize the social optimum. Formally, this amounts to choosing T^* in such a way that the optimality conditions (32)-(34) match the conditions for a social optimum (11) and (12). The following proposition characterizes the optimal tax rule:

Proposition 3. *The optimal subsidization scheme that decentralizes the socially optimal time-path of expenditure in public infrastructure capital as a markov-perfect Nash equilibrium is*

$$\tau_i^\star(K_i,\, K_j){=}m^\star + n^\star K_i + q^\star K_j, \quad \forall i\,(i{\neq}j){=}1,\, 2,$$

where

$$q^\star \;=\; \frac{(4\,p_5 - c_2\,(2\,r-\rho_1-3\rho_2)\,(2\,r-3\,\rho_1-\rho_2))\,(\rho_1-\rho_2)+8\,p_4\,(-r+\rho_1+\rho_2)}{4\,(r-\rho_1-\rho_2)\,(2\,(r+\delta)-\rho_1-\rho_2)}, \tag{35}$$

$$n^\star \;=\; -\frac{2\,(p_4-p_5)\,(r-2\,\rho_1)}{(r+2\delta)\,(2r-3\rho_1-\rho_2)} + q^\star\left(1-\frac{2\,(r-2\rho_1)\,(\delta+\rho_1)}{(r+2\delta)\,(2r-3\rho_1-\rho_2)}\right), \tag{36}$$

and

$$\begin{aligned} m^\star{=}\frac{1}{(r+\delta)}&\left\{((r+\delta)\;c_1-p_1)+\frac{p_2\;(\rho_1-\rho_2)}{(2\,r-\rho_1-\rho_2)}\right\}\\ &+\frac{K^\infty}{(r+\delta)}\left\{(\delta\;(r+\delta)\;c_2-p_3+p_5)-\frac{2\;(p_4+p_5)\;(r-\rho_1)}{(2\,r-\rho_1-\rho_2)}\right.\\ &\left.\qquad -q^\star\left(r+\frac{2\,\delta\;(r-\rho_1)}{(2\,r-\rho_1-\rho_2)}\right)-n^*\;(r+2\delta)\right\} \end{aligned} \tag{37}$$

Proof. See Appendix B. □

Elucidating the dynamic properties of the optimal subsidization scheme in this general model is a non-trivial task due to the complex interactions between stocks of green infrastructure. Indeed, these dynamic properties will obviously change depending on whether stocks of infrastructure are independent, strategic complements or strategic substitutes. Furthermore, they are also be affected by the strengh of the assumed relationship. Accordingly, in the next section, we first analyse three simple examples in which both the nature of the strategic interaction and its strengh are fixed. In the subsequent section, we elicit to what extent the insights from the three auxiliary models carry over to the general case.

5 Three limit cases

We now present a detailed analysis of the dynamic properties of the optimal subsidization scheme in three limit cases. In this section, we assume that $p_1{=}p_2$ and $p_3{=}p_5$. Our three examples are then obtained by setting successively p_4 equal to either 0, p_3 or $-p_3$. Obviously, they correspond respectively to situations of strategic independence ($p_4{=}0$), strategic substitutability ($p_4{<}0$) and strategic complementarity ($p_4{>}0$). In an effort to simplify the discussion, we shall further assume a context of capital accumulation; i.e., $K_i^0{<}K^\infty$, $i({\neq}j){=}1,\,2$. Finally, we adopt the convention that $K_1^0{>}K_2^0$.

Before we turn to the analysis of our three examples, the following remark is worth mentionning.

Remark 4. *sign*$(\delta+\rho_2)$=*sign*(p_3+p_4) *and sign*$(\delta+\rho_1)$=*sign*(p_3-p_4).

Proof. From the expressions of ρ_1 and ρ_2 given in equations 18 and 19. □

5.1 Strategic Independance p_4=0

Of special interest is the solution to the investment regulation game when p_4=0. In this case, note that $\partial^2 P_i(K_i,\, K_j)/(\partial K_j \partial K_i)$=0, $i(\neq j)$=1, 2. In other words, an increase in the stock of green infrastructure held by one jurisdiction has no impact on the marginal productivity of its rival's investment. Such a strategic independance between the two stocks of infrastructure implies that the regulation of each jurisdiction can be considered as a separate task and conducted independantly. Accordingly, optimal investment rules require that each jurisdiction conditions its current level of investment exclusively on the current value of its own stock of infrastructure capital. Given p_4=0 note that ρ_1=ρ_2 so that the following corollary can be stated[7] :

Corollary 5. *Given p_4=0, the socially optimal time-paths of investment in public infrastructure are*

$$e_i(t)=(\delta+\rho)\left(K_i^0-K^\infty\right)e^{\rho t}+\delta\, K^\infty,\quad \forall i(\neq j)=1,\,2, \tag{38}$$

where

$$\rho=\frac{1}{2}\left[r-\sqrt{(r+2\,\delta)^2-8\,p_3/c_2}\right]. \tag{39}$$

Stocks of green infrastructure evolve along the following trajectories

$$K_i(t)=\left(K_i^0-K^\infty\right)\,e^{\rho t}+K^\infty,\quad \forall i(\neq j)=1,\,2 \tag{40}$$

which converge to the unique steady-state

$$K^\infty=\left((r+\delta)\,c_1-2\,p_1\right)/\left(2\,p_3-\delta(r+\delta)c_2\right). \tag{41}$$

The feedback representation of socially optimal time-paths of investment is $e_i(K_i)=(\rho+\delta)K_i-\rho K^\infty$, i=1, 2. As expected, Jurisdiction i's investment behaviour along the optimal path depends exclusively on $K_i(t)$. Observe that $\dot{e}_i(t)=\rho(\rho+\delta)(K_i^0-K^\infty)e^{\rho t}<0$ and $\dot{K}_i(t)=\rho(K_i^0-K^\infty)e^{\rho t}>0$. Therefore, both jurisdictions reduce their investments over time until the steady-state is reached. Correspondingly, both stocks of infrastructure increase monotonically and converge to their steady-state level K^∞ from below. Finally, note that $e_1(t)-e_2(t)=(\delta+\rho)(K_1^0-K_2^0)e^{\rho t}<0$. Hence, Jurisdiction 1 invests less than its rival during the transition phase.

Let us now turn to the characterization of the optimal subsidization scheme. An obvious consequence of strategic independance is that the subsidy faced by Jurisdiction i at each instant t is independant of the stock of infrastructure held by its rival, $K_j(t)$. If p_4=0, then it can be checked that $q^\star$=0 so that the optimal subsidization scheme rewrites as $\tau_i^\star(K_i)=m^\star+n^\star K_i(t)$, $i(\neq j)$=1, 2. The following proposition holds:

Proposition 6. *Given p_4=0, the optimal subsidization scheme is given by $\tau_i^\star(K_i)=m^\star+n^\star\,K_i(t)$, i= 1, 2, where $m^\star=p_2/(r+\delta)>0$ and $n^\star=p_5/(r+2\delta)<0$. Since $\tau^\infty=\tau^\star(K^\infty)>0$, both jurisdictions are granted a subsidy at the steady-state.*

Proof. Note that

$$\begin{aligned}
\tau^\infty>0 \quad &\Leftrightarrow \quad K^\infty<-m^\star/n^\star,\\
&\Leftrightarrow \quad \tfrac{(r+\delta)c_1-2\,p_1}{2\,p_3-\delta(r+\delta)c_2}<-\tfrac{p_1(r+2\,\delta)}{p_3(r+\delta)},\\
&\Leftrightarrow \quad \left((r+\delta)c_1-2p_1\right)p_3(r+\delta)<-p_1(r+2\delta)\left(2\,p_3-\delta(r+\delta)c_2\right),\\
&\Leftrightarrow \quad p_3(r+\delta)^2c_1+2\,\delta\,p_1p_3-p_1\delta(r+2\delta)(r+\delta)c_2<0.
\end{aligned}$$

□

[7] It can be easily checked that the two eigenvectors corresponding to the degenerate eigenvalue ρ are orthogonal.

When stocks are independent $n^\star$ is negative. The subsidy granted to each jurisdiction decreases monotonically over time as its stock of infrastructure converges to the steady-state level K^∞. This pattern of evolution combined with the fact that $\tau_i^\infty{>}0$ implies that $\tau_i(t){>}0$, $\forall t{\in}[0,+\infty[$; i.e., the two jurisdictions are subsidized over the whole horizon of the game. However, Jurisdiction 1 receives a lower subsidy all along transition to the steady-state. Indeed, we have $\tau_1^\star(t)-\tau_2^\star(t){=}n^\star(K_1^0-K_2^0){<}0$.

Finally, it is important to note that the optimal markovian policy differs from the optimal open-loop policy. By direct comparison, we obtain:

$$\tau^\star(K_i(t)){=}\tau_i^o+\gamma \quad \text{where} \quad \gamma{=}-\frac{\delta\, p_5}{(r+\delta)(r+2\delta){>}0}. \tag{42}$$

When $K_i(t){>}0$, the markovian time-path of subsidization is always located above the one corresponding to the optimal open-loop policy. Indeed, in the latter case, the subsidy granted to jurisdiction i decreases more rapidly as its stock of capital converges to the steady state.

This result might seem counterintuitive at first. As shown by Figuières (2009), when $p_4{=}0$ the MPE equilibrium and the Open-Loop Nash Equilibrium of the 'duopoly' subgame coincide.

5.2 Cas $p_3{=}p_4{=}p_5$

Given $p_3{=}p_4{=}p_5{<}0$, the model describes a context in which stocks are strategic substitutes: $\partial^2 P_i(K_i,\,K_j)/(\partial K_j \partial K_i){<}0$. Namely, an increase in the stock of green infrastructure held by one jurisdiction reduces the marginal productivity of its rival's investment. Furthermore, note that the environmental quality index can be rewritten as $P_i(\mathbb{K}){=}p_0+p_1\mathbb{K}+p_3\mathbb{K}^2$, $i{=}1,\,2$, where $\mathbb{K}(t){=}K_1(t)+K_2(t)$. It appears clearly that the relevant variable is the aggregate stock of infrastructure $\mathbb{K}$. In fact, the rate of investment at any time t and the resulting distribution of capital between the two jurisdictions is determined solely by cost minimization conditions ($e_i(t){=}(\lambda_i(t){-}c_1)/c_2)$, $i{=}1,\,2$). In the long-run, since the cost of investment is assumed to be quadratic, these conditions require that $e_1^\infty{=}e_2^\infty{=}\delta K^\infty$[8] Finally, given $p_3{=}p_4{=}p_5$ note that $\rho_1{=}-\delta$ so that the following proposition holds:

Proposition 7. *Given $p_3{=}p_4{=}p_5$, the socially optimal aggregate time-path of investment in public infrastructure is*

$$E(t){\equiv}e_1(t)+e_2(t){=}(\delta+\rho)\,\left(\mathbb{K}^0-\mathbb{K}^\infty\right)e^{\rho t}+\delta\,\mathbb{K}^\infty, \tag{43}$$

where ρ is given by

$$\rho{=}\frac{1}{2}\left[r-\sqrt{(r+2\,\delta)^2-16\,p_3/c_2}\right]. \tag{44}$$

The aggregate stock of green infrastructure evolves along the following trajectory

$$\mathbb{K}(t){=}\left(\mathbb{K}^0-\mathbb{K}^\infty\right)e^{\rho t}+\mathbb{K}^\infty, \tag{45}$$

which converges to the unique steady-state :

$$\mathbb{K}^\infty{=}2\,K^\infty{=}2\,((r+\delta)\,c_1-2\,p_1)\,/\,(4\,p_3-\delta(r+\delta)c_2)\,. \tag{46}$$

Individual trajectories of investment and capital accumulation are given by $e_i(t){=}{}^1\!/_2\,E(t)$ and $K_i(t){=}{}^1\!/_2\,\mathbb{K}(t)+{}^1\!/_2\,(K_i^0-K_j^0)\,e^{-\delta t}$, $i({\neq}j){=}1,\,2$.

[8]Note that these properties carry over to the more general case where $p_1{\neq}p_2$. Indeed, p_1 and p_2 appears only in the expression for K^∞.

Observe that $\dot{E}(t)=\rho\,(\delta+\rho)\left(\mathbb{K}_i^0-\mathbb{K}_i^\infty\right)e^{\rho t}<0$. Hence, both jurisdictions follow the same time-path of investment which is monotonically decreasing and converges to e^∞ from above. Furthermore, since $\dot{\mathbb{K}}(t)=\rho(\mathbb{K}_i^0-\mathbb{K}^\infty)\,e^{\rho t}>0$, the aggregate stock of green infrastructure increases monotonically and converges to $\mathbb{K}^\infty$. However, $K_2(t)$ increases monotonically and converge to K^∞ from below whereas $K_1(t)$ overshoots the steady state and converge to K^∞ from above.

Let us now turn to the properties of the optimal policy instrument. From Equation 35, observe that $p_4=p_5$ and $\rho_1=-\delta$ imply $n^\star=q^\star$. Therefore, the optimal subsidization rule faced by Jurisdiction i rewrites as $\tau_i^\star(\mathbb{K})=m^\star+n^\star\mathbb{K}$. Hence, the evolution of the subsidy received by each jurisdiction is driven by that of the aggregate stock of infrastructure. We obtain the following proposition:

Proposition 8. *The optimal subsidization scheme is given by* $\tau_i^\star=m^\star+n^\star\,\mathbb{K}(t)$, $i=1,\,2$, *where*

$$
\begin{aligned}
m^\star &= \frac{\left(2\,n\,(r+\delta)\,(-2\,(r+\delta)+\rho)+(r-\rho)\left(\delta\,(r+\delta)-r\,\rho+\rho^2\right)c_2\right)K_\infty}{(r+\delta)\,(2\,r+\delta-\rho)}+\frac{2\,(r-\rho)\,((r+\delta)\,c_1-p_1)}{(r+\delta)\,(2\,r+\delta-\rho)},\\
n^\star &= \frac{(r-2\,\rho)\,(\delta+\rho)\,c_2}{4\,(r+\delta-\rho)}.
\end{aligned}
$$

At the steady-state, both Jurisdictions receive a subsidy, $\tau^\infty=\tau^\star(\mathbb{K}^\infty)>0$.

Proof. We prove that the optimal subsidization scheme stipulates that the two firms should receive a subsidy at any time t. Since the sign of $m^\star$ is not known, we use the following indirect argument. To begin with, recall that $\dot{\mathbb{K}}(t)=\rho(\mathbb{K}_i^0-\mathbb{K}^\infty)\,e^{\rho t}>0$. Because $n^\star<0$ the subsidy granted to Jurisdiction i decreases monotonically as the aggregate stock of infrastructure increases and converges to the steady-state. To prove that $\tau_i(t)>0$ for all $t\in[0,+\infty[$ (and, accordingly, that $m^\star>0$), then it is sufficient to show that $\tau(K^\infty)>0$. Note that $\tau(\mathbb{K})=\frac{1}{R}\,(V+W)$ where

$$
\begin{aligned}
R &= 2\,(r-\rho)\,(r+\delta-\rho)\,(2\,r+\delta-\rho)\,\rho,\\
V &= (\delta+\rho)\left(r\,(r+\delta)\,(2\,r+\delta)-6\,r\,(r+\delta)\,\rho+2\,(3\,r+2\,\delta)\,\rho^2-2\,\rho^3\right)c_1,\\
W &= -2\,\delta\left(r\,(2\,r+\delta)-5\,r\,\rho+4\,\rho^2\right)p_1.
\end{aligned}
$$

Observe that $(\delta+\rho)<0$ and $\rho<0$ implies that R, V and W are negative so that $\tau(K^\infty)>0$. We conclude that $\tau_i(t)>0$ for all $t\in[0,+\infty[$. □

Finally, note that this case in which the relevant variable is the aggregate stock of capital $\mathbb{K}$ presents some similarities with the model considered by Benchekroun and Long (1998). The analogy is even more striking if we further assume that $K_i^0=K_j^0$ so that both jurisdictions follow the same path of investment. One could then view our model as a counterpart of the one studied by Benchekroun and Long (1998) where player are linked by positive rather than negative externalities. Accordingly, one would be led to conclude that there are some instances in which the optimal policy requires that the jurisdictions be taxed before being subsidized. However, we have just shown that this cannot be the case.

At this point, it is important to emphasize a major difference between our model and that of Benchekroun and Long (1998). In our 'duopolity' model, players compete indirectly through their stocks of capital whereas in the duopoly model considered by Benchekroun and Long (1998) players' control variables interact explicitly in the revenue functions $p_i=(a-e_i-e_j)\,e_i$ where e_i is firm i's control variable (quantity). In other words, there is no term featuring the product e_ie_j in our model. Naturally, this is a concious modelling decision designed to eliminate the strategic effect uncovered by Benchekoun and Long from the analysis. This choice allows us to focus on the strategic role of initial conditions.

5.3 Cas $p_3 = p_5 = -p_4$

Given $p_3 = -p_4 = p_5$, the model describes a context in which stocks are strategic complements: $\partial^2 P_i(K_i,\, K_j)/(\partial K_j \partial K_i) > 0$. Namely, an increase in the stock of green infrastructure held by one jurisdiction increases the marginal productivity of its rival's investment. Now, intuition suggests that the relevant variable will be the difference in the stock of infrastructure held by the two jurisdictions. Given $p_3 = p_5 = -p_4$ note that $\rho_2 = -\delta$ so that the following corollary can be stated:

Corollary 9. *Given $p_3 = -p_4 = p_5$, the socially optimal time-paths of investment in public infrastructure are*

$$e_i(t) = (1/2)\,(\delta + \rho)\,\left(K_i^0 - K_j^0\right) e^{\rho t} + \delta\, K^\infty, \quad \forall i (\neq j) = 1,\, 2, \tag{47}$$

where

$$\rho = \frac{1}{2}\left[r - \sqrt{(r + 2\,\delta)^2 - 16\, p_3/c_2}\right]. \tag{48}$$

The stocks of green infrastructure evolve along the following trajectories

$$K_i(t) = \frac{1}{2}\left(K_i^0 - K_j^0\right) e^{\rho t} + \frac{1}{2}\left(K_i^0 + K_j^0 - 2\,K^\infty\right) e^{-\delta t} + K^\infty, \forall i (\neq j) = 1,\, 2 \tag{49}$$

which converge to the unique steady-state

$$K^\infty = (2\, p_1 - (r + \delta)\, c_1)\, /\, (\delta (r + \delta) c_2)\,. \tag{50}$$

Proof. Straightforward. □

Observe that the aggregate investment remains constant over time and equal to $E(t) = e_1(t) + e_2(t) = 2\delta K^\infty$. Moreover, note that $e_1(t) - e_2(t) = (\delta + \rho)(K_1^0 - K_2^0) e^{\rho t} > 0$; i.e., the jurisdiction with the higher initial endowment in infrastructure capital invests more than its rival over the whole transition period. Finally, the aggregate stock of capital is given by $\mathbb{K}(t) = (K_i^0 + K_j^0 - 2\,K^\infty) e^{-\delta t} + 2\,K^\infty$ and converges to $\mathbb{K}^\infty = 2K^\infty$ from below.

Let us now turn to the characterization of the optimal subsidization scheme. From Equation 35, observe that $p_3 = p_5 = -p_4$ and $\rho_2 = -\delta$ imply $n^\star = -q^\star$. Therefore, the optimal subsidization rule for Jurisdiction i rewrites as $\tau_i^\star(K_i,\, K_j) = m^\star + n^\star(K_i - K_j)$. The evolution of the subsidy received by each jurisdiction is thus driven by that of the difference in the stock of infrastructure capital held by the two jurisdictions. The characterization of the optimal subsidization scheme is provided by the following proposition:

Proposition 10. *The optimal subsidization scheme is given by $\tau_i^\star = m^\star + n^\star\,\left(K_i - K_j\right)$, $i = 1,\, 2$, where*

$$m^\star = \frac{4}{\left(3\,r + 2\,\delta + \sqrt{(r + 2\,\delta)^2 - 16\, p_3/c_2}\right)}\left(p_1 - \frac{2\, p_3\, \delta \sqrt{c_2\left((r+2\delta)^2 c_2 - 16 p_3\right)}}{(r+2\delta)\left((r+2\delta) c_2 + \sqrt{c_2\left((r+2\delta)^2 c_2 - 16 p_3\right)}\right) - 8 p_3}\right)$$

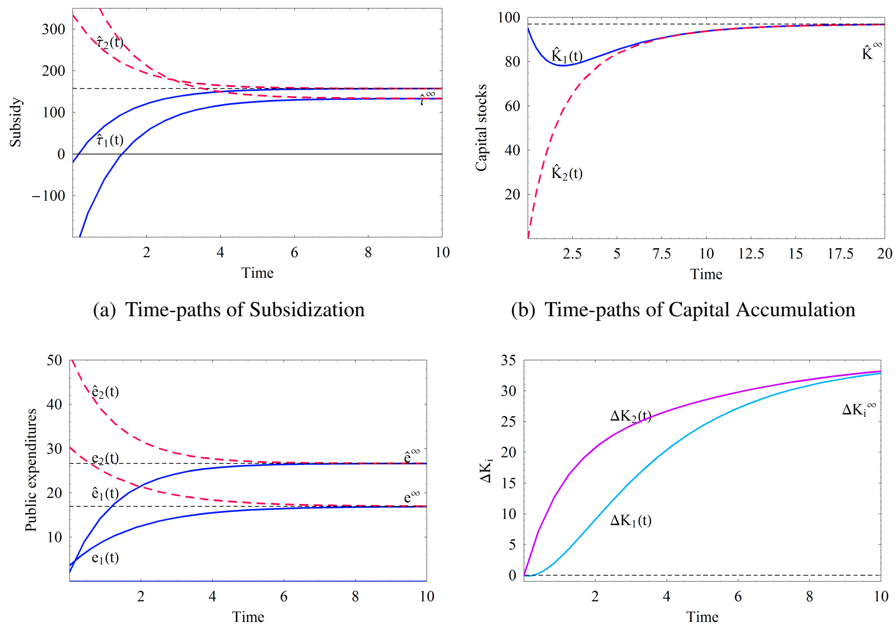
and

$$n^\star = \frac{4\, p_3 \sqrt{c_2}\, \sqrt{(r+2\,\delta)^2\, c_2 - 16\, p_3}}{\left((r+2\,\delta) \sqrt{c_2} + \sqrt{(r+2\,\delta)^2\, c_2 - 16\, p_3}\right)^2} < 0.$$

At the steady-state, $\tau^\infty = m^\star > 0$.

Proof. Straightforward. □

At time $t = 0$, note that Jurisdiction i faces a lower rate of subsidization than its rival. In the short-run, this difference in fiscal treatment increases as the gap between the two stocks increases. Indeed, the subsidy faced by jurisdiction 1 decreases whereas that of jurisdiction 2 increases. In the long-run, both subsidies converge to $\tau^\infty > 0$. In the short-run however, Jurisdiction 1 may be taxed while Jurisdiction 2 is subsidized. A numerical example is illustrated in Figure 1, using the following parameter values: p_0=0, p_1=p_2=50, p_3=p_5=$-p_4$=-1.5, r=0.1, c_1=0, c_2=10, δ=0.275, K_1^0=95>K_2^0=0.



(a) Time-paths of Subsidization

(b) Time-paths of Capital Accumulation

(c) Time-paths of Investment

(d) Infrastructure Gap

Figure 1: Carrot and stick policy with complementarity

6 An attempt at generalization

The following general picture emerges from the preceding section. First, in the markovian scenario the one-to-one relationship between the sign of the externality generated by jurisdiction i's investment in infrastructure capital and its tax treatment is broken. Second, the subsidy granted to jurisdiction i decreases as its own stock of infrastructure increases and converges to the social optimum (m>0, n<0). This is consistent with intuition, since additional capital accumulation renders the underinvestment problem less severe. Third, there seems to be a strong and stable relationship between the sign of p_4 and the sign of $q^\star$. Indeed, recall that we obtained $q^\star$<0 when p_4=p_3<0, $q^\star$=0 when p_4=0 and $q^\star$>0 when p_3=$-p_4$>0. In other words, the subsidy received by each jurisdiction seems to be decreasing, constant or increasing in the stock of infrastructure held by its rival depending on whether stocks are substitutes, independent or complements. In section 6.1, it is shown that this property holds in the general case. Therefore, the dynamic properties of the optimal subsidization scheme can be deduced directly from the payoff structure of the 'duopolity' subgame.

Finally, in our three examples, under the assumption of capital accumulation (K_1^0<K_2^0), the optimal instrument remains constant in sign over the whole horizon of the game under both strategic independence and strategic substitutability. By contrast, Jurisdiction 1 may be taxed in the short-run in a context of strategic complementarity. A natural question to ask is whether or not these properties carry over the general case. These three conjectures deserve further scrutiny. They are now considered in turn.

6.1 Strategic properties of the optimal subsidization scheme

For the class of capital accumulation games considered in this paper, Figuieres (2009) shows that the nature of strategic interaction at the markov perfect equilibrium can be deduced directly from the payoff structure. The 'duopolity' subgame is characterized by markov substitutability, complementarity or independence depending on whether p4 is negative, positive or zero. Similarly, the following proposition shows that the value of p_4 allows us to infer the qualitative properties of the optimal markovian subsidization rule whithout having to compute it.

Proposition 11. *Suppose that the stocks are* strategic complements $(p_4{>}0)$ *then the subsidy received by each jurisdiction is* increasing *in the stock of infrastructure held by its rival* (*i.e.*, $q^{\star}{>}0$). *Suppose that the stocks are* strategic substitutes $(p_4{<}0)$ *then the subsidy received by each jurisdiction is* decreasing *in the stock of infrastructure held by its rival* (*i.e.*, $q^{\star}{<}0$).
Suppose that the stocks are independent $(p_4{<}0)$ *then the subsidy received by each jurisdiction is* unaffected *by a change in the stock of infrastructure held by its rival* (*i.e.*, $q^{\star}{=}0$).

Proof. The following identities can be checked by direct computations:

$$p_4 = \frac{1}{2}\left[-(p_3+p_5)+(r+\delta-\rho_2)(\delta+\rho_2)\,c_2\right], \tag{51}$$

$$p_4 = -\frac{1}{2}\left[-(p_3+p_5)+(r+\delta-\rho_1)(\delta+\rho_1)\,c_2\right]. \tag{52}$$

Using these values to eliminate p_4 from Equation 35, we obtain two alternative expressions for $q^{\star}$. Let us denote these expressions by $q_1^{\star}$ and $q_2^{\star}$. Then, we have: $q^{\star}{=}(q_1^{\star}+q_2^{\star})/2{=}-(\rho_1-\rho_2)\,A$ with

$$A{=}\frac{\left(2\,r^2\,c_2-4\,p_5-4\,r\,c_2\,\rho_1+c_2\,\rho_1^2-4\,r\,c_2\,\rho_2+6\,c_2\,\rho_1\,\rho_2+c_2\,\rho_2^2\right)}{4\,\left(r-\rho_1-\rho_2\right)\,\left(2\,\left(r+\delta\right)-\rho_1-\rho_2\right)}{>}0.$$

From Remark 4, we conclude that $q^{\star}$ of the same sign as p_4. Finally observe that $p_4{=}0$ implies $\rho_1{=}\rho_2$ and $q^{\star}{=}0$. □

6.2 Sign of $\tau_i(K_i^0, K_j^0)$ under strategic substitutability

We proceed by considering the second conjecture. Assuming strategic substitutability, this conjecture is supported by the following proposition:

Lemma 12. *Given* $p_5{=}p_3{<}p_4{<}0$*, then* $n^{\star}{<}q^{\star}{<}0$.

Proof. Recall that $n^{\star}{=}A+q^{\star}\,[1+B]$ where

$$A{=}\frac{2(p_4-p_5)(r-2\rho_1)}{(r+2\delta)(3\rho_1+\rho_2-2r)}, \quad B{=}\frac{2(\delta+\rho_1)(r-2\rho_1)}{(r+2\delta)(3\rho_1+\rho_2-2r)}.$$

Observe that

$$(1+B){=}\frac{2\,r\,\left(r+\delta\right)-\left(5\,r+2\,\delta\right)\,\rho_1+4\,\rho_1^2-\left(r+2\,\delta\right)\,\rho_2}{\left(r+2\,\delta\right)\,\left(2\,r-3\,\rho_1-\rho_2\right)}{>}0 \tag{53}$$

for all admissible values of p_4. Furthermore, since $p_5{=}p_3{<}p_4{<}0$ implies $A{<}0$ and $q^{*}{<}0$, we have $n^{\star}{<}0$. In order to prove that $n^{\star}{<}q^{\star}{<}0$, it remains to show that $(n^{\star}-q^{\star}){<}0$. Note that $\text{sign}(n^{\star}-q^{\star}){=}-\text{sign}\left[(p_4-p_5)+q^{\star}(\delta+\rho_1)\right]$ with $\text{sign}\,(\delta+\rho_1){=}\text{sign}(p_3-p_4)$. We obtain $(n^{\star}-q^{\star}){<}0$. □

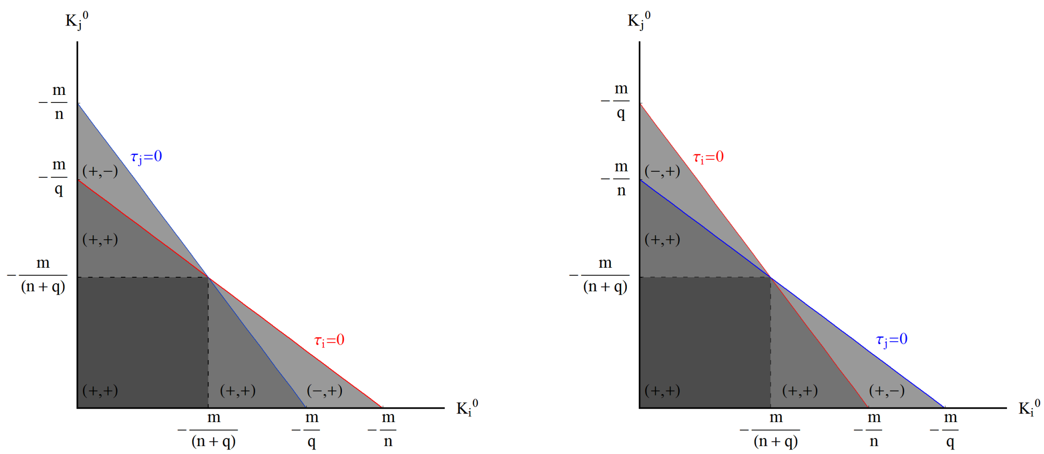
In other words, under mild substitutability, the subsidy received by each jurisdiction is decreasing in the level of both stocks of infrastructure.

Lemma 13. *Given* $p_4{<}0{<}p_5{=}p_3{<}0$*, then* $(n-q){>}0$.

Proof. From

$$(n^\star - q^\star) = \frac{2\,(p_4 - p_5)\,(r - 2\rho_1)}{(-2r + 3\rho_1 + \rho_2)\,(r + 2\delta)} + q^* \frac{2\,(r - 2\rho_1)\,(\delta + \rho_1)}{(-2r + 3\rho_1 + \rho_2)\,(r + 2\delta)}$$

note that $sign\,(n^* - q^*) = -sign\,[(p_4 - p_5) + q^*\,(\delta + \rho_1)]$. Given $p_4 < p_5 = p_3 < 0$, we have $q^\star < 0$, $(\delta + \rho_1) > 0$ and $(p_4 - p_5) < 0$. Hence, we get $(n^\star - q^\star) > 0$. We conclude that $K_i^\star(t) > K_j^\star(t) \Leftrightarrow \tau_i^\star(t) > \tau_j^\star(t)$ and thus $K_i^\star(0) > K_j^\star(0) \Leftrightarrow \tau_i^\star(0) > \tau_j^\star(0)$. □



(a) $m>0$, $q<n<0$

(b) $m>0$, $n<q<0$

Figure 2: Signs of $\tau_i^\star(K_i^0, K_j^0)$ and $\tau_j^\star(K_j^0, K_i^0)$ under strategic substitutability

Then, the following questions arise: when does the optimal policy scheme require to tax one jurisdiction and subsidize the other ? What is the economic factor that determines which jurisdiction is taxed or subsidized ? Under strategic substitutability, we have the following answers.

Proposition 14. *Given $p_5 = p_3 < p_4 < 0$. Assume that Jurisdiction i is initially taxed whereas Jurisdiction j is subdivised ($\tau_j^\star(0) < 0$ and $\tau_i^\star(0) > 0$) then $K_j^\star(0) > K_i^\star(0)$.*

Observe that $\tau_i^\star(t) - \tau_j^\star(t) = (n^\star - q^\star)\left(K_i^\star(t) - K_j^\star(t)\right)$. Lemma 13 ensures that $(n^\star - q^\star) < 0$ for $p_5 = p_3 < p_4 < 0$. We conclude that $K_i^\star(t) > K_j^\star(t) \Leftrightarrow \tau_i^\star(t) < \tau_j^\star(t)$. Proposition 11 follows directly from this relation.

Proposition 15. *Given $p_4 < p_5 = p_3 < 0$. Assume that Jurisdiction j is initially taxed whereas Jurisdiction i is subsidized ($\tau_j^\star(0) < 0$ and $\tau_i^\star(0) > 0$) then $K_j^\star(0) < K_i^\star(0)$.*

In other words, under 'mild' substitutability $(p_4 < p_5 = p_3)$ when there is an initial taxation, then it applies to the jurisdiction with the largest initial capital stock. By contrast, when the substitutability is strong enough $(p_4 > p_5 = p_3)$ then it applies to the jurisdiction with the lower initial capital stock.

6.3 Sign of $\tau_i(K_i^0, K_j^0)$ under strategic complementarity

At this point, it is natural to ask whether it is possible to establish a counterpart of Lemma 13 under strategic complementarity. Recall that we expect the parameters $n^\star$ and $q^\star$ to have opposite signs in this case. Furthermore, we expect $n^\star$ to be strictly negative because $q^\star$ is strictly positive. Hence, the natural counterpart of Lemma 13 would read as : given $-p_5{=}-p_3{>}p_4{>}0$ then $q^\star{>}0{>}n^\star$.

Unfortunately, we are unable to sign $n^\star$ from its expression given in 36. Admittedly, we know that $n^\star$ is negative if $q^\star{>}-A/[1+B]{>}0$ but this interval may be empty. One way to circumvent this difficulty would be to prove that $(n^\star + q^\star)$ is negative for $-p_5{=}-p_3{>}p_4{>}0$. At this point, an important remark is in order. Recall that the values of $q^\star$ and $n^\star$ at the boundaries of the interval $-p_5{=}-p_3{>}p_4{>}0$ are already known. In the previous section, it was shown that $p_4{=}0$ implies $q^\star{=}0$ and $n^\star{<}0$; i.e., $(n^\star + q^\star){<}0$. In addition, it was found that $p_4{=}-p_3$ implies $q^\star{=}-n^\star$ or $(n^\star + q^\star){=}0$. Then, using a continuity arguments, it can be argued that there exists at least one sub-interval of $p_5{=}p_3{>}p_4{>}0$ for which $q^\star{>}0{>}n^\star$. The following numerical illustration is offered as a typical example.

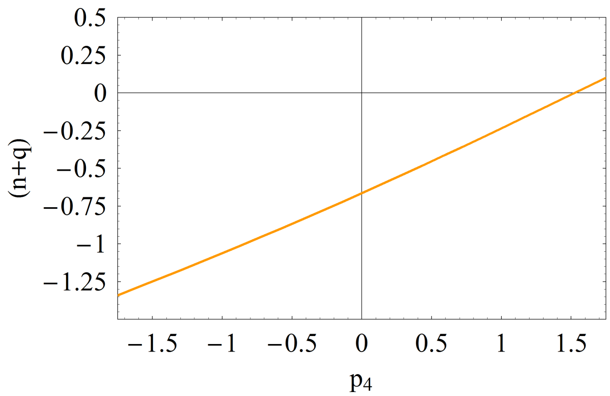
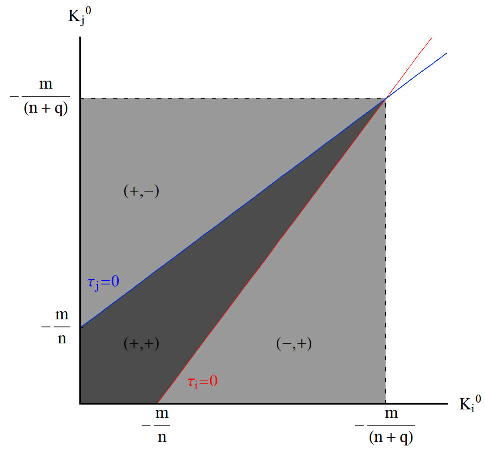


Figure 3: $(n^\star + q^\star)$ as a function of p_4 (parameter values: $p_0{=}0$, $p_1{=}50$, $p_2{=}50$, $p_3{=}-1.525$, $p_5{=}-1.525$, $r{=}0.1$, $c_1{=}0$, $c_2{=}10$, $\delta{=}0.275$, $K_1^0{=}90$, $K_2^0{=}0$)

Given the chosen parameter values, we have $(n^\star + q^\star){<}0$ for all $p_4{\in}]0, -p_3[$. Such an example is, if not a definitive proof, at least an indication of the broad validity of our second conjecture in the case of mild strategic complementarity[9]. We conclude this section by a discussion of the validity of our last conjecture: both firms face a subsidy over the whole transition period.

[9] We were unable to find a numerical counter-example.



(a) $m>0$, $(m+q)<0$

Figure 4: Signs of $\tau_i^\star(K_i^0, K_j^0)$ and $\tau_j^\star(K_j^0, K_i^0)$ under strategic substitutability

Given the chosen parameter values, Jurisdiction 1 - the one with the higher initial stock of infrastructure - is initially taxed whereas jurisdiction 2 is subsidized. However, it is possible to construct numerical examples in which the converse holds[10].

Let us turn to the complementarity case. Proofs of proposition 11 and 12 rely on the sign of $(n^\star - q^\star)$ which is ambiguous under the assumption of complementarity. However, we provided broad evidence above to argue that we have $q^\star > 0 > -q^\star > n^\star$ for $-p_5 = -p_3 > p_4 > 0$. In this case, we have $(n^\star - q^\star) < 0$. Consequently, it is reasonable to think that the following statement is correct: under mild complementarity $(-p_5 = -p_3 > p_4)$ when there is an initial taxation, then it applies to the jurisdiction with the largest initial capital stock.

Finally, note that such a differential treatment cannot occur when both jurisdictions have identical initial endowments in infrastructure capital.

7 Conclusion

This paper complements earlier contributions on price-based policies in a dynamic setting by investigating how differences in initial conditions alter the dynamic properties of the optimal tax or subsidy policy. Specifically, we concentrate on a model of competition in public infrastructure between two jurisdictions. Each jurisdiction invests over time in green infrastructure to provide environmental services to its own residents. However, once supplied, it is supposedly impossible to exclude the residents of the other jurisdiction from the consumption of these environmental services. We assume that the two jurisdictions are identical in all respects except (perhaps) their initial stocks of green infrastructure capital. As is well known, this is a dynamic type of heterogeneity that disappears in the long run. Consequently, at the steady state, usual intuitions from static settings apply. Due to the presence of positive inter-jurisdictional spillovers, both jurisdictions will inefficiently under-invest, which calls for the implementation of a green capital subsidization scheme. In the short run, however, counterintuitive properties are established:

i) the pigovian scheme is not necessarily a subsidy. This finding confirms that the sign of the instrument, negative for a tax, positive for a subsidy, in a dynamic context is relatively

[10] Examples are available from the authors upon request

unrelated to the immediate goals of reducing or increasing the incentives. Intuitions gained from static settings cannot be transposed into dynamic frameworks without care; important qualifications are often required. For instance in situations where the goal is to encourage investments, to some extent it does not matter whether the incentive instrument is a tax rather than a subsidy, provided that the tax is a decreasing function of the investment.

ii) One jurisdiction can be temporarily taxed, even though at those taxation dates its investments should be increased, whereas the other is subsidized. It is shown how these phenomena are related to initial conditions and to the kind of technological link between stocks of infrastructure (complementarity or substitutability). Put differently, initial conditions can be important drivers for the qualifications alluded to above.

A follow-up research of the present analysis would be to investigate the pigouvian regulation of infrastructure competition when public capitals generate negative externalities. One may expect in this context that, despite the needs to discourage non cooperative investments, one jurisdiction might be subsidized at early dates.

Appendices

A Optimal time-paths of investment

In this appendix we characterize the socially optimal time-paths of investment in public infrastructure. From the theory of differential equations, solutions to the system of differential equations (14) are of the form

$$K_i = \alpha_i e^{\rho_1 t} + \beta_i e^{\rho_2 t} + K^\infty, \quad i(\neq j)=1,\ 2. \tag{54}$$

where the parameters $\alpha_1, \alpha_2, \beta_1, \beta_2$ are constant coefficients to be determined. Differentiating (54) with respect to time yields

$$\dot{K}_i = \alpha_i \rho_1 e^{\rho_1 t} + \beta_i \rho_2 e^{\rho_2 t}, \quad i(\neq j)=1,\ 2. \tag{55}$$

Also, we know that when optimal time-paths of public expenditure exist, they can be written in feedback form as $e_i(t)=\phi_1 + \phi_2 K_i(t) + \phi_3 K_j(t)$, $\forall i(i\neq j)=1,2$. Substituting these strategies into the Nerlove-Arrow equations yields an alternative (feedback) representation of optimal capital stock trajectories:

$$\dot{K}_i = (\phi_2 - \delta)K_i + \phi_3 K_j + \phi_1, \quad i(\neq j)=1,\ 2. \tag{56}$$

This system can be rewritten in matrix form as $\dot{K}=BK+h$. The coefficient matrix B admits two distinct real roots:

$$\rho_1 = \phi_2 - \delta - \phi_3, \quad \rho_2 = \phi_2 - \delta + \phi_3. \tag{57}$$

We are now in a position to determine the values of the coefficients $\{\alpha_1, \alpha_2, \beta_1, \beta_2\}$ by identifying equations (54) with (56). From (56) we know that $\phi_3 K_2 = \dot{K}_1 - (\phi_2 - \delta)K_1 - \phi_1$. Plugging (54) and (55) into this expression, and rearranging terms yields:

$$\phi_3 K_2 = \alpha_1(\rho_1 - (\phi_2 - \delta))e^{\rho_1 t} + \beta_1(\rho_2 - (\phi_2 - \delta))e^{\rho_2 t} - (\phi_2 - \delta)K^\infty - \phi_1. \tag{58}$$

Now, from (57) we know that $(\rho_1 - (\phi_2 - \delta)) = -\phi_3$ and $(\rho_2 - (\phi_2 - \delta)) = \phi_3$. Plugging this into (58) and rearranging terms yields

$$K_2 = -\alpha_1 e^{\rho_1 t} + \beta_1 e^{\rho_2 t} - \phi_3^{-1}\left((\phi_2 - \delta)K^\infty + \phi_1\right), \tag{59}$$

By identification of (54) and (59), it comes that $\alpha_1 = -\alpha_2 = \alpha$, $\beta_2 = \beta_1 = \beta$ and $K^\infty = -\phi_1/(\phi_3 + \phi_2 - \delta)$. From (57), it comes that $\phi_2 = (\rho_1 + \rho_2 + 2\delta)/2$ and $\phi_3 = (\rho_2 - \rho_1)/2$. Plugging this in K^∞ yields $\phi_1 = -\rho_2 K^\infty$. Now,let us denote $\Delta K_i(t) = K_i(t) - K^\infty$. Observe that $K_1(0) = \alpha_1 + \beta_1 + K^\infty$ and $K_2(0) = \alpha_2 + \beta_2 + K^\infty$ so that we have a system of equation

$$\alpha + \beta = \Delta K_1(0), \quad \beta - \alpha = \Delta K_2(0), \tag{60}$$

which can be solved to get the values of the coefficients α and β :

$$\alpha = \frac{1}{2}(\Delta K_1(0) - \Delta K_2(0)) = \frac{1}{2}(K_1^0 - K_2^0), \quad \beta = \frac{1}{2}(\Delta K_1(0) + \Delta K_2(0)) = \frac{1}{2}(K_1^0 + K_2^0 - 2K^\infty). \tag{61}$$

Substituting α for α_1, $-\alpha$ for α_2 and β for β_1 and β_2 into (54) yields Equations (20). Finally, Equations (17) easily follow from Equation (3) by observing that $e_i = \dot{K}_i + \delta K_i$ yields

$$e_1 = \alpha(\delta + \rho_1)e^{\rho_1 t} + \beta(\delta + \rho_2)e^{\rho_2 t} + \delta K^\infty, \tag{62}$$

$$e_2 = -\alpha(\delta + \rho_1)e^{\rho_1 t} + \beta(\delta + \rho_2)e^{\rho_2 t} + \delta K^\infty. \tag{63}$$

B Optimal tax/subsidy policy

Let us recall that jurisdiction i's optimal time-path of investment in public infrastructure is given by (62). Plugging e_1^c into the short-run equilibrium condition (32) and rearranging terms yields $\mu_i = v_1 + v_2\alpha e^{\rho_1 t} + v_3\beta e^{\rho_2 t}$ where $v_1 = -m + c_1 + (\delta c_2 - n - q)\,K^\infty$, $v_2 = -n + q + c_2\,(\delta + \rho_1)$ and $v_3 = -(n+q) + c_2\,(\delta + \rho_2)$. Differentiating with respect to time, we get $\dot{\mu}_i = v_2\,\alpha\,\rho_1\,e^{\rho_1 t} + v_3\,\beta\,\rho_2\,e^{\rho_2 t}$. Using this equation to eliminate $\dot{\mu}_i$ from Equation (33), solving for σ^i and rearranging terms yields

$$\sigma^i = w_1 + w_2\alpha e^{\rho_1 t} + w_3\beta e^{\rho_2 t} \tag{64}$$

where

$$w_1 = -\frac{1}{\phi_3}\left(p_1 + K^\infty\,(n\,\delta + p_3 + p_4) - (r+\delta)\,v_1\right), \tag{65}$$

$$w_2 = \frac{1}{\phi_3}\left((p_4 - p_3) + v_2\,(r + \delta - \rho_1) - n\,(\delta + \rho_1)\right), \tag{66}$$

$$w_3 = -\frac{1}{\phi_3}\left((p_3 + p_4) - v_3\,(r + \delta - \rho_2) + n\,(\delta + \rho_2)\right). \tag{67}$$

Finally, using Equation (64) to eliminate σ_i from (34) yields

$$\dot{\sigma}^i = z_1 + z_2\alpha e^{\rho_1 t} + z_3\beta e^{\rho_2 t} \tag{68}$$

with

$$z_1 = p_2 + K^\infty\,(q\,\delta + p_4 + p_5) - w_1\,(r + \delta - \phi_2), \tag{69}$$

$$z_2 = (p_4 - p_5) + q\,(\delta + \rho_1) + w_2\,(-r - \delta + \rho_1 + \phi_2), \tag{70}$$

$$z_3 = (p_4 + p_5) + q\,(\delta + \rho_2) + w_3\,(-r - \delta + \rho_2 + \phi_2). \tag{71}$$

We now replace the coefficients w_1, w_2, w_3 and $v_1,\ v_2,\ v_3$ by their respective values into Equations (69) -(71) to get

$$\begin{aligned}
z_1 &= \frac{1}{\phi_3}\left[\left((r+\delta)(m-c_1)+p_1\right)(r+\delta-\phi_2)+p_2\phi_3\right]+K^\infty(q\delta+p_4+p_5) && (72)\\
&\quad+\frac{K^\infty}{\phi_3}\left[\left((n+q)r+(2n+q)\delta-\delta(r+\delta)c_2+p_3+p_4\right)(r+\delta-\phi_2)\right] && (73)\\
z_2 &= \frac{X}{\phi_3}\left(-(q(r+\delta))+n(r+2\delta)+p_3-p_4+q\rho_1-c_2(r+\delta-\rho_1)(\delta+\rho_1)\right) && (74)\\
&\quad+\left[(p_4-p_5)+q(\delta+\rho_1)\right], && (75)\\
z_3 &= \frac{Y}{\phi_3}\left((n+q)r+(2n+q)\delta+p_3+p_4-q\rho_2-c_2(r+\delta-\rho_2)(\delta+\rho_2)\right) && (76)\\
&\quad+\left[(p_4+p_5)+q(\delta+\rho_2)\right], && (77)
\end{aligned}$$

$$\text{where}\quad X = (r + \delta - \rho_1 - \phi_2),\ \text{and}\ Y = (r + \delta - \rho_2 - \phi_2). \tag{78}$$

Finally, we solve the algebraic system $\{z_1 = 0,\ z_2 = 0,\ z_3 = 0\}$ for the parameters $\{m,\ n,\ q\}$. After tedious but straightforward manipulations, one obtains expressions (35)- (37).

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