Defining sustainability objectives

Vincent MARTINET[*]

September 17, 2008

Abstract

Defining sustainability consists in choosing between conflicting objectives, with a strong intergenerational equity concern. In this paper, we propose a way to select sustainability objectives. Our approach is based on the definition of minimal rights to be guaranteed to any generation, in a Rawlsian equity perspective. We develop a framework that describes the trade-offs between minimal rights given the initial endowment of the economy, and apply a criterion that select sustainability objectives among feasible ones. The resulting criterion is a "generalized" maximin. We illustrate this approach by applying it to a canonical model in the literature on sustainability (Dasgupta-Heal-Solow model; Review of Economic Studies 1974). Last, we discuss on the possibility to apply such an approach to real-life sustainability issues.

Key-words: sustainability, intergenerational equity, minimal rights, viability.
JEL Classification: Q01, Q32, O13, C61.

---

[*] Economie Publique - UMR INRA-AgroParisTech. 78850 Thiverval-Grignon, France. Email: vincent.martinet@grignon.inra.fr Phone: +33 1 30 81 53 57.

Contents

1 Introduction

Robert Solow (1993, p.167-168) claimed that

> *"If the sustainability means anything more than a vague emotional commitment, it must require that something be conserved for the very long run. It is very important to understand what that thing is."*

Sustainability objectives consist in preserving something for future generations. According to the famous Brundtland report, *Our Common Future* (WCED, 1987), a *sustainable development* is a "development that meets the needs of the present without compromising the ability of future generations to meet their own needs". This explicit reference to the "needs" of the various generations can be a way to address the issue. An important challenging question is then the definition of the needs to be met by generations for a sustainable development.

The economic theory addresses the sustainability issue by defining general sustainability criteria (Heal, 1998) that take into account environmental concerns and intergenerational equity issues (Beltratti et al., 1995; Cairns and Long, 2006). A strong body of literature adopts an axiomatic approach to determine if there exist preference rules over infinite stream of utilities that satisfy equity and efficiency properties (Diamond, 1965; Sakai, 2006). Chichilnisky (1996) proposes a very interesting approach defining a (family of) criterion satisfying a set of axioms representing intergenerational equity requirements.

The link between these theoretical approaches and the real world issues is based on the definition of a comprehensive utility function that reflects all sustainability objectives (Hediger, 2000). From a practical point of view, it is often hard to do: as sustainability has to encompass various objectives, it requires to define multi-attribute utility functions and develop a multiobjective optimization (Béné and Doyen, 2000). Such optimal control problems are hard to solve and often require strong simplifications of the objectives function (for example to linearize the utility function by weighting all the criteria in order to materialize the trade-offs between the conflicting objectives; see Riesgo and Gómez-Limón (2006) for agricultural economics case-studies). We should qualify these approaches of "top-down", going from theory and general criteria to the real issues and applications.

Sustainable development encompasses several objectives, often classified under the usual three categories: Economic, environmental and social. From an operational perspective, sustainability assessment is generally based on a set of indicators (UN, 2001). Sustainability indicators differ from a situation to another. For example, U.S. are much more concerned with security issues and crime rates than European countries (U.S., 2001; European Commission, 2004) ; Developing countries focus more on development and economic issues than developed countries, these latter emphasizing environmental issues. From that point of view, sustainability concerns and objectives change over space (or geographical scale). However, in a given ecological economic system, a set of sustainability indicators is used to reflect sustainability objectives. But an indicator by itself is neither a policy nor an objective. It is just a measure of something.

Using indicators with thresholds to represent sustainability is a way to define the boundaries in which the system should stay, and what are the minimum standards a generation should pass on to future generations. However, the definition of the thresholds is not always straightforward. And objectives can be conflicting each other, imposing trade-offs in the definition of sustainability objectives.

Anyway, sustainability indicators, that are the base of policy making, can be viewed at the "bottom" of the issue; and this paper proposes a "bottom-up" approach of sustainability:

In this paper, we provide a theoretical reflexion on the way to use sustainability indicators and to define sustainability objectives. Based on the Rawlsian conception of justice[1], our approach consists in defining minimal rights to be guaranteed to all generations, in an intergenerational equity perspective. This minimal rights are represented by thresholds resulting in constraints on sustainability indicators. From that point of view, a sustainable development path is an economic trajectory for which all the sustainability objectives are met at all time.

From the methodological point of view, our approach is based on two steps. The first step consists in defining what are the sustainability objectives that are achievable in a dynamic perspective, given the initial economic endowments. Basically, this step describes the trade-offs between sustainability objectives. The second step consists in choosing sustainability objectives within the set of achievable ones, with respect to a preference function. This step is a static optimization problem that is much more simpler to develop than dynamic optimization problems associated with usual criterion.

We argue that our approach can be a practical step toward sustainability. In particular, it can be applied to real cases sustainability problem, and adequate numerical and algorithmic methods exist for application studies. This framework can even be extended to address the issue of reaching sustainability objectives that are not achievable with the initial endowment of the economy.

To illustrate our approach, we apply it to an intertemporal resource allocation model with a manufactured capital stock and a non renewable natural resource. It is a canonical model severally used in the literature on sustainability (Dasgupta

---

[1] According to John Rawls' conception of justice (Rawls, 1971), the first requirement for equity is to choose the allocation of resources that provides the maximal number of minimal rights every one can enjoy. This result comes from the allocation of rights one would made under the "veil of ignorance". Rawls argues that justice should be based on two principles, with a priority order. The first principle is the definition of fundamental rights every one can enjoy ("*each person is to have an equal right to the most extensive scheme of equal basic liberties compatible with a similar scheme of liberties for others*"). The second principle is based on (with here again a priority order) "fair equality of opportunity" to a social position and on the "difference principle" that stipulates that inequality in the wealth distribution is justified if it is beneficial for the poorest individual, i.e. if the poorest individual in this configuration in richer than the poorest individual in all other possible allocations. This last statement leads to the maximin criterion, which is thus the less important point in Rawls' theory of justice.

and Heal, 1974; Solow, 1974; Heal, 1998).

The remainder of the paper is organized as follows. The general approach is presented in section 2. An application to the canonical Dasgupta-Heal-Solow model is provided in section 3. A discussion on the proposed approach, its applicability and possible extensions is proposed section 4.

2 Defining minimal rights for sustainability

Consider an economy with n capital stocks represented by the vector $X \in \mathbb{X} \subseteq \mathbb{R}^n$. Each capital stock can either be a reproducible man-made capital, or a natural resource (renewable or not). Let us define the decision vector $u \in \mathbb{U} \subseteq \mathbb{R}^m$. Each component of u can be interpreted as a consumption/extraction or investment modifying the stocks. In a continuous time dynamic framework, we represent the dynamics of the economy by function F, such that

$$\dot{X} = F(X, u) \tag{1}$$

F represents production functions and natural resources stocks dynamics.

Let the initial endowments of the economy be denoted by the initial stock levels $X(0) = X_0$.

2.1 Sustainability objectives

We assume that sustainability has to encompass a finite number I of (potentially conflicting) objectives of different nature (ecological, economic and social), in an intertemporal framework.

Sustainability indicators

To address the sustainability issue, we consider a finite number I of sustainability "goods", each good being associated with a sustainability objective. Example of sustainability goods can be consumption, environmental quality,... The availability of these goods depends on the state of the economy and on applied decisions. We consider I indicators that measure the quantity of the sustainability goods available for given economic state and decisions, and define these indicators as follows.

Definition 1 *A* sustainability indicator *is a function* $\mathcal{C}_i : \mathbb{X} \times \mathbb{U} \to \mathbb{R}$, *for* $i = 1, \ldots I$, *that provides a measure of the quantity of sustainability good* i *available in economic state* X *when decisions* u *are applied.*

Using that formalism, we postulate that, at a given time t, the indicators $\mathcal{C}_{i=1,\ldots I}(X, u)$ only depend on the stocks and decisions at the considered time t.[2]

---

[2] As a remark, note that the generation living at time t benefits from quantities $\mathcal{C}_{i=1,\ldots,I}(X_t, u_t)$ of sustainability goods.

Moreover, we assume that sustainability indicators are defined in such a way that a larger value is preferred to a smaller one. This means that indicators representing “bads” (pollution...) are represented by negative values. This latter assumption implies that, broadly speaking, a sustainability objective is to increase the quantity of the associated sustainability good.

Minimal rights seen as sustainability constraints
We associate a targeted minimal level $\underline{c}_i$ to each sustainability good. We assume that sustainability objective is to maintain the level of the sustainability good i above the threshold c_i. It leads to the sustainability constraint $\mathcal{C}_i \geq c_i$ that represents the right for any generation to benefit from achievement of objective i at least at a level $\underline{c}_i$.

Definition 2 *A* sustainability objective *is an indicator $\mathcal{C}_i(X, u)$ associated with a threshold $\underline{c}_i$.*

From that definition, given a set of sustainability indicators, defining sustainability objectives consists in defining thresholds for the I sustainability indicators. An important thing at that stage is that the thresholds $\underline{c}_i$ are not defined exogenously, and that the purpose of the analysis is to define them (define sustainability objectives). We are interested in a way to *i)* examine trade-offs between sustainability objectives (how much one can increase the quantity of a sustainability good w.r.t. the others), and *ii)* to represent preferences on these objectives (if one could increase the consumption of a sustainability good, which one would be preferred).

Sustainable development paths Using the previous definition, we define a sustainable development trajectory as follows.

Definition 3 *A economic trajectory $(X(.), u(.))$ is sustainable if at any time t, $\mathcal{C}_i(X(t), u(t)) \geq \underline{c}_i$, for all $i = 1, \ldots, I$.*

As from objectives thresholds $\underline{c}_i$ are defined, the problem of defining sustainable trajectories is a viability problem (Aubin, 1991). The viability framework studies the consistency between a dynamic system and so-called viability constraints. From that point of view, in definition 3, sustainability objectives are viability constraints to be satisfied at any time by the economy. The aim of a viability problem is to define if there are intertemporal paths starting from the initial economic state X_0 that satisfy all of the viability constraints forever. In that sense, the viability framework does not allow trade-offs neither between constraints (all constraints must be satisfied for the system to be said viable), nor between time periods (the constraints must be satisfied at any time for the system to be said viable). In our sustainability framework, it means that all the objectives and all the generations are treated equally, and that if one (or more) of the objectives is not satisfied at some time, the economy is not sustainable. In such a situation (when a viability constraint is not satisfied), the system is said to face a crisis.

In other words, the viability focuses on crisis, and aims at defining decisions to avoid them. By introducing the function $\mathbf{1}(\mathcal{C}, \underline{c})$ that is equal to one if the condition $\mathcal{C} \geq \underline{c}$ holds and zero otherwise, one can define the instantaneous viability value function $v(t) = \prod_{i=1}^{I} \mathbf{1}(\mathcal{C}_i(X(t), u(t)), \underline{c}_i)$ that is equal to 1 if all the constraints are satisfied at time t and 0 otherwise. At any instant during which one of the constraint is not respected, the system is said to face a crisis. In such a case, $(1 - v(t))$ is equal to 1. A viability problem consists in defining paths that don't face crisis. It can be stated as the following minimization problem[3]

$$\begin{aligned} V_{(\underline{c}_1,\ldots,\underline{c}_i,\ldots,\underline{c}_I)}(X_0) &= \min_{u(.)} \int_0^{+\infty} (1 - v(s))ds \\ s.t. \;\; & \dot{X} = F(X, u) \\ & X(0) = X_0 \end{aligned} \tag{2}$$

If $V_{(\underline{c}_1,\ldots,\underline{c}_i,\ldots,\underline{c}_I)}(X_0)$ is equal to zero, there exist decisions $u(.)$ generating a trajectory $X(.)$ starting from X_0 that satisfies all of the viability constraints at any time. It means that sustainability objectives $(\underline{c}_1, \ldots, \underline{c}_i, \ldots, \underline{c}_I)$ are achievable from the initial state X_0.

If $V_{(\underline{c}_1,\ldots,\underline{c}_i,\ldots,\underline{c}_I)}(X_0)$ is greater than zero, there are no intertemporal decisions $u(.)$ resulting in economic trajectories satisfying the sustainability objectives at any time. It means that, given the initial economic endowments, the sustainability objectives can not be achieved for all generations and that some of the constraints will be violated for some of the generations. In such a situation, there are two possibilities to recover the sustainability of the economy:

On one hand, one can drive the system toward a new economic state $\tilde{X}$ such that $V_{(\underline{c}_1,\ldots,\underline{c}_i,\ldots,\underline{c}_I)}(\tilde{X}) = 0$ (if such states exist[4]). It means that during some transition period, some of the constraints will be violated. This case will be briefly discussed in Section 4, but we do not treat it here as we are concerned with the intergenerational equity issue, and want to define minimal rights to be guaranteed to all generation, excluding the sacrifice of some generations to improve the sustainability of the economy.

On the other hand, one can modify the viability constraint levels and choose new sustainability objectives $(\tilde{c}_1, \ldots, \tilde{c}_i, \ldots, \tilde{c}_I)$ such that $V_{(\tilde{c}_1,\ldots,\tilde{c}_i,\ldots,\tilde{c}_I)}(X_0) = 0$. One then faces the problem of choosing the levels of the sustainability objectives. This is the problem we address here.

---

[3] Since $1 - v(s)$ is only lower semicontinuous, the usual tools of optimal control in an infinite horizon do not generally apply. The purpose of the viability theory is to solve such problems.

[4] The set of states X such that $V_{(\underline{c}_1,\ldots,\underline{c}_i,\ldots,\underline{c}_I)}(X) = 0$ is called the viability kernel of the problem (Aubin, 1991). It is the set of states from which sustainable economic trajectories (as defined by definition 3) start. If that set is empty, there are no economic states that allow to satisfy the constraints in the long run. Only the second possibility is possible to reach a sustainable economy.

2.2 Maximizing minimal rights for sustainability

In this section, we propose a way to determine sustainability objectives, when they are defined like minimal rights to be guaranteed to all generations and represented by constraints on indicators and associated thresholds.

Preferences

We assume here that there is a preference function $\mathcal{P}(\underline{c}_1, \ldots, \underline{c}_i, \ldots, \underline{c}_I)$ associated with minimal rights levels. $\mathcal{P}$ is increasing with respect to all its arguments under our assumption that sustainability indicators represent "goods".[5]

Note that $\mathcal{P}$ is not the instantaneous utility function as it does not depend on the actual consumption levels $\mathcal{C}_i(X(t), u(t))$, but on the sustainability objectives thresholds $\underline{c}_i$.[6]

The MMR criterion

To define sustainability objectives, we introduce the following criterion.

Definition 4 *We define the* Maximization of Minimal Rights *problem (MMR hereafter):*

$$\begin{array}{rl} \max_{\underline{c}_i,\ i=\{1,I\}} & \mathcal{P}(\underline{c}_1, \ldots, \underline{c}_i, \ldots, \underline{c}_I) \\ s.t \quad \mathcal{C}_i(X(t), u(t)) \geq \underline{c}_i & \forall i = \{1, I\}, \forall t \in \mathbb{R}^+ \\ \dot{X} = F(X(t), u(t)) & \\ X(0) = X_0 & \end{array} \tag{3}$$

A generalized maximin problem

To link our approach to the existing literature, we argue that the MMR criterion introduced in definition 4 is a generalized *maximin* criterion (Solow, 1974; Cairns and Long, 2006). The general criterion defined by eq.(3) is actually a maximin problem when $I = 1$, considering only one sustainability objective. For example, if the objective is to sustain the current utility level $U(\mathcal{C}_1, \ldots, \mathcal{C}_i, \ldots, \mathcal{C}_I)$ (assuming that the utility depends on the I sustainability goods), such a standard maximin "utilitarian" problem would have the form

$$\begin{array}{rl} \max_{\underline{U}} \underline{U} & \\ s.t \quad U(X_t, u_t) \geq \underline{U} & \forall t \in \mathbb{R}^+ \\ \dot{X} = F(X(t), u(t)) & \\ X(0) = X_0 & \end{array} \tag{4}$$

---

[5] At that stage, we do not need further assumptions on the preferences. In particular case-studies, such as the one presented in Section 3, we should stipulate some properties of $\mathcal{P}$. Nevertheless, we will discuss in Section 4 how our framework can be used without knowing explicitly $\mathcal{P}$.

[6] Usually in economics, one considers a utility function $U(\mathcal{C}_1, \ldots, \mathcal{C}_i, \ldots, \mathcal{C}_I)$ that associates an utility level to consumption levels. This is not the case here. We will discuss the implications of such a choice in Section 4.

It means that the preference function is reduced to the simple one argument linear form $\mathcal{P}(\underline{U}) = \underline{U}$, and that all sustainability objectives are encompassed in the utility function $U(X_t, u_t)$[7] and the threshold value $\underline{U}$.

In our approach, the sustainability objectives are taken as separate constraints, instead of being grouped in the constraint on the minimal utility. The preference function is thus a combination of the thresholds, instead of the sole minimal utility. We should give an interpretation of our approach with respect to the more usual maximin approach (with an utility) in the discussion part (section 4). But first, let us focus on the technical issue of solving the problem (3).

2.3 A methodology

As our problem is a generalized maximin problem, one could extend the direct approach proposed by Cairns and Long (2006). Their approach to solve problem (4) consists in developing a time autonomous optimization problem in which $\underline{U}$ is a control parameter. They introduce an adjoint variable for each of the n capital stocks (each adjoint variable being interpreted like the shadow value of the associated state variable), and another adjoint variable for the equity constraint $U \geq \underline{U}$. This last adjoint variable is interpreted as the shadow-value of equity.

Doing the same for our problem would require to consider I control parameters (all the $\underline{c}_i$), and to introduce I adjoint variables on the respective sustainability constraints, being interpreted as the shadow-values of the satisfaction of each sustainability objective.

Notwithstanding the fact that it would result in a high dimensional dynamic optimization problem, the major drawback from our point of view is that it is likely to result in our equivalent of what Cairns and Long (2006) call a non-regular maximin problem: in the classical maximin formulation, when the equity constraint is ineffective, i.e. when $U(X_t, u_t) > \underline{U}$ for some t, the associated shadow value is equal to zero and the solution of the problem is extremely hard to compute (see Cairns, 2008b). In our case, there are no reasons for all the constraints to be effective at all time. If one consider a large number of sustainability objectives, it is even more likely that only a part of them will result in an effective constraint. Addressing our problem by extending the maximin approach may thus be impossible.

In order to avoid such difficulties, we propose to split the problem in the two following steps.

Set of achievable sustainability objectives

First, we define the set of reachable minimal rights, given the initial state of the

---

[7] Note that the function $U(\mathcal{C}_1(X, u), \ldots, \mathcal{C}_i(X, u), \ldots, \mathcal{C}_I(X, u))$ is a function $U(X, u)$ as all its arguments are functions depending on the state X and decisions u. It can thus be interpreted as a sustainability indicators, according to definition 1.

economy X_0.

$$\mathcal{S}(X_0) = \left\{ (\underline{c}_1, \ldots, \underline{c}_i, \ldots, \underline{c}_I) \left| \begin{array}{l} \text{there exist decisions } u(.) \text{ such that} \\ \text{given } X_0 \text{ and the dynamics } \dot{X} = F(X, u) \\ \mathcal{C}_i \geq \underline{c}_i,\ \forall i \in \{1, I\}, \forall t \in \mathbb{R}^+ \end{array} \right. \right\} \quad (5)$$

This first step is an extension of a viability problem. It consists in defining the set of all combinations of $\underline{c}_i$ that are achievable given the initial state of the economy X_0: From the initial economic endowments, it is possible to define a sustainable development trajectory, as introduced in definition 3, for any $(\underline{c}_1, \ldots, \underline{c}_i, \ldots, \underline{c}_I) \in \mathcal{S}(X_0)$. On the contrary, there is no sustainable trajectory from X_0 for any combination of sustainability objectives that is not in $\mathcal{S}(X_0)$.

This set of achievable sustainability objectives describes the trade-offs between the conflicting objectives. Once the trade-offs between sustainability objectives are described, it is easier to choose sustainability objectives among feasible ones.

Static optimization problem on preferences

Second, we solve the static optimization problem of defining within the set of all feasible minimal rights what is the vector of minimal rights $(\underline{c}_1, \ldots, \underline{c}_i, \ldots, \underline{c}_I)$ that maximizes the MMR criterion. This problem simply reads

$$\max_{(\underline{c}_1, \ldots, \underline{c}_i, \ldots, \underline{c}_I) \in \mathcal{S}} \mathcal{P}(\underline{c}_1, \ldots, \underline{c}_i, \ldots, \underline{c}_I) \quad (6)$$

3 Application to the Dasgupta-Heal-Solow model

In order to describe the implication of such an approach, we apply it to the seminal two stock model by Dasgupta and Heal (1974) and Solow (1974).

3.1 A consumption-production economy with non-renewable resources

Consider an economy with infinitely many generations, and make the simplifying assumption that each generation is composed by an unique representative agent. A non renewable resource x_t is extracted (at a rate r_t) and used with capital k_t to produce capital. The production function is of the Cobb-Douglas form $k^\alpha r^\beta$, with $\alpha > \beta$. The capital can be ever consumed (c_t) or invested ($\dot{k}$). Such a model has been studied in Dasgupta and Heal (1974); Solow (1974), and is an useful stylized model for addressing the sustainability issue: the intertemporal allocation of the exhaustible resource, and the stream of consumption through time make intertemporal comparisons possible, in an intergenerational equity perspective.
This model has been widely used in the sustainable development literature (Heal, 1998) to compare various sustainability criteria.

The dynamics are

$$\begin{aligned} \dot{k} &= k_t^{\alpha} r_t^{\beta} - c_t, \quad (7) \\ \dot{x} &= -r_t. \quad (8) \end{aligned}$$

To illustrate our approach, we will consider two sustainability objectives in that model. On the one hand, we want to sustain a consumption level $\underline{c}$. On the other hand, we want to preserve a part of the stock $\underline{x}$. We thus consider the following sustainability constraints

$$\begin{aligned} c_t &\geq \underline{c}, \quad (9) \\ x_t &\geq \underline{x}. \quad (10) \end{aligned}$$

The MMR approach consists in defining the level of guaranteed consumption $\underline{c}$ and preserved resource stock $\underline{x}$.

3.2 Admissible sustainability objectives for a given initial state

Following the methodology presented in the previous section, we will define the set of objectives $(\underline{c}, \underline{x})$ that are reachable from the initial capital stocks of the economy (k_0, x_0). Doing that, we define the set of minimal rights that could be guaranteed to any generation:

$$\mathcal{S}(k_0, x_0) = \left\{ (\underline{c}, \underline{x}) \;\middle|\; \begin{array}{l} \text{there exist paths } (k(.), x(.)) \text{ starting from } (k_0, x_0) \\ \text{that satisfy the constraints (9) and (10)} \end{array} \right\}. \quad (11)$$

To determine that set, we define the maximal sustainable consumption with respect to a preservation objective $\underline{x}$

$$c^{+}(k_0, x_0, \underline{x}) = \max\Big(c^{\sharp} \;\Big|\; \text{given } (k_0, x_0), \text{there exists } (c(.), r(.)) \text{ such that } \forall t \geq 0, c_t \geq c^{\sharp} \text{ and } x_t \geq \underline{x} \Big). \quad (12)$$

Solow (1974) first studied the sustainability of this economic model in the maximin framework. He proved that the maximal sustainable consumption associated with the initial endowments of the economy (k_0, x_0) is

$$c^{+}(k_0, x_0) = (1-\beta)\big(x_0(\alpha-\beta)\big)^{\frac{\beta}{1-\beta}} k_0^{\frac{\alpha-\beta}{1-\beta}}. \quad (13)$$

Martinet and Doyen (2007) described the relationship between consumption and preservation goals in the same model. They examined in the viability framework the conditions for a minimal consumption $\underline{c}$ to be guaranteed when there is also a constraint on the preservation of the resource $\underline{x}$. They extended Solow's result by determining the maximal sustainable consumption given a preservation objective $\underline{x}$, which is

$$c^{+}(k_0, x_0, \underline{x}) = (1-\beta)\big((x_0 - \underline{x})(\alpha - \beta)\big)^{\frac{\beta}{1-\beta}} k_0^{\frac{\alpha-\beta}{1-\beta}}. \tag{14}$$

Fig. 1 represents that result. Eq. (14) is the upper bound of the set of all reachable goals $\mathcal{S}(k_0, x_0)$ and represents the trade-offs in sustainability objectives.

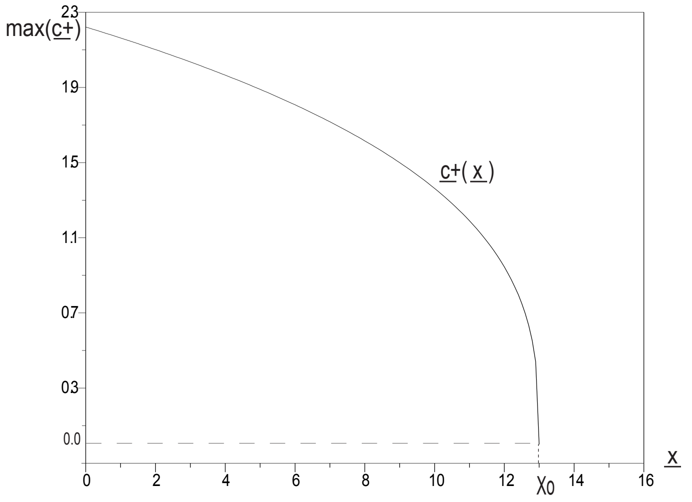

Figure 1: Substitution between guaranteed consumption and resource conservation for a Cobb-Douglas technology.

$\mathcal{S}(k_0, x_0)$ is the set of all reachable goals for sustainability. Any inner pair $(\underline{c}, \underline{x})$ such that $\underline{c} \leq c^{+}(\underline{x})$ is feasible.[8] Note that on the border, a rise of resource preservation implies a fall of sustainable consumption. The two sustainability objectives are in conflict.

A question that arises now is to define sustainability goals in order to satisfy some intergenerational equity.

3.3 Defining sustainability objectives

In this section we introduce the preference function to define which sustainability rights should be chosen among the possible set $\mathcal{S}(k_0, x_0)$. We assume that preferences among the minimal rights to be guaranteed are represented by the preference function $\mathcal{P}(\underline{c}, \underline{x})$, with the conditions

---

[8] For the sake of simplicity, we will omit the initial state in the notation of function $c^{+}(k_0, x_0, \underline{x})$.

- $\underline{c} \in \mathbb{R}^+$; $\underline{x} \in \mathbb{R}^+$
- $\mathcal{P}(\underline{c}, \underline{x}) : \mathbb{R}^+ \times \mathbb{R}^+ \mapsto \mathbb{R}^+$
- $\mathcal{P}_c \geq 0$; $\mathcal{P}_x \geq 0$
- $\mathcal{P}_{c,x} \leq 0$.

We apply the Maximizing Minimal Rights criterion define by eq.3:

$$\max_{\underline{c},\underline{x}} \mathcal{P}(\underline{c}, \underline{x}) \tag{15}$$

subject to

$$\begin{aligned}
(k_0, x_0) & \quad \textit{given} && (16)\\
c_t & \geq \underline{c} && (17)\\
x_t & \geq \underline{x} && (18)\\
\dot{k} & = k_t^{\alpha} r_t^{\beta} - c_t && (19)\\
\dot{x} & = -r_t && (20)
\end{aligned}$$

Using the result 14, this problem is equivalent to the maximization of this criterion among the possible pairs $(\underline{c}, \underline{x}) \in \mathcal{S}(k_0, x_0)$, i.e.

$$\begin{aligned}
& \max_{\underline{c},\underline{x}} \mathcal{P}(\underline{c}, \underline{x}) \\
& s.t. \\
0 & \leq \underline{x} && (21)\\
0 & \leq \underline{c} && (22)\\
0 & \leq x_0 - \underline{x} && (23)\\
0 & \leq c^+(\underline{x}) - \underline{c} && (24)
\end{aligned}$$

This problem is a classical static optimization problem under inequality constraints (Léonard and Long, 1992).

To solve this problem, we define the following functional form

$$\phi(\mu_1, \mu_2, \mu_3, \mu_4, \underline{c}, \underline{x}) = \mathcal{P}(\underline{c}, \underline{x}) + \mu_1 \underline{x} + \mu_2 \underline{c} + \mu_3 (x_0 - \underline{x}) + \mu_4 (c^+(\underline{x}) - \underline{c}) \tag{25}$$

where the μ_i are the dual variables of the problem.

According to the Khun-Tucker theorem, the optimality conditions of the problem are[9]

$$\phi_{\mu_1} = \underline{x} \geq 0, \qquad \mu_1 \geq 0, \qquad \mu_1 \underline{x} = 0 \tag{26}$$
$$\phi_{\mu_2} = \underline{c} \geq 0, \qquad \mu_2 \geq 0, \qquad \mu_2 \underline{c} = 0 \tag{27}$$
$$\phi_{\mu_3} = x_0 - \underline{x} \geq 0, \qquad \mu_3 \geq 0, \qquad \mu_3 (x_0 - \underline{x}) = 0 \tag{28}$$
$$\phi_{\mu_4} = c^+(\underline{x}) - \underline{c} \geq 0, \qquad \mu_4 \geq 0, \qquad \mu_4 (c^+(\underline{x}) - \underline{c}) = 0 \tag{29}$$
$$\phi_{\underline{x}} = \mathcal{P}_{\underline{x}} + \mu_1 + \mu_4 \frac{dc^+(\underline{x})}{d\underline{x}} \leq 0, \qquad \underline{x} \geq 0, \qquad \underline{x} \left(\mathcal{P}_{\underline{x}} + \mu_1 + \mu_4 \frac{dc^+(\underline{x})}{d\underline{x}} \right) = 0 \tag{30}$$
$$\phi_{\underline{c}} = \mathcal{P}_{\underline{c}} + \mu_2 - \mu_4 \leq 0, \qquad \underline{c} \geq 0, \qquad \underline{c}(\mathcal{P}_{\underline{c}} + \mu_2 - \mu_4) = 0 \tag{31}$$

Strictly positive solutions

First assume that, at the optimum, both the optimization variables $\underline{x}$ and $\underline{c}$ are strictly positives. From eq. (26) and (27), we get $\mu_1 = \mu_2 = 0$. Moreover, if consumption is positive, the preserved resource stock will be lower than the initial stock x_0. Eq. (28) then leads to $\mu_3 = 0$. We thus get a system from equations (29), (30) and (31), in which there are three equations and three variables.

$$\mu_4 (c^+(\underline{x}) - \underline{c}) = 0 \tag{32}$$
$$\underline{x} \left(\mathcal{P}_{\underline{x}} + \mu_4 \frac{dc^+(\underline{x})}{d\underline{x}} \right) = 0 \tag{33}$$
$$\underline{c}(\mathcal{P}_{\underline{c}} - \mu_4) = 0 \tag{34}$$

As we have assumed that $\underline{c} \neq 0$, eq. (34) leads to $\mu_4 = \mathcal{P}_{\underline{c}}$. We thus get from eq. (32) and (33) the conditions

$$\underline{c} = c^+(\underline{x}) \tag{35}$$
$$\frac{dc^+(\underline{x})}{d\underline{x}} = -\frac{\mathcal{P}_{\underline{x}}}{\mathcal{P}_{\underline{c}}} \tag{36}$$

It leads to the following result

$$\frac{d\underline{c}^+(\underline{x})}{d\underline{x}} = -\frac{\mathcal{P}_{\underline{x}}}{\mathcal{P}_{\underline{c}^+(\underline{x})}} \tag{37}$$

Fig. 2 illustrates this result.

Corner solution $\underline{x} = 0$

Assume now that $\underline{x} = 0$. It implies that $\mu_3 = 0$ (from eq. 28).

If $\underline{c} = 0$, eq. (29) would require $\mu_4 = 0$. But it is in contradiction with relation (31) which requires $\mathcal{P}_{\underline{c}} + \mu_2 - \mu_4 \leq 0$. Thus, we have $\mu_4 > 0$ and $\underline{c} = c^+(0)$, from eq. (29).

---

[9] The variables are at optimal values. In order to get simple notations, we do not note them $\underline{x}^*$ and $\underline{c}^*$ but simply $\underline{x}$ and $\underline{c}$

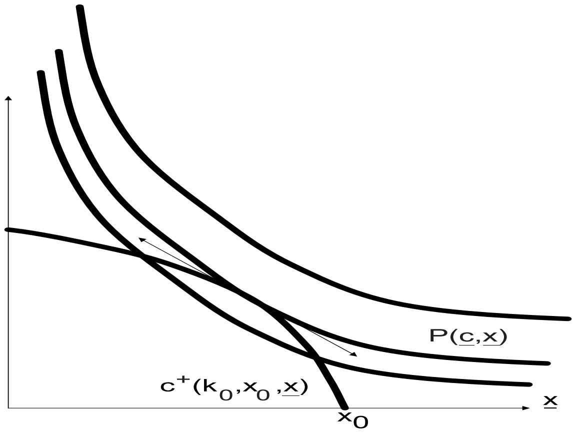

Figure 2: Optimal minimal rights when $\underline{c} > 0$ and $\underline{x} > 0$.

As $\underline{c} \neq 0$, we get $\mu_2 = 0$ from eq. (27). We then get $\mu_4 = U_{\underline{c}}$ from eq.(31). Finally, the inequality condition (30) requires

$$\mathcal{P}_{\underline{x}|(\underline{x}=0)} + \mu_1 + \mathcal{P}_{\underline{c}|(\underline{c}=c^+(0))} \left(\frac{dc^+(\underline{x})}{d\underline{x}} \right)_{|(\underline{x}=0)} \leq 0 \tag{38}$$

This equation can be expressed with respect to μ_1

$$\mu_1 \leq -\mathcal{P}_{\underline{x}|(\underline{x}=0)} - \mathcal{P}_{\underline{c}|(\underline{c}=c^+(0))} \left(\frac{dc^+(\underline{x})}{d\underline{x}} \right)_{|(\underline{x}=0)} \tag{39}$$

As $\mu_1 \geq 0$, it is possible only if $-\mathcal{P}_{\underline{x}|(\underline{x}=0)} - \mathcal{P}_{\underline{c}|(\underline{c}=c^+(0))} \left(\frac{dc^+(\underline{x})}{d\underline{x}} \right)_{|(\underline{x}=0)} \geq 0$, or equivalently if

$$0 \leq \frac{\mathcal{P}_{\underline{x}|(\underline{x}=0)}}{\mathcal{P}_{\underline{c}|(\underline{c}=c^+(0))}} \leq -\left(\frac{dc^+(\underline{x})}{d\underline{x}} \right)_{|(\underline{x}=0)} \tag{40}$$

It means that such a corner solution is possible if the slope of the preference function is smaller than that of the function $c^+(\underline{x})$. Such a corner solution is then possible only if the marginal benefit of preservation for a nil resource stock is small with respect to the marginal benefit of an extra unit of guaranteed consumption. Obviously, it requires that $\mathcal{P}_{\underline{x}|(\underline{x}=0)} < \infty$.

Fig. 3 illustrates this result.

Corner solution $\underline{c} = 0$

We now turn toward the other case : $\underline{c} = 0$. The inequality from eq. (31) implies

$$\mathcal{P}_{\underline{c}|(\underline{c}=0)} + \mu_2 - \mu_4 \leq 0 \quad \Longrightarrow \quad \mu_4 \geq \mathcal{P}_{\underline{c}|(\underline{c}=0)} + \mu_2 > 0 \tag{41}$$

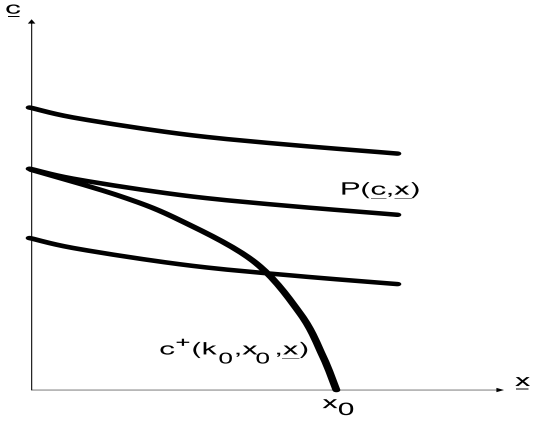

Figure 3: Optimal minimal rights when $\underline{x} = 0$.

It is only possible if $\mathcal{P}_{\underline{c}|(\underline{c}=0)} < \infty$.

As $\mu_4 > 0$, we know from eq. (29) that $\underline{c} = c^+(\underline{x})$ which means, as $\underline{c} = 0$, that $\underline{x} = x_0$. We have $\mu_1 = 0$ from eq. (26). Thus, $\underline{x} > 0$ requires from eq. (30) that

$$\mu_4 = -\frac{\mathcal{P}_{\underline{x}|(\underline{x}=x_0)}}{\left(\frac{dc^+(\underline{x})}{d\underline{x}}\right)_{|(\underline{x}=x_0)}} \tag{42}$$

Combining this condition with eq. (41), we get

$$0 \leq \mu_2 \leq -\mathcal{P}_{\underline{c}|(\underline{c}=0)} - \frac{\mathcal{P}_{\underline{x}|(\underline{x}=x_0)}}{\left(\frac{dc^+(\underline{x})}{d\underline{x}}\right)_{|(\underline{x}=x_0)}} \tag{43}$$

We thus have a condition on the marginal preferences in $(\underline{c} = 0, \underline{x} = x_0)$:

$$0 \leq -\left(\frac{dc^+(\underline{x})}{d\underline{x}}\right)_{|(\underline{x}=x_0)} \leq \frac{\mathcal{P}_{\underline{x}|(\underline{x}=x_0)}}{\mathcal{P}_{\underline{c}|(\underline{c}=0)}} \tag{44}$$

The slope of the preference function must be greater than that of the function $c^+(\underline{x})$ in $\underline{x} = x_0$. In particular, the marginal preference for guaranteed consumption, when consumption is zero, must be finite (and small with respect to the marginal preference of preservation).

Fig. 4 illustrates this result.

Interpretation of the results in that case-study

The application of the MMR criterion to that canonical model allows us to

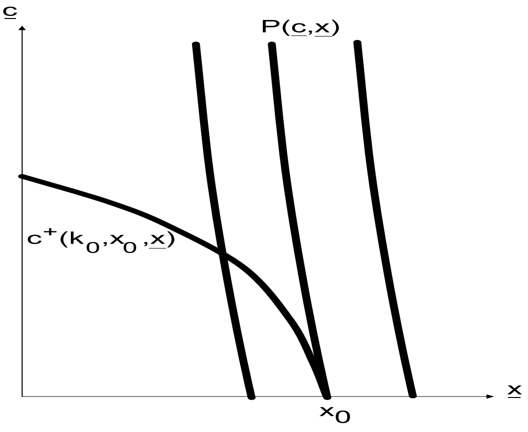

Figure 4: Optimal minimal rights when $\underline{c} = 0$.

emphasize its relationship with the maximin approach as it is presented in Solow (1974) and Cairns and Long (2006).

Whatever the case, the optimal solution always satisfies $\underline{c} = c^+(\underline{x})$. The only intertemporal path that maximizes minimal rights is a thus maximin under constraints, and is efficient from an economic point of view (no resource is wasted).

Cairns and Long (2006) argued that there are two ways to take into account environmental and natural resources issues in the viability approach. One can include them as arguments of the utility function, or one can define constraints to restrict the maximin path. But they don't address the issue of defining the level of that constraints. Our approach results in the definition of optimal level of such constraints.

The level of the resource preservation will depend on the marginal preferences on guaranteed consumption and resource stock preservation objectives. If we assume that the consumption marginal preference for a nil consumption $\mathcal{P}_{c|(c=0)}$ is infinite, and that that of the resource for a nil resource stock $\mathcal{P}_{x|(x=0)}$ is also infinite, the result will satisfy $\underline{c} > 0$ and $\underline{x} > 0$, and sustainability objectives will be to sustain a positive consumption and to preserve a positive part of the stock natural resource stock.

4 Another almost practical step toward sustainability?

In that last section, we develop a discussion on the interpretation of our approach and its links with usual approaches, mainly the maximin one.

4.1 What does the MMR criterion mean?

Now that we have described the principle of our approach, the methodological steps to address problem (3) and an illustration in a standard model, we can give interpretations of both the criterion and the methodology.

Our criterion allows us to define minimal rights to be guaranteed to any generation. Each of the minimal right is represented by a constraint on a sustainability indicator representing a sustainability objective. The purpose of our approach is to define the level of the constraint which represents the sustainability threshold for the indicator.

In our framework, all sustainability objectives must be achieved at all time, which means that once the thresholds are determined, there are no priorities between the objectives. In the same way, there are no priorities between generations. Intergenerational equity is thus strongly taken into account in our framework as all the generations benefit from the same minimal rights.

To define the minimal rights for sustainability, we maximize what we call the MMR criterion. This criterion is based on a preference function $\mathcal{P}$ that represents the social preferences on the sustainability objectives. It is a function of the constraints thresholds for sustainability indicators. From that point of view, $\mathcal{P}$ is not an utility function as it is not related to the actual levels of the sustainability indicators.

Once the sustainability objectives are defined, the only thing we know is that along a sustainable economic intertemporal path the actual utility will be greater than that associated to the minimal rights. In particular, although our criterion is a generalized form of the maximin criterion, the following condition holds:

$$\begin{array}{lll} & \forall i, \forall t, & \mathcal{C}_i(X_t, u_t) \geq \underline{c}_i \\ \Rightarrow & \forall t, & U(\mathcal{C}_1(X_t, u_t), \ldots, \mathcal{C}_i(X_t, u_t), \ldots, \mathcal{C}_I(X_t, u_t)) \geq U(\underline{c}_1, \ldots, \underline{c}_i, \ldots, \underline{c}_I) \\ \Rightarrow & & \min_t U_t \geq U(\underline{c}_1, \ldots, \underline{c}_i, \ldots, \underline{c}_I) \\ \Rightarrow & & \underline{U} \geq U(\underline{c}_1, \ldots, \underline{c}_i, \ldots, \underline{c}_I) \end{array}$$

It means that the utility associated with the minimal rights is lower than (or equal to) the maximal sustainable utility defined by a classical maximin program. What is sustained (and optimized) in our model is thus not a utility level, but a set of sustainability rights. And as it is likely that not all the sustainability constraints will be effective, almost all generations are likely to have higher an utility than that associated with the minimal rights.

Many existing approaches to sustainability rely on sustainability constraints: The neo-classical discounted utility approach can be extended with sustainability constraints to take into account environmental issues; The maximin approach can also be completed with constraints (Cairns and Long, 2006); The viability approach is based on such constraints (Martinet and Doyen, 2007); Strong sustainability conception advocates for critical natural goods to be preserved (Daly, 1974).

We have developed here a framework to set the level of such sustainability constraints. Our approach is somewhere between weak sustainability and strong sustainability: trade-offs occur in the social preferences between sustainability objectives, but they result in a definition of critical values for sustainability indicators that can be interpreted as conservation objectives. And according to Gerlagh and Keyser (2003), conservationist policies can be Pareto efficient, and strict resource conservation is equivalent to non-dictatorship of the present, as defined by Chichilnisky (1996). This emphasizes the intergenerational equity aspect of the present approach.

An interesting point is that further interpretations can be drawn from the methodological part of our framework. The fact that the methodological approach is divided in two steps allows us to interpret each step.

By defining the set of achievable sustainability objectives, one represents the trade-offs between these objectives. The borders of the set can be interpreted as a particular production frontier of the sustainability goods representing the possibilities to sustain production levels of that goods forever. It represents what is possible to do in our sustainability framework. This is thus a technical analysis describing in a positive way the necessary trade-offs between conflicting sustainability objectives. There is nothing normative in that, and this can be the purpose of a scientific work.

On the contrary, the static optimization part is based on a social preference function. The preferences on sustainability objectives represent the trade-offs on what is better to do. The definition of such preferences depends on a normative choice, which may not be the role of scientist (Béné and Doyen, 2000).

To finish the interpretation of our approach, let us stress some of its strengths and weaknesses.

Our approach is much more simpler to carry on than standard approaches based on optimal control problems with multi-attribute utility functions. It result in a static optimization problem. But it has a major drawback: it does not lead to the definition of some decision rule, but only to the definition of some sustainability objectives. One can see our approach like a loosen form of the maximin. Even if it is not the purpose of our approach to define some "optimal intertemporal path", a sustainable intertemporal economic trajectory (according to definition 3) is dominated by the maximin path (from the utilitarianism point of view). It can be interpreted as follows. If the sustainability objectives we consider are the arguments of the utility function, the maximin approach defines some optimal sustainable path. But if it is not possible to define such an utility function and/or solve the associated maximin problem, a weaker version of it is to maximize each of the sustainability objectives on its side (given the necessary trade-offs between objectives). In that sense, our approach would represent a sub-optimal sustainability approach. However, this drawback may make it closer to real life issues.

4.2 Is it possible to apply the MMR criterion?

Our approach is based on a set of sustainability objectives represented by constraints on indicators. As emphasized in the introduction, it is close from the way the sustainability issue is addressed in real-life. Referring to indicators and objectives on their levels has the great advantage to be easier to implement than defining intertemporal decisions resulting in trajectories sustaining an utility. This last choice could require strong variations in the indicators levels, implying permanent compensation between the ones being decreasing and the ones being increasing. This is what is stated in Gerlagh and Keyser (2003, p.312): *"strict conservationist policies that impose explicit exploitation constraints ensure sustainability, and are far simpler to implement, compared to the more complex resource management rules that aim at a careful balancing of costs and benefits"*. Our approach is based on such a simpler idea. But does it mean that it can be applied?

Until now, we have presented an approach to define minimal rights for sustainable development. However, any reader would argue that the use of our criterion relies on the definition of the set of all reachable goals, and that all the difficulty of addressing the dynamic aspect of the problem is in the first step of the analysis. Defining achievable minimal rights is not an easy task, and one can argue that the approach is not easier than maximizing the value of a multi attribute utility function in a multicriteria context.

However, there is an increasing body of literature using viability approach in complex bioeconomic systems, emphasizing its applicability if one accepts to use numerical methods to solve the problem. The criterion we propose can be applied to real bioeconomic problems, and help choosing between reachable sustainability objectives. It can also be applied in a stochastic framework, taking into account uncertainty.

In fact, when usual criteria do not have solutions, a possibility is to reduce the set of admissible path (by selecting paths that satisfy a set of constraints) and then to apply a criterion. For example, the green golden rule criterion has no solution in the canonical model presented in Section 3. Our criterion is an alternative approach that guarantees that the long run utility is greater than $U(\underline{c}, \underline{x})$.

Moreover, the second step of the analysis also provide an opportunity to a simple application. The function $\mathcal{P}$ is a static preference ordering. It is much more easy to define than an an intertemporal utility function. If the role fo defining preferences is devoted to some policy maker, it is possible to determine preferences using graphical representations of the achievable goals (Choi et al., 2007).

4.3 On the possibility to reach highest a sustainability in the future

To conclude, we can discuss a last limitation of the approach and present how it can lead to an extension of the original framework.

The maximin approach has been criticized because it may maintain the economy in a poor situation if the initial endowments are low. This critics also apply to our approach as the set of achievable sustainability objectives is defined with respect to the initial economic endowments. It is possible that none of the reachable objectives are relevant for sustainability. For example, the level of sustainable consumption may be lower that a subsistence level, or basic needs.

It is the purpose of the sufficientirianism approach to determine how to reach economic development paths that satisfy basic needs. In the late 70's, the economic development issues where much more concerned with the intragenerational equity issue, and the development of poor countries. It has been argued (Chichilnisky, 1977) that aggregated economic indicators like GDP were not able to encompass the development issue, as it neglects an important dimension of development: the satisfaction of basic needs. In fact, the scarcity of some basic goods can jeopardize economic development, or at least, modify the priority of the social planner: the maximization of Present Net Value of the economy may not be the primary goal of development. Chichilnisky (1977) argued that economic development must be consistent with the attainment of adequate levels of *per capita* consumption of basic goods, a primary goal to achieve intergenerational equity can be define as a minimal *per generation* access to basic goods. She proposed criterion minimizing the time needed to reach an economic path that satisfies the basic needs. Alvarez-Cuadrado and Long (2007) defined a minimal level of utility $\hat{u}$ to be reached, while Chichilnisky (1977) considered a multiobjective approach in which various basics needs have to be fulfilled, without considering an associated utility. Chichilnisky (1977) gives a mathematical formalization of economic efficiency in an economy concerned with the satisfaction of the consumption of basic goods in some near future. Efficiency is defined with respect to the minimization of the time horizon at which the minimal consumption level is reached. The criterion can be written as

$$\min_{u(.)} T \;\Big|\; u_\tau \geq \hat{u} \ , \quad \forall \tau > T \tag{45}$$

In a similar spirit, it is possible to complete the MMR approach in order to define how to reach high sustainability rights in the future, and find a way to escape from poverty traps when initial low economic endowments only make low sustainability objectives achievable. Such an extension is based on the concept of "minimal time of crisis" (Doyen and Saint-Pierre, 1997) which is the cumulated duration of crisis during which the constraints are not met in a viability problem. This extension come directly from the interpretation of the viability criterion (2) that we recall here:

$$V_{(\underline{c}_1,\ldots,\underline{c}_i,\ldots,\underline{c}_I)}(X_0) = \min_{u(.)} \int_0^\infty \left(1 - \prod_{i=1}^{I} \mathbf{1}\left(\mathcal{C}_i(X(t),u(t)),\underline{c}_i\right) \right) dt$$

Assume that the objectives $(\underline{c}_1,\ldots,\underline{c}_i,\ldots,\underline{c}_I)$ are outside the achievable set $\mathcal{S}(X_0)$, this criterion minimizes the intertemporal sum of time periods during which some of the viability constraints are not satisfied. It leads to optimal intertemporal decisions $u^\star(.)$ that minimize the number of time periods during which a generation does not benefit from the sustainability objectives. V is the crisis duration (the number of generations that do not achieve sustainability if the time unit is a generation). It is different from the Chichilnisky (1977) formulation as nothing requires that the first generations are the ones that do not benefit for sustainability minimal rights. It is thus not a time horizon like in eq.(45) (which would imply to make the first generations do sacrifices to improve the sustainability of the economy), but the minimal number of generations (anywhere in time, it depends on the optimal solution) that will face an unsustainable situation.

References

Asheim G., Buchholz W. and Tungodden D. (2001) Justifying sustainability. *Journal of Environmental Economics and Management*, 41, pp. 252-268.

Aubin J.P. (1991) *Viability Theory.* Birkhäuser, Springer Verlag.

Beltratti A., Chichilnisky G. and Heal G. (1995) The Green Golden Rule. *Economics Letters*, 49(2), pp. 175-179.

Béné C. and Doyen L. (2000) Storage and Viability of a Fishery with resource and market dephased seasonalities. *Environmental and Resource economics*, 15, pp.1-26.

Cairns R. (2008a) Value and Income. *Ecological Economics*, 66, pp.417-424.

Cairns R. (2008b) Sustained development of Easter Island?, EAERE 2008 (Gothenburg) Conference Proceedings.

Cairns R. and Long N.V. (2006) Maximin, a direct approach to sustainability. *Environmental and Development Economics*, 11, pp. 275-300.

Chichilnisky G. (1977) Economic Development and efficiency criteria in the satisfaction of basic needs. *Applied Mathematical Modelling*, 1, p.290-298.

Chichilnisky G. (1996) An axiomatic approach to sustainable development. *Social Choice and Welfare*, 13(2), pp. 219-248.

Choi S., Fisman R., Gale D. and Kariv S. (2007) Revealing Preferences Graphically: An old method gets a new tool kit. *The American Economic Review*, 97, pp.153-158.

Daly H. (1974) The economics of the steady state. *American Economic Review*, 64, pp.15-21.

Dasgupta P. and Heal G. (1974) The Optimal Depletion of Exhaustible Resources. *Review of Economic Studies. Symposium on the Economics of Exhaustible Resources*, 41, pp. 1-28.

Dasgupta P. and Heal G. (1979) *Economic Theory and Exhaustible Resources.* Cambridge University Press.

Diamond P. (1965) The evaluation of infinite utility streams. *Econometrica*, 33, pp.170-177.

Doyen L. and Saint-Pierre P. (1997) Scale of viability and minimal time of crisis. *Set-Valued Analysis*, 5, p.227-246.

Dixit A., Hammond P. and M. Hoel (1980) On Hartwick's Rule for Regular Maximin Paths of Capital Accumulation and Resource Depletion, *Review of Economic Studies*, 47, pp.551-556.

Engelmann D. and Strobel M. (2004) Inequality Aversion, Efficiency, and Maximin Preferences in Simple Distribution Experiments. *The American Economic Review*, 94, pp.857-869.

European Commission (2004) EU member states experiences with sustainable development indicators. 95p.

Gerlagh R. and Keyser M.A (2003) Efficiency of conservationist measures: an optimistic viewpoint. *Journal of Environmental Economics and Management*, 46, pp. 310-333.

Hartwick J. (1977) Intergenerational Equity and the Investing of Rents from Exhaustible Resources. *American Economic Review*, 67, pp. 972-974.

Heal G. (1998) *Valuing the Future, Economic Theory and Sustainability.* Columbia University Press, New York.

Hediger W. (2000) Sustainable development and social welfare. *Ecological Economics*, 32, pp.481-492.

Hotelling H. (1931) The Economics of Exhaustible Resources. *Journal of Political Economy*, 39, pp. 137-175.

Howarth R. (2007) Towards an operational sustainability criterion, *Ecological Economics*, 63, pp.656-663.

Krautkraemer J. (1985) Optimal Growth, Resource Amenities and the Preservation of Natural Environments. *Review of Economic Studies*, 52, pp.153-170.

Krautkraemer J. (1998) Nonrenewable resource scarcity, *Journal of Economic Literature*, XXXVI, pp. 2065-2107.

LÉONARD D. AND LONG N.V. (1992) *Optimal control theory and static optimization in economics*, Cambridge University Press.

ALVAREZ-CUADRADO F. AND LONG N. (2007) A mixed Bentham-Rawls Criterion for Intergenerational Equity: Theory and Implications. *working paper.* McGill University.

MARTINET V. AND DOYEN L. (2007) Sustainability of an economy with an exhaustible resource: a viable control approach, *Resource and Energy Economics*, 29, pp.17-39.

NOWAK M. (2004) Preference and veto thresholds in multicriteria analysis based on stochastic dominance, *European Journal of Operational Research*, 158, pp.339-350.

PEZZEY J. AND TOMAN M. (2002) Progress and Problems in Economics of Sustainability. In *International Yearbook of Environmental and Resource Economics 2002-3*, p. 165-232, T. Tietenberg and H. Folmer (eds), Edward Algar, UK.

RAWLS J. (1971) *A theory of Justice.* Oxford, England: Clarendon.

Riesgo L. and Gómez-Limón J. (2006) Multi-criteria policy scenario analysis for public regulation of irrigated agriculture. *Agricultural Systems*, 91, pp.1-28.

SAKAI T. (2006) Equitable intergenerational preferences on restricted domains, *Social Choice and Welfare*, 27, pp.41-54.

SOLOW R.M.(1974) Intergenerational Equity and Exhaustible Resources. *Review of Economic Studies. Symposium on the Economics of Exhaustible Resources*, 41, pp. 29-45.

SOLOW, R. (1993) An Almost Practical Step Towards Sustainability, *Resources Policy*, 19, pp.162-172.

United Nations - Commission on Sustainable Development (2001) Indicators of Sustainable Development: Guidelines and Methodologies, 315p.

U.S. Interagency Working Group on Sustainable Development Indicators (2001) Washington, D.C.

WCED (1987). “Our common Future”, World Commission on Environment and Development. *Oxford University Press.*