

A Column Generation Approach for the post Enrolment Course Timetabling Problem of the ITC

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Input

- set E of events.
- set T of timeslots (5 days of 9 hours each).
- set R of rooms with $\forall r \in R$:
 - C_r = seating capacity of room r .
 - F_r = set of features satisfied by room r .
- set S of students with $\forall s \in S$:
 - set E_s of events that student s is attending.

Input

$\forall e \in E$:

- F_e = set of features required by event e .
- N_e = number of students attending event e .
- T_e = set of available timeslots for event e .
- R_e = set of allowed rooms for event e .
 - $F_e \subseteq F_r$
 - $N_e \leq C_r$

Precedence requirements:

$\forall e, f \in E : p_{ef} = 1$ if event e has to be scheduled before event f , zero otherwise.

Hard Constraints:

1. No student can attend more than one event at the same time.
2. An event e can only be assigned to a room $r \in R_e$.
3. Only one event is assigned to each room in any timeslot.
4. An event e can only be assigned to a time slot $t \in T_e$.
5. Events have to be scheduled in the prescribed order in the week.

Soft Constraints:

1. Events/students should not be assigned in the last timeslot of a day.
2. Students should not have to attend three or more events in successive timeslots on the same day.
3. Students should not be required to attend only one event a day.

How to compare solutions?

Valid timetable → no hard constraint violations,
unplaced events allowed.

Feasible timetable → no hard constraint violations and
all events in timetable

The quality of solutions is evaluated with two measures:

1. Distance to feasibility (dtf)
2. Total number of violated soft constraints.

Collisions

Two events *collide* if they have:

- a student in common,
- only one possible room that is the same,
- a precedence relation between the two events

c_e = the number of events colliding with event e .

Slot-schedule

A *slot-schedule* k has a timeslot t_k and a set E_k of events.

A slot-schedule is feasible if:

- $\forall e, f \in E_k : e, f$ are not colliding.
- $\forall e \in E_k : t_k \in T_e$.
- $\forall e \in E_k : \text{event } e \text{ is assigned to a room } r \in R_e$.
- At most one event is assigned to each room.

$$a_{ke} := \begin{cases} 1 & \text{if } e \in E_k \\ 0 & \text{otherwise} \end{cases}$$

Room Assignment Generator (RAG)

Input: a set E^p of events and corresponding weights $w_e \in E^p$.

Goal: Determine feasible room assignments with the sum of the weights of the assigned events maximized.

Constraints: Events are assigned to allowed rooms.
Only one event is assigned to each room
No two events are colliding.

Output: a set of feasible room assignments for events in E^p .

Heuristic for RAG

- Sort events in:
1. Decreasing order of w_e .
 2. Increasing order of c_e .
 3. Increasing order of $|R_e|$.

To generate the $p - th$ room assignment:

1. Select event e on position p of the sorted list of events.
2. If $\exists r \in R_e$ that has no event assigned, then assign e to room r and go to 5.
3. Try to find an augmenting path.
4. If augmenting path found, then assign all events to the rooms found in the matching. Otherwise event e can not be assigned.
5. If there are rooms and events left, $p := p + 1$, go back to 1.

Master Problem

$$\min \sum_{e \in E} y_e + \sum_{e, f \in E | p_{ef}=1} z_{ef}$$

$$y_e + \sum_{k \in K} a_{ke} x_k \geq 1 \quad \forall e \in E \quad (1)$$

$$\sum_{k \in K | t_k=t} x_k \leq 1 \quad \forall t \in T \quad (2)$$

$$z_{ef} + \sum_{k \in K} t_k (a_{kf} - a_{ke}) x_k \geq 1 \quad \forall e, f \in E | p_{ef} = 1 \quad (3)$$

$$x_k \geq 0 \quad \forall k \in K \quad (4)$$

$$y_e \geq 0 \quad \forall e \in E \quad (5)$$

$$z_{ef} \geq 0 \quad \forall e, f \in E | p_{ef} = 1 \quad (6)$$

The Pricing Problem

Weighting factor $w_e, \forall e \in E^p$ is equal to:

$$w_e := \begin{cases} (\alpha_e - \gamma_{ef}) & \exists f \in E | p_{ef} = 1 \\ (\alpha_e + \gamma_{fe}) & \exists f \in E | p_{fe} = 1 \\ \alpha_e & \nexists f \in E | p_{ef} = 1 \end{cases}$$

Then the value of the generated column ($= c^k$) is:

$$c^k = \sum_{e \in E^p} w_e y'_e + \beta_t$$

The Column Generation Procedure

1. Initialize period p and the set of columns and set $t = 0$.
2. Solve RMP $\rightarrow, \alpha_e, \beta_t, \gamma_{ef}$ (shadowprices) and obj^{rmp} .
If $obj^{rmp} \leq 0$, then quit.
3. Generate columns for timeslots $t, \dots, t + p - 1$.
4. Add k if c^k is larger than 0.85 times the average reduced costs over the last 40 added slot-schedules.
5. $t = t + p(\text{mod}|T|)$
6. If no new slot-schedules found for a number of periods, then quit.
7. Go to step 2.

Heuristic based on LP-solution

1. Initialize $T_c = T \setminus \{8, 17, 26, 35, 44\}$ and $E_c = E$.
2. Apply column generation procedure $\rightarrow K$.
3. Solve MP as IP, break after five seconds if no optimal solution found.
4. $\forall k \in K, obj_k = \sum_{e \in E} a_{ke} c_e$ - penalties.
5. Fix slot-schedule k' with maximum obj_k .
6. $T_c = T_c \setminus t_{k'}$, $E_c = E_c \setminus E_{k'}$ and delete all columns that are infeasible.
7. If $|T_c| > 0$ and $|E_c| > 0$, then go to step 2.
8. If $|E_c| > 0$, then solve an IP to assign as much as possible of the events in E_c to $t \in \{8, 17, 26, 35, 44\}$.

Computational Results

I	E	R	F	S	c.t.(s)	dtf	3 e in row	1 e a day	eod e	s.c.
1	400	10	10	500	316	0	1882	34	508	2424
2	400	10	10	500	324	0	1755	38	529	2322
3	200	20	10	1000	55	0	850	776	0	1626
4	200	20	10	1000	57	0	884	700	0	1584
5	400	20	20	300	209	0	1026	24	213	1263
6	400	20	20	300	218	0	1111	26	232	1369
7	200	20	20	500	29	0	387	321	0	708
8	200	20	20	500	44	0	428	330	0	758
9	400	10	20	500	328	0	1928	33	735	2696
10	400	10	20	500	331	0	1621	38	730	2389
11	200	10	10	1000	43	0	939	713	0	1652
12	200	10	10	1000	64	0	960	552	306	1818
13	400	20	10	300	200	0	1182	31	223	1436
14	400	20	10	300	215	0	1013	13	249	1275
15	200	10	20	500	73	0	497	338	108	943
16	200	10	20	500	40	0	553	340	0	893

Comparison with the best results of the finalists

I	dI	s.c.I	d2	s.c.2	d3	s.c.3	d4	s.c.4	d5	s.c. 5	d us	s.c. us
1	o	61	o	571	o	1482	o	1482	o	1861	o	2424
2	o	547	o	993	o	1635	o	1755	39	2174	o	2322
3	o	382	o	164	o	288	o	850	o	272	o	1626
4	o	529	o	310	o	385	o	884	o	425	o	1584
5	o	5	o	5	o	559	o	1026	o	8	o	1263
6	o	o	o	o	o	851	o	1111	o	28	o	1369
7	o	o	o	6	o	10	o	387	o	13	o	708
8	o	o	o	o	o	o	o	428	o	6	o	758
9	o	o	o	1560	o	1947	o	1928	162	2733	o	2696
10	o	o	o	2163	o	1741	o	1621	161	2697	o	2389
11	o	548	o	178	o	240	o	939	o	263	o	1652
12	o	869	o	146	o	475	o	960	o	804	o	1818
13	o	o	o	o	o	675	o	1182	o	285	o	1436
14	o	o	o	1	o	864	o	1013	o	110	o	1275
15	o	379	o	o	o	o	o	497	o	5	o	943
16	o	191	o	2	o	1	o	553	o	132	o	893

Conclusions

- The heuristic finds a feasible timetable for all instances.
- The number of violated soft constraints is large in comparison with the 5 finalists.

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Wake UP!