Optimal Placement of Tsunami Warning Buoys using Mesh Adaptive Direct Searches

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Avant-propos

My main research interest is nonsmooth optimization:

$$(NLP) \qquad \begin{array}{ll} \text{minimize} & f(x) \\ \text{subject to} & x \in \Omega, \end{array}$$

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- accurate approximation of derivatives is problematic
- the constraints defining Ω may be nonlinear, nonconvex, nonsmooth and may simply return '`yes/no`'.

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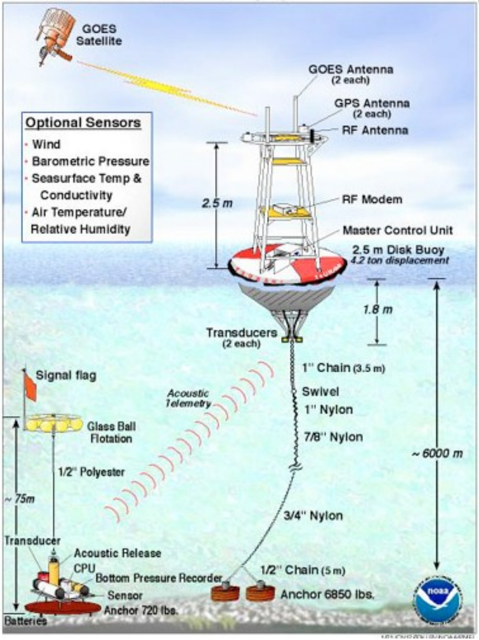
DART mooring system

- *Deep ocean Assessment and Reporting of Tsunamis* (DART) buoys are sensors on the ocean floor with a communication connection to a surface buoy. The tsunami amplitude they detect feeds prediction.
- DART buoys cost about 250,000$US + the cost of deployment and maintenance.



Tsunami reporting responsibility within NOAA (National Oceanic and Athmospheric Administration)

This is my personal understanding of the NOAA structure: there are surely subtleties I am missing, but for the purposes of this talk

- PMEL (Pacific Marine Environmental Lab) developed the buoys and recommends where they are deployed.
- NDBC (National Data Buoy Center) manufactures, deploys, and maintains the buoys
- PMEL monitors the buoy data and provides forecasts to the National Weather Service (NWS).
- NWS issues warnings and alerts to the public.

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A budget for 35-40 buoys was given to PMEL. They quickly realized that positioning them in the vast Pacific involved optimization, and contacted members of the optimization community.

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  - How does the underwater landscape affect the detection amplitude of the DART buoy ?
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John Dennis spent two months at the PMEL headquarters learning about the problem, and teaching them notions of optimization.

PMEL's perspective

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- Optimization modeling requires specifying appropriate decision variables, objective function, and constraints so that the formalism models the real-world problem adequately and provides a solvable problem.
- Modeling is inherently interdisciplinary, and it is not easy.

Optimization format for NOMAD

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where $\Omega \equiv \{x \in X : C(x) \leq 0\} \subset \mathbb{R}^n$.
The constraints are partitioned into two groups.

- X contains the *closed* constraints.
- $C(x) \leq 0$ are called the *open* constraints.

Open, closed and hidden constraints

Consider the toy problem:

$$\begin{array}{ll} \min\limits_{x \in \mathbb{R}^2} & x_1^2 - \sqrt{x_2} \\ \text{s.t.} & -x_1^2 + x_2^2 \leq 1 \\ & x_2 \geq 0 \end{array}$$

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  DART placement has nasty closed and hidden constraints.

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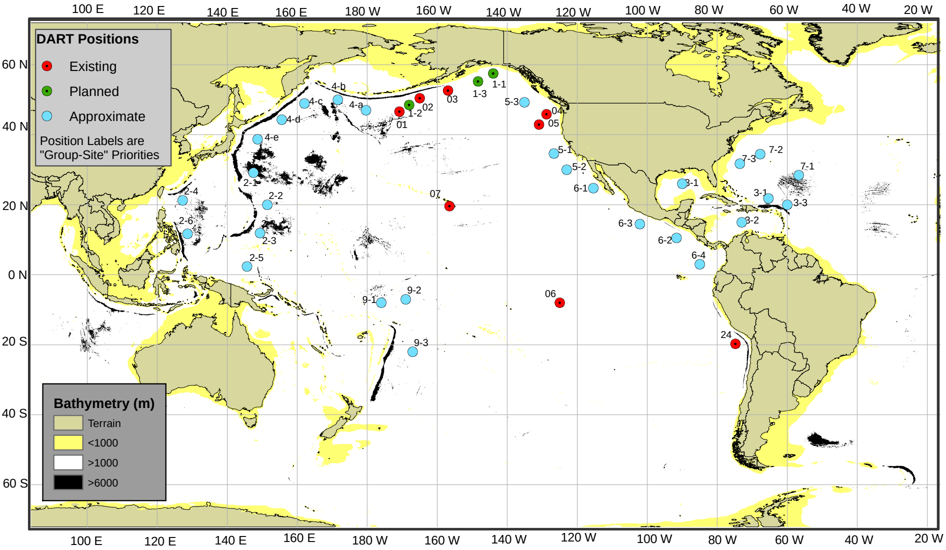
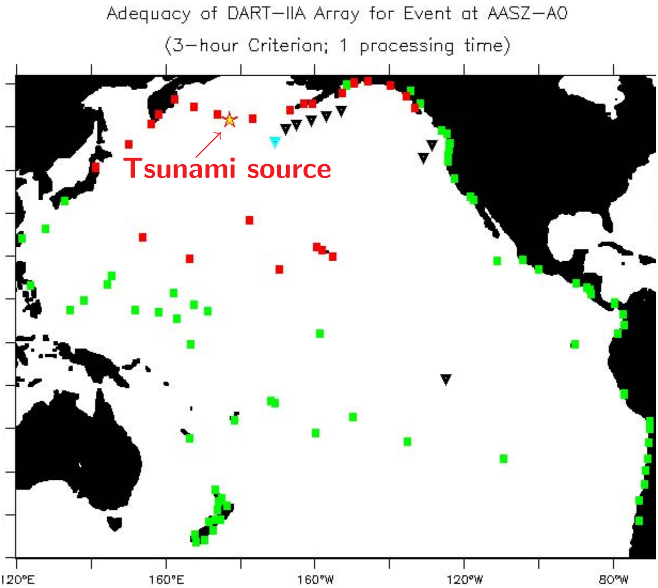
- The PMEL scientists possess a lot of data on tsunamis but it is not organized in the form of an optimization problem.
- The followings slides represent examples of the raw data.

Preliminary placement by a panel of experts



PMEL scientists can forecast arrival time given the source



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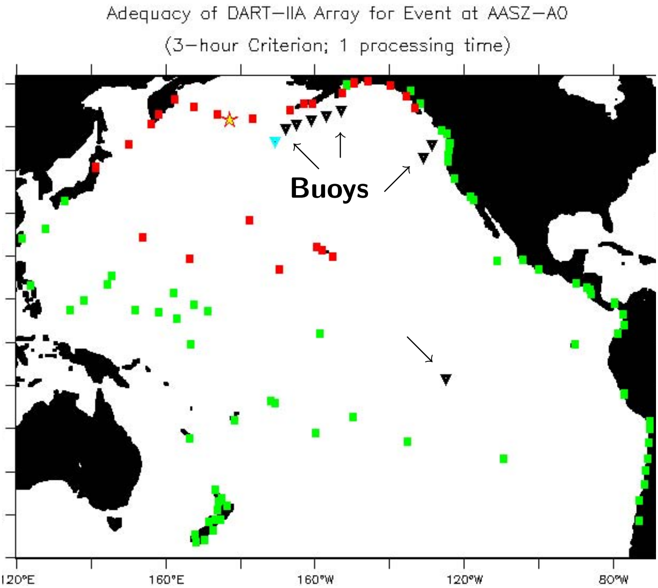

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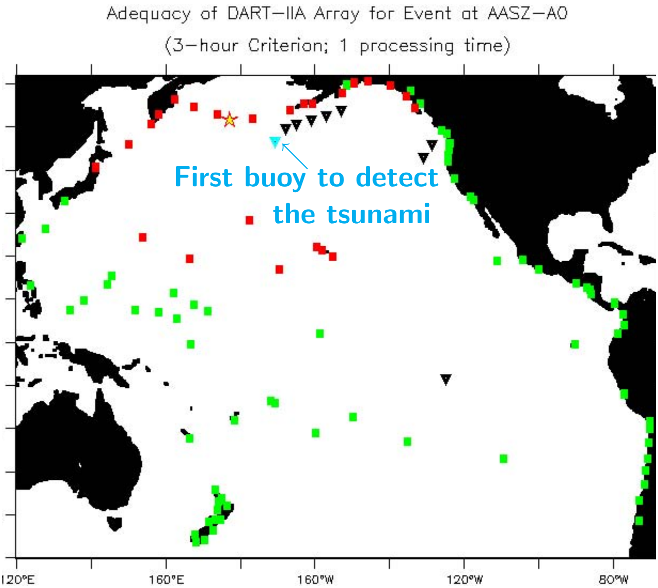

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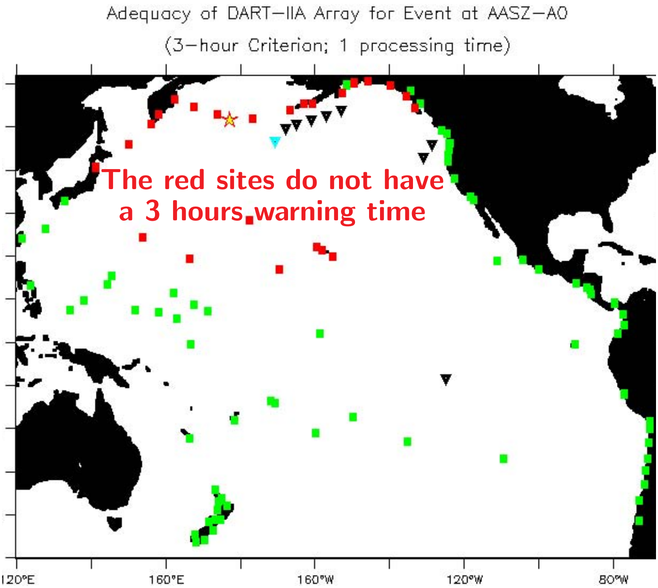

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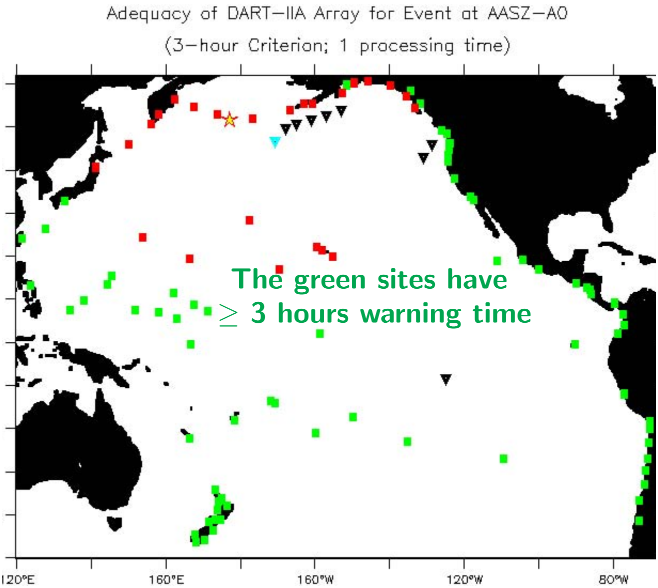
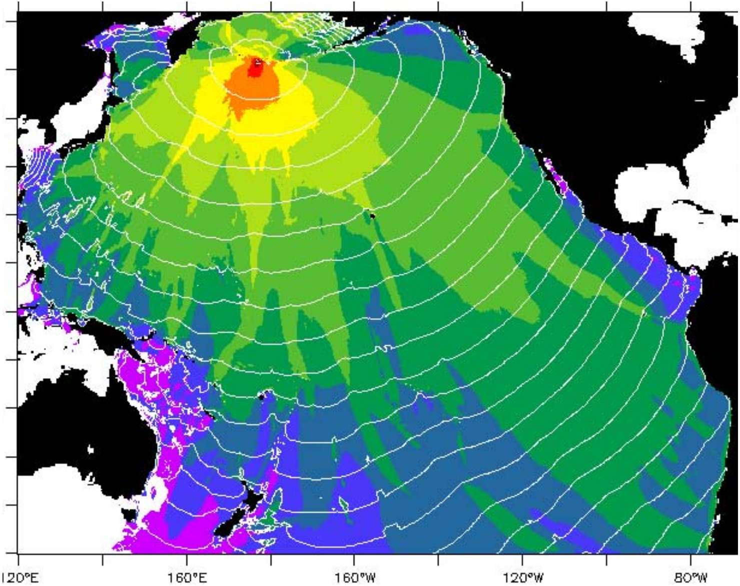


PMEL scientists can predict intensity given the source



Level sets of the intensity of a tsunami wave and of travel time.

A non-smooth problem

Source – http://nctr.pmel.noaa.gov/Mov/andr1.mov

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- The preliminary placement can obviously serve as a starting point for our method.
- Travel time of the wave can be turned into a function
- Intensity of the wave can be turned into a function
- Warning time can be turned into a function
- ...

Building blocks (computer codes that return various function values) can be elaborated.

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Given some buoys positions, PMEL produced software that measures these quantities. The CPU time for these computations is of the order of 30 seconds.

Presentation Outline

First problem formulation

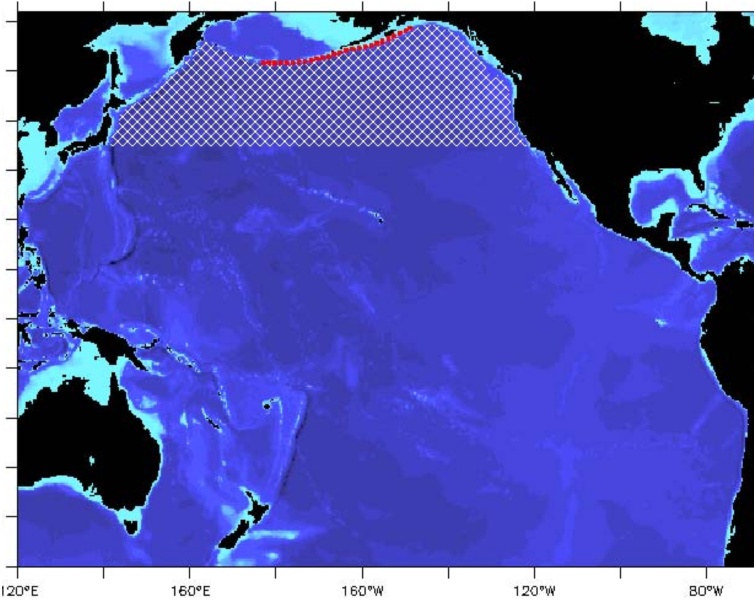
First we tried to optimize:

$\min$ (time to detection)

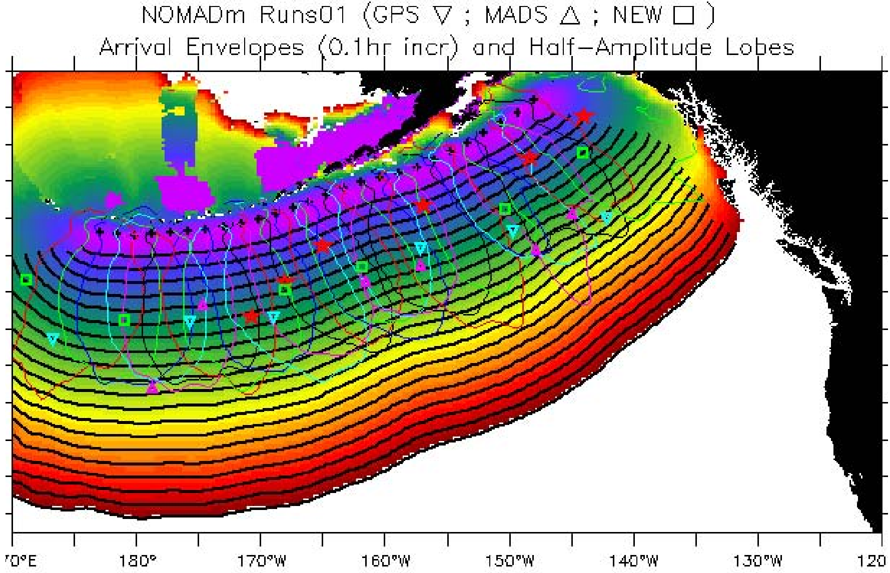
subject to buoy placements that satisfy:

- the closed constraints:
    - bottom conditions
    - not too close
- the open constraints:
    - sufficient detection amplitude
    - sensor coverage

First test problem - the domain



First test problem - NOMADm results



What did we learn from the first test problem ?

- Two NOMAD variants converged in an hour to reasonable solutions for this test problem
- The detection time was adequate (the objective function)
- Unfortunately, the solutions did not satisfy every constraints of the initial model. To satisfy the open sensor coverage constraint, we had to loosen the required tsunami detection amplitude constraints to lower levels

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The conclusion of this first model is that we do not have enough buoys to achieve the specified tsunami detection amplitude and sensor coverage constraints for 7.5 earthquakes.
The objective was satisfactory. So tried a second test problem:

Second problem formulation

To nail down how much we miss the data quality requirement we solved:

$\max$ (tsunami detection amplitude) $\Leftarrow$ was a $\geq$ constraint
subject to buoy placements that satisfy:

- the closed constraints:
  - bottom conditions
  - not too close
- the open constraints:
  - adequate time to detection $\Leftarrow$ was the objective
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- Again MADS converged in an hour.
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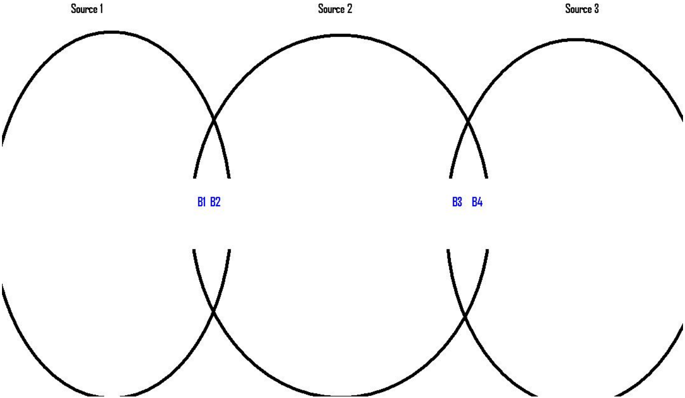

- The extra buoys wandered off in the feasible region, clearly

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In summary, NOMAD is used as a tool by the decision makers.
The solutions provided by NOMAD allow the user to refine the model, and his interpretation of objectives and constraints.

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The Mesh Adaptive Direct Search algorithm

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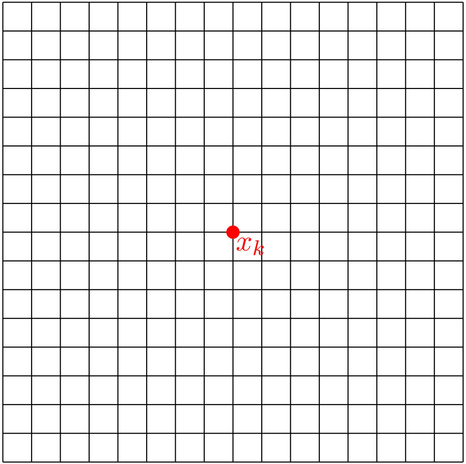
The optimality conditions that MADS guarantees on $\hat{x}$ are 'proportional' to the smoothness of f and Ω.

A MADS iteration



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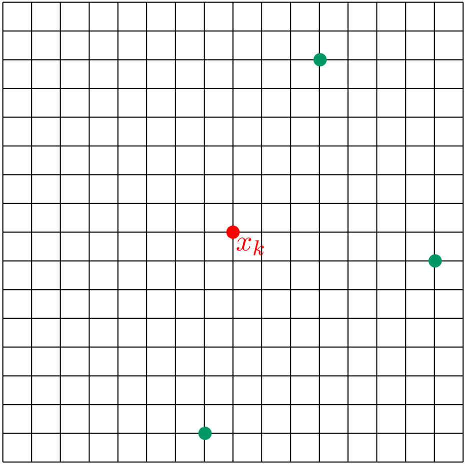

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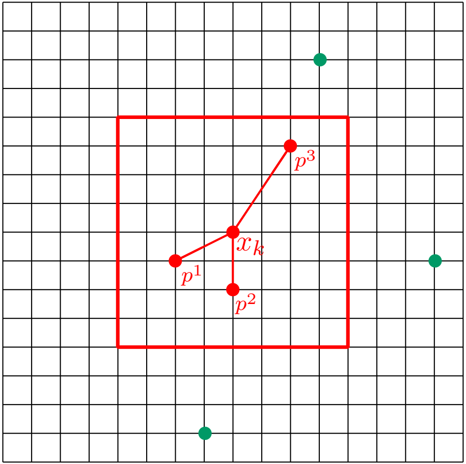
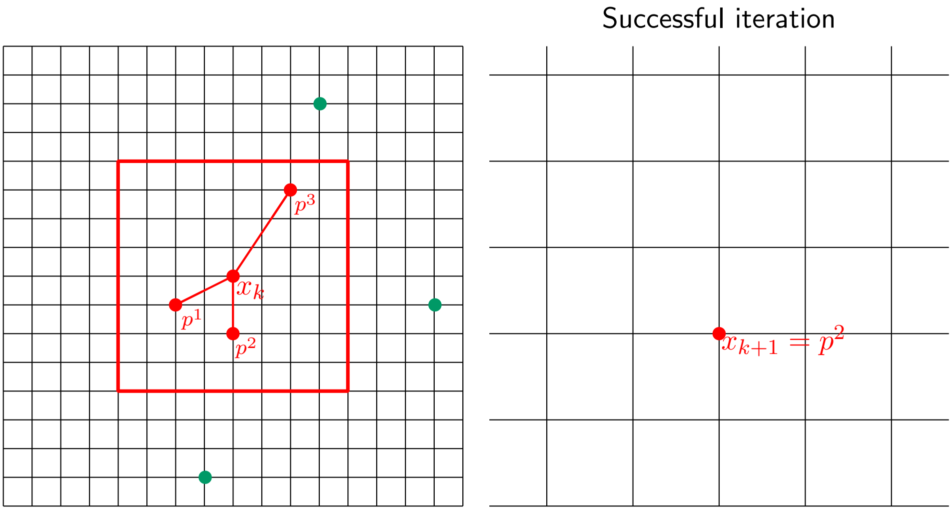
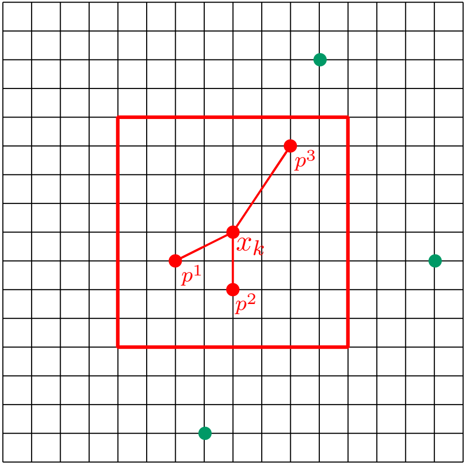



Successful iteration
p^3
x_k
p^1
p^2
x_{k+1} = p^2

A MADS iteration



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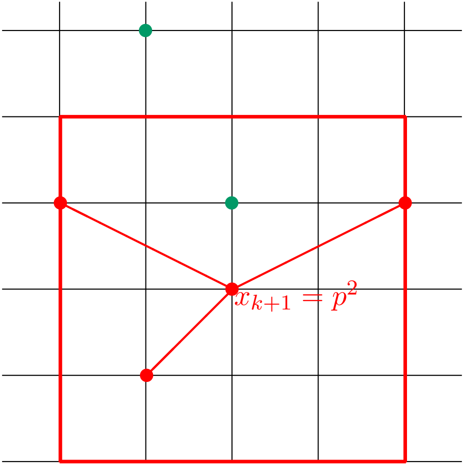

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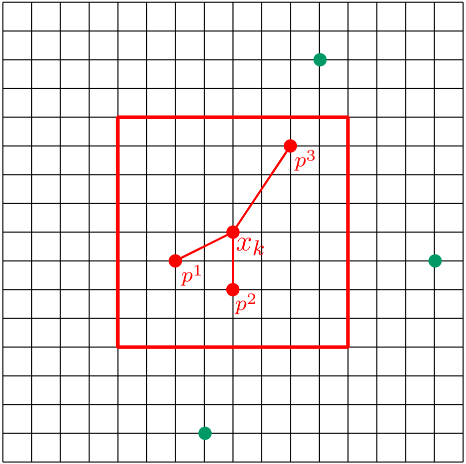
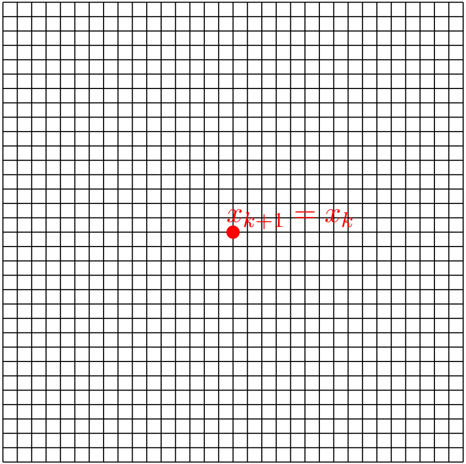


unsuccessful iteration



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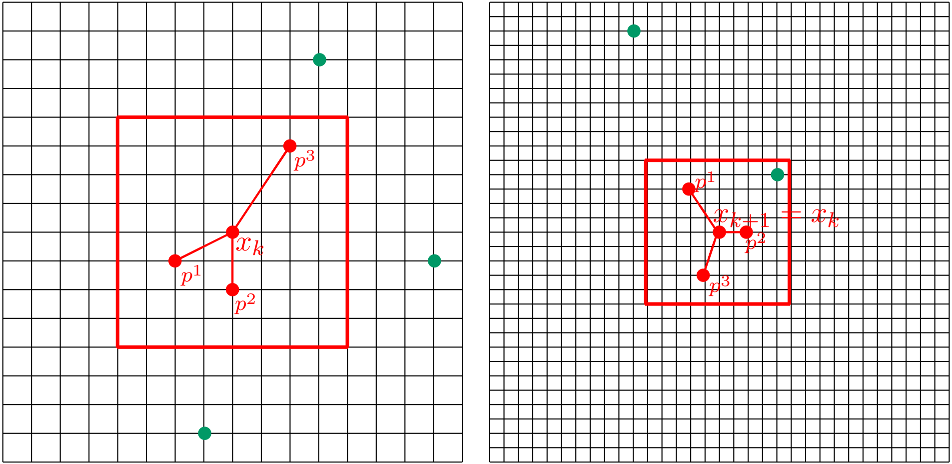

Barrier approach to closed constraints

To enforce X constraints, replace f by a barrier objective

$$f_X(x) := \begin{cases} f(x) & \text{if } x \in X, \\ +\infty & \text{otherwise.} \end{cases}$$

Then apply the unconstrained algorithm to f_X.

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Then apply the unconstrained algorithm to f_X.
Remarks :

- The quality of the limit solution depends the local smoothness of f, not of f_X.
- This approach can handle strict inequalities.
- Expensive evaluations of f are saved when x is found to be infeasible.

Filter approach to open constraints

Define the nonnegative constraint violation function

$$h(x) \ := \ \sum_j \max(0, c_j(x))^2$$

Remarks :

- $h(x) = 0$ if and only if all open constraints are satisfied.
- Accept a new trial points if it is feasible and improves f or if it is infeasible but improves h.

Presentation Outline

Hierarchy of MADS convergence results

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- Furthermore, if f is twice strictly differentiable at $\hat{x}$ and $\nabla^2 f(\hat{x})$ is non-singular, and if Ω locally convex near $\hat{x}$, then with probability 1, $\hat{x}$ is local minimizer of f over Ω:
$$\exists \epsilon > 0 \text{ such that } f(\hat{x}) \leq f(y), \forall y \in \Omega \cap B_\epsilon(\hat{x}).$$

Presentation Outline

1. Tsunamy warning buoys

2. Buoy placement optimization
   - Initiating the collaboration
   - The building blocks of an optimization model
   - Playing with model formulations

3. A direct search algorithm
   - The Mesh Adaptive Direct Search algorithm
   - Summary of convergence analysis

4. Conclusions and plans

Conclusions

- NOMADm solved several tweaks of the first two test problem easily and quickly.
- Collaboration is an iterative process.

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0-     Learn each other's language
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6-     UNTIL a satisfactory solution is found.
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- Step 0 is hard. But once it is done, things progress rapidly.
- Collaboration between both groups is essential in steps 0,1,4 and 5.
  PMEL is the judge for step 6.
  Our optimization team handles step 3 using NOMAD.

Plans

- Continue to work with NOAA/PMEL tsunami experts to refine the formulation to get answers they like.
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Thank you for your attention.